Mathematics and Non-School Gameplay

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Abstract This chapter investigates the mathematics in the gameplay of three popular games (Angry Birds, Plants vs. Zombies and The Sims) that are unlikely to be played in mathematics lessons. The three games are different but each has been observed to provide opportunity for mathematical activity in gameplay. After describing each game, and the mathematics that can arise in gameplay, the chapter explores two questions: What kind of mathematics is afforded in these games? Can these games be used in/for school mathematics? Issues considered under the first question include: the nature of mathematics and the difficulty of isolating the mathematics in non-school gameplay; players' strategic actions as mathematical actions; and 'truth' and its warrants in different mathematical worlds. Issues considered under the second question include: tensions between curricular expectations and the mathematics that arise in gameplay; and possible changes in gameplay when a game is moved from a leisure to an educational setting.

Keywords mathematics/Mathematics · Non-school gameplay · Strategies · Abstraction-in-context · Theory of didactical situation · Three worlds of mathematics

Introduction

Gameplay can be used to present and structure mathematical activities in classrooms: Nim, for example, has been used extensively in French primary mathematics lessons (see Brousseau 1997); in England teachers have used the Shell Centre

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(1987–1989) *Design a Board Game* resource box in their lessons; in North America, the National Council of Teachers of Mathematics [NCTM] (2004) claim that mathematical games "can foster mathematical communication…can motivate students and engage them in thinking about and applying concepts and skills" <http://www.nctm.org/fractiontrack/>. Research, however, reminds us that learning mathematics through gameplay is not automatic: "games can be used to teach a variety of content in a variety of instructional settings…there is no guarantee that every game will be effective" (Bright et al. 1985, p. 133); "it appears that assumptions that students will see the usefulness of mathematics games in classrooms are problematic" (Bragg 2006, p. 233). However, these examples focus on mathematical games used in classroom settings which leaves a question about games that are not deemed appropriate for classrooms.

By a *non-school game* we mean a game that is unlikely to be offered for students to play in a mathematics lesson. It has been argued that non-school games can have beneficial impact on players' problems solving skills (Chuang and Chen 2009) and spatial ability (Dye et al. 2009); and Gee and Hayes (2010) claim that some games require a considerable knowledge of geometry. The adoption of non-school games in a classroom largely depends on the classroom teacher (Bakar et al. 2006). When a digital game is used in a mathematics lesson, it is likely that the game meets a teacher's interpretation of a curriculum objective (NCTM 2004). When a student chooses to play a new non-school game, they are highly unlikely to play this for reasons that a teacher might have in introducing the game in a lesson, such as curriculum content. Studies have shown that the content of non-school games is often irrelevant or not aligned with that of school curricula (Egenfeldt-Nielsen 2005). Further to this, students do not necessarily appreciate it when non-school games are used for education rather than fun (Bourgonjon et al. 2010). The issue of mathematics and non-school gameplay is, thus, far from straightforward. We restrict our attention, unless otherwise stated, to digital games, and all references below to *game* or *gameplay* may be assumed to concern digital games.

This chapter investigates the question: What mathematics is there in non-school gameplay[1]? How one understands and addresses such a question depends, amongst other factors, on one's theoretical framework. Our framework is sociocultural in as much as we view mathematics as a cultural practice and doing mathematics as an artefact, person and sign mediated, object-oriented activity. From this position, our understanding of the question is that mathematics resides in mathematical activity and the answer to the question depends on the game, the player and the context of the gameplay.

To address the question, we focus on three popular (circa 2013) games: Angry Birds, Plants vs. Zombies and games in The Sims series. The next section presents these games and discusses mathematics that can arise in gameplay. This is followed by a discussion of two further questions arising from our considerations of the three games: What kind of mathematics is afforded in gameplay? Can these games be used in/for school mathematics?

[1] Note that we use the word *gameplay* and not *games* in this question. This reflects an ontological assumption that mathematics, if it exists at all, does not reside in the game itself but in the gameplay.

Three Games

We focus closely on three games, rather than surveying a large number, because of a conviction that the detail of gameplay is important in a consideration of mathematics in gameplay. We chose the three games below because: they are clearly non-school games; they have each given rise to observed gameplay which can, in a sense to be discussed in this chapter, be viewed as mathematical activity; there are differences in the nature of the mathematical activity in these three games; and they are popular games. For each game, we first describe the game and then raise issues concerned with mathematics.

Angry Birds

Angry Birds is a *casual game* developed by Rovio Entertainment which was first issued for the Apple iPhone and is now available for a range of iOS and Android devices, including high-definition versions for tablet devices such as the Apple iPad. An underpinning principle of casual games is that they can be played in very small blocks of time such as a 10-min bus journey (although some players may devote more time to the game). Typically, each level takes a short time to complete. Angry Birds begins with the narrative premise that the pigs stole eggs from the birds. The birds are consequently angry and take revenge on the pigs by firing themselves from catapults to destroy the pigs and their shelters. The task of the player is to aim the catapult to fire the birds at the pigs. As the game progresses, the shelters in which the pigs take refuge become increasingly complex and incorporate a wider variety of materials which present different constraints (for example, stone is more difficult to destroy than wood). In addition the structures often require a chain of actions so that the bird cannot be fired directly at the target but needs to hit, for example, a boulder which will strike a pedestal at the bottom of a structure and knock away support for higher levels. The birds also change as the game progresses with new attributes triggered by swiping the screen during the flight. A small blue bird, for example, splits into three smaller birds each flying at a different height whereas a white bird drops an egg when the screen is swiped. The player cannot choose which bird to deploy but is presented with a fixed number, type and sequence for each level. In order to achieve successful destruction of a pig, the player has to think about the nature of the structure and which part of the structure to target. The player then has to consider the flight path curve that the bird needs to take and so the angle at which the catapult must be pulled back in order to achieve the required trajectory. In addition, the further the catapult is pulled, the further the bird will travel, although speed is constant (whereas in real life, the further the catapult is drawn back, the greater the speed of the projectile/bird). The game draws the flight path for the current bird as it travels and the player can use this as a guide when launching the next bird.

In the following two subsections we recount two instances of individuals playing Angry Birds. The first arose from a chance encounter with a young person playing it. The second was an attempt to replicate the first encounter with a very different person, a mature mathematician. In both cases the (same) observer simply made notes on the gameplay.

Emily Plays Angry Birds

Emily is 4 years old. Her older sister has an iPod Touch so Emily is familiar with touchscreen games, although she is not often allowed (by her sister) to play them. She is familiar with the Angry Birds concept but has not previously played the game. She is excited to be playing games on an iPad. It is briefly explained to Emily that she needs to fire birds from the catapult to hit the pigs but she is given no direction about how best to achieve pig destruction. Emily fires a bird but the flight path is too low so the bird hits the ground before it reaches the pigs' shelter. When asked what happened, Emily says "I needed to go upper". The second shot is successful. Emily chooses higher levels to play and these require a strategic approach. Emily plans her attack by tracing the prospective flight path of the first bird. It might be expected that a 4-year-old would aim for the *easy* birds but Emily does not do this. Instead she aims the bird high, so that it will knock down the coping stones (Fig. 1) which fall behind the structure thus destroying two pigs. The bird falls forward and catches one pig.

It might be assumed that this was simply a lucky shot had Emily not carefully traced the arc before aiming the catapult. It should be noted that to an observer it seemed as though the shot would simply bounce off the structure and be wasted. However, Emily's reaction made clear that she had achieved the intended result. In order to plan the shot, Emily needed to consider how the blocks were arranged, the shapes of the blocks, the direction in which blocks would fall, the optimum point at which the bird should hit the structure and, finally, the flight path and the angle/distance at which the catapult should be released.

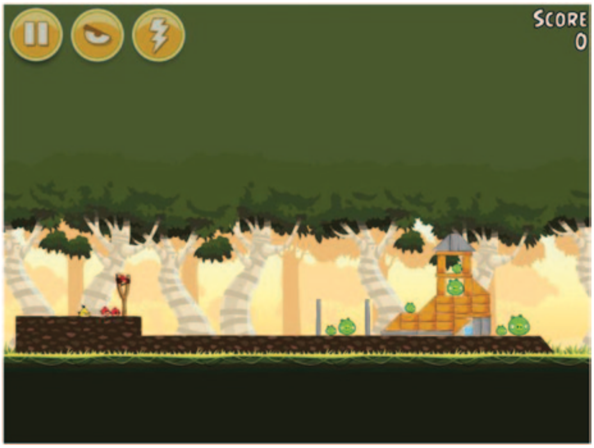

Fig. 1 Emily aims for the top

We believe that Emily's strategic thinking is mathematical (and we provide an argument that this is so in the Discussion section below); we also feel that Emily's strategic thinking is pretty impressive for a 4-year-old. Emily navigates this *mathematics* effortlessly but without analysis, which may be expected in a classroom. Her intention was to perform the necessary moves to destroy the pigs and she did this easily. However, she was not able to explain what she had done; she could give only simple description. Her lack of explicit knowledge of the mathematics is made clear by her inability to put in words the decisions that she has made.

Rich Plays Angry Birds

Rich is an adult, an academic in the field of mathematics education. Although a confident user of digital tools, Rich is not a player of electronic games and had not previously encountered Angry Birds. Rich takes aim and fires the first bird at the structure but the bird falls short. The same happens with the second bird. The third (of three) overshoots. Rich becomes frustrated with the game and gives up, saying, "As a mathematician and a scientist, this makes no sense to me". The problem for Rich is that although the game is mathematically accurate in some respects, for example, in terms of angles and curves, it does not completely replicate real-world physics. In real life, the further the catapult is drawn back the greater the speed of the projection of the bird. In Angry Birds, pulling the catapult back further increases the distance that the bird will travel but does not increase either the speed of projection or the force with which the bird strikes the structure. Rich is correct; in this respect the game makes no sense. Unlike Emily, he is able to explain the mathematics (and physics) of the game. However, Emily is able to use the mathematics within the game whereas Rich cannot.

The 'Magic Circle' and Mathematics

As with many games, the gameplay of Angry Birds takes place within a closed environment. Moore (2011) calls this the *magic circle* and relates it to the spaces in which traditional games are played, for example chessboards or card tables. Within the *magic circle* the rules of everyday life are suspended and replaced by the rules of the game. With traditional games the boundaries of the *magic circle* are clear and the rules are explicit; all players know how and when behaviours within the *magic circle* diverge from everyday life. Moore argues that the ubiquitous nature of digital gaming, especially on mobile devices, blurs the distinction between the *magic circle* and everyday life because the games do not have to be played in special places but are available everywhere. However the boundaries are blurred in other, perhaps more important ways. With traditional games it is obvious that the games operate in specialised contexts. For example, a game board clearly delineates the space in which the game is played and it is obvious to the players that the board is not real life. With Angry Birds there are aspects of the game which are clearly artificial such as the cartoon characters. There is no attempt to replicate reality with the birds and pigs, indeed, there seems to be a clear attempt to make sure that nobody could confuse them with real creatures as that could be distressing. The birds and pigs are clearly *magic circle* characters. However, the materials used in the structure are designed to look similar to real-world wood and stone and, to a certain extent, share the characteristics of their real-world counterparts. Wood is much easier to break than stone. The parabolas of the birds also appear to be real-world rather than *magic circle*.

The mathematics of Angry Birds is real and is explained clearly by Chartier (2012) and by teaching websites such as InThinking Teach Maths (2013). For example, InThinking Teach Maths provides resources for working with quadratic equations based on Angry Birds. Clearly, Rich is capable of understanding these equations where Emily is not. Yet Emily can play the game whereas Rich is puzzled by the mechanics. Because Emily does not yet have any real-world understanding of the mathematics employed in Angry Birds, she is able to enter the *magic circle* of Angry Birds completely and therefore can make the practical calculations that she needs to play the game successfully. In future years, when she reaches the curriculum stage that addresses the mathematics employed in Angry Birds she may be able to relate the skills she has developed inside the 'magic circle' to the abstract concepts of real-world mathematics.

Plants vs. Zombies

Plants vs. Zombies (PvZ) is another casual game: a *tower defence* real-time strategy game where you, the player, plant plants in your garden to repel zombies from entering your house (where they promptly eat your brains and you lose). There are a variety of plants and zombies with different defensive and attack attributes. The basic game has five levels: front garden by day/night; back garden by day/night; and roof. Each level has ten *adventures* (zombie attacks). Collecting suns allows plants to be planted. Successful planting strategies vary with the adventure as the zombies vary. In addition to the basic game, there are a variety of *puzzles*. We present the *last stand—roof* puzzle. Last stand puzzles have *onslaughts* (each with several *waves* of zombie attacks) and you successfully complete the puzzle when you have withstood five onslaughts.

Figure 2 shows the screen at the beginning of the puzzle (where plants are inserted into flower pots) of *last stand—roof* with the *plants* available to use (and their individual costs, measured in *suns*) displayed on the left and zombies (who will start their attack after the *set up*) in the inset. Going down from the top: plants 2, 3, 4, 5 and 8 are attacking plants (plants 2 and 4; 4 is an upgrade of plant 3, which also slow zombies down); plants 6 and 7 are defensive plants ('tall nuts' and 'umbrella leaves'); plant 1 is actually a plant pot (only needed on roof levels as there is no soil as there is in garden levels). To the right of the plant pot are the available suns (in *last stand* puzzles, most of the suns available during an adventure are available at the outset). The zombies (not present at this stage in this game) come in *waves*,

Fig. 2 The start of *last stand—roof*

Fig. 3 A possible configuration of plants in *last stand—roof*

mainly from the right hand side of the screen; the exceptions to this are *bungee jumping* zombies who can *land* zombies to, or steal plants from, the left hand side of the screen. Once an onslaught has been successfully defended, the player gets an additional 500 suns.

Figure 3 shows a possible configuration of plants and the start of the first wave of zombies. It is not a particularly good configuration but serves initial explanatory purposes at this point in this section. Rows (of 5) of plants 2, 3 and 7 have been planted. The cost of these rows is $5\times100+5\times300+5\times125=2675$ (suns) and

Fig. 4 Missing plants in *last stand—roof*

there are 5000−2675=2375 suns remaining. Note that the player does not need to do this arithmetic her/himself as once a plant is planted (or a wave withstood), the cost is automatically deducted from (or added to) the available suns. But although this automatic update of available suns means that mental or pencil-and-paper calculations are not necessary, the player must do some serious estimates because the initial 5000 suns (with additional suns after withstanding waves of zombies) is not generous—surviving until the end of the puzzle is just possible with careful use of plants/suns.

A problem with the configuration shown in Fig. 3 is apparent if we compare it with Fig. 4, which shows what happens in the Fig. 3 situation after a couple of minutes. There are missing plants. Some of the plants have been stolen by bungee jumping zombies, some have been destroyed by catapult zombies (the ones in little golf carts), and it can be seen that some are being eaten by zombies. These plants can be replaced but they cost suns, and it is not possible to survive for long with this configuration. Survival requires more strategic planning using powerful attacking plants (plants 3 and 4), the occasional chilli pepper (which clears a line of zombies but can only be used once) and, crucially, strategically positioned 'umbrella leaves'.

For reasons of space we skip to an initial configuration (Fig. 5) from which it is possible to survive the final wave of zombies.

We say *initial* because there is more to come but we need to wait until we have more suns from surviving waves of zombies. There are two spatial strategies behind the configuration in Fig. 5. The first is simply that we have positioned the plants in the first three rows, that is, we have kept them as far to the left as possible so that the zombies have to cover a lot of open ground (and they can be picked off in this open ground, at least in the first wave). The second is the use of umbrella plants to

Fig. 5 A possible winning initial configuration of plants in *last stand—roof*

protect the other plants from bungee jumping and catapult zombies; an umbrella plant (marked by U) in Fig. 6, will protect plants in all the other squares in the grid (so all the plants shown in Fig. 5 are safe).

Figure 7 shows an update of Fig. 5 that has a 'tall nut' at the right end of each row. This is needed in the second level since *pogo zombies* (see the top line of Fig. 7) travel fast but are brought to a halt by tall nuts. Notice that the two spatial strategies referred to above are used in this update: the plants are kept as far to the left as possible; an extra umbrella plant has been used to protect the central tall nuts.

Figure 8 shows the configuration moments before the successful end of the puzzle with just three zombies left. Extra tall nuts and umbrella plants have been used and a plant pot, which held a chilli pepper, has been destroyed by the dying large zombie in line 2.

Fig. 6 Positioning umbrella plants

	U	

Fig. 7 An update of Fig. 5 that has a tall nut at the right end of each row

Fig. 8 The configuration moments before the successful end of the puzzle

Comment on the Mathematics

We comment on mathematical content in this puzzle and then consider the puzzle in terms of Brousseau's (1997) Theory of Didactical Situations (TDS).

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