

Preface

The aim of this book is to present a formal theory of action and to show the relations of this theory with logic and other disciplines. The book concerns the semantic, mathematical, and logical aspects of action.

In contemporary logic, reflections on actions and their performers (agents) have assumed a growing and expanding importance. The theme of action, particularly that of effective and rational action, is heavily rooted in the praxeological tradition. From the viewpoint of logic, the problem of action goes beyond traditional branches of logic such as syntactics and semantics. The center of gravity of the issues the problem of action raises is situated on the borderline between logical pragmatics and praxeology.

The book focuses on the following tasks:

A. Description and Formalization of the Language of Action

In the contemporary literature, one can distinguish at least seven approaches to action. Each of them outlines a certain perspective of action theory by bringing out some specific aspects of human actions.

1. *The linguistic framing*, initiated by Maria Nowakowska. Atomic (in other words: elementary) actions (procedures), as well as compound actions are distinguished. Atomic actions are primitive and non-reducible to others. Compound actions are sets of finite sequences of atomic actions. If one identifies the set of atomic actions with an alphabet, in the sense of formal linguistics, each compound action becomes a language over this alphabet, that is, it becomes a set of words. For a description of compound actions one can then apply the methods of mathematical linguistics. Compound actions can be—in particular cases—regular languages, context-free languages, etc. This formulation goes deeply into the theory of algorithms and is appropriate for describing routine, algorithmizable actions, such as the manufacture of cars or the baking of bread.

2. *The dynamic logic approach.* The view that human action is modeled on their resemblance of computer programs can be found in the works of many researchers (Boden, Segerberg, Suppes to mention a few). In this formulation, an action is identified with a binary relation defined on a set of states. This relation, called the resultant relation, assigns to each state a set of possible outcomes of the action, when the action is being performed in this state. Each pair of states belonging to this relation is called a possible performance of the action. The formulation, in a natural way, links action theory with (fragments of) set theory, whose main components are graph theory and the theory of relations.
3. *Stit semantics* gives an account of action from a perspective in which actions are seen *not* as operations performed in an action system and which yield new states of affairs, but rather as selections of pre-existent histories (or trajectories) of the system in time. Stit semantics is therefore time oriented, and time, being a situational component of action, plays a special role in it.
4. A special framing of the subject area of action is offered by *deontology* and *deontic logic*. It is from the deontological perspective that a typology of actions is determined; here, forbidden, permitted, and obligatory actions are distinguished. This formulation binds action theory with jurisprudence and the theory of norms.
5. The fifth perspective of action originates from *Dynamic Epistemic Logic* (DEL), the logic of knowledge change. DEL is concerned with actions which change states of agents' knowledge and beliefs. DEL builds models of the dynamics of inquiry and accompanying flows of information. It provides insight into the properties of individual or group agents (knowers) and analyzes consequences of epistemic or verbal actions. Public announcements may serve as an example [see van Benthem (2011); van Benthem et al. (2013)].
6. A pragmatic approach to action is developed by *decision theory*. From the perspective of this theory, 'decision making is a cognitive process resulting in the selection of a course of actions usually among several alternative scenarios'—see Wikipedia. Decision theory therefore differentiates between problem analysis, which is a part of the cognitive process, and the selection of an appropriate course of actions by the agent(s). The information gathered in problem analysis at each stage of decision making is then used toward making further steps. Decision theory is not concerned with the performability of actions but rather with their costs—some actions turn out to be less or more profitable than others. In other words, decision theory views actions as decisions and assesses the latter in terms of costs or losses.
7. *Game theory* is a study of strategies and decision making. There is no strict division line between game theory and decision theory. It is said that game theory may be viewed as *interactive* decision theory because it builds mathematical models of conflict and cooperation between rational decision-makers.

B. Models of the Action Theory

In view of the difficulty in determining the adequate language of actions, one should not expect a theory to be defined in an axiomatic way. The natural compromise consists in defining some intended models of action. In this book, two classes of models are discussed:

- the class of elementary action systems,
- the class of situational action systems.

The second of the classes includes the first one as a limit case. The models allow for unification of most of the formulations of action theory mentioned in (A). On the ground of the above-mentioned models, one can define compound actions (as it is done in the models considered by Nowakowska); likewise, one can reconstruct models for dynamic logic. In terms of action systems it is possible to determine notions of permitted, obligatory, and forbidden actions that are fundamental to deontology.

The book also outlines the relations obtaining between situational action systems and situational semantics.

C. Performability of Actions

The central problem that action theory poses for itself to solve is to provide an adequate concept of the performability of action. The performability of actions depends on the parameter which is the state of an action system (see point B). What is more, the very notion of performability itself is not of an absolute character but is relativized to a possible manner (aspect) of performing an action: for example, an action can be physically performable (e.g., driving a car along a one-way road in the opposite direction), when one takes into account technical limitations while being legally non-performable—if one takes into account (as in the example given) the limitations arising from the regulations contained in the highway code (actions *in fraudem legis*). (In this example, the non-performability of action in the legal sense means that the action is forbidden.) One of the aims of the book is to present the definition of performability (atomic and compound ones) formulated in terms of elementary and situational action systems.

D. Actions and Their Agents

Actions are performed by single persons (individuals), teams of people (collective groups), robots, and groups, these being combinations of collective bodies, robots, and machines. Performers of actions are referred to with the collective term of *agent*

of action. The literature on the subject is quite extensive and is focused on providing truth-conditions for sentences of the form: *a is the agent of the action A*.

The above problem has been analyzed by Brown, Chellas, Horthy, Kanger, Pacuit, Segerberg, van Benthem, von Wright, and many others. An initial discussion of the issue requires accepting certain ontological assumptions first. Actions (and acts) are correlated with changes in states of affairs. Besides the states, categories of actions and their agents are distinguished. Moreover, in this book, there is introduced the notion of the situational envelope of an action that takes into account such parameters accompanying an action as: time, location, order of actions, etc.

A difficult part of the theory is the question of the intentionality of an action, e.g., when the intention of the agent is to perform an action, yet—for a variety of reasons—the agent desists from performing it. The notional apparatus permits the introduction of clear-cut criteria for differentiating between single actions (when the agent is an individual) and collective actions (when the agent is a collective body), as well as between one-time actions (such as the stabbing of Julius Caesar) and actions understood as a type (e.g., stabbing as a type of criminal action).

The problem area of verbal actions and models of information flow which accompanies actions requires special treatment. This question is not studied in the book; nevertheless, the models of action to be developed allow for their extension over verbal actions. In this context, one can modify the existing models of belief systems deriving from Alchourrón, Gärdenfors, and Makinson.

E. Probabilistic Models of Action

The notion of performability mentioned in C is not probabilistic but binary: a given action *A* is either performable or not in a definite state *u*. This notion does not encompass some aspects of the performability of actions as, e.g., quality grading (poor, medium, good performance, etc.). One framework that brings theories of action closer to probability calculus and decision theory introduces a quantitative measure of the degree of performability of an action. It is the probability of performability of an action in a given state *u*. Also introduced are other measures of performability such as the probability of the transition of the system from one state to another on the condition that an action is performed. The measure is, to use the simplest example, the probability of hitting—in the determined initial conditions *u*—the ‘bull’s-eye’ with an arrow shot from a bow (the intended state *v*). The performed action is here shooting an arrow at a target.

Two types of probabilistic models of action are distinguished. The emphasis is put on their practical applications. The relevant models are constructed from elementary systems (point B) by introducing (conditional) probabilities of transition from one state to another, under the assumption that the given action is performed. (The notion of the performability of an action is distinguished from a possible performance as well as from a performance of an action—the latter being a one-time act, belonging to the situational surrounding of the system of action.)

F. Relationship with Deontic Logic

A significant feature of action theory is its firm rooting in the theory of law and theory of legal and moral norms. This part of action theory is called deontology. The central place in it is occupied by deontic logic. This is still an area which is characterized by the existence of disparities concerning fundamental matters, the proliferation of formal logical systems, as well as a lack of mathematical and logical results of generally recognized significance and depth. In the formulations of deontic logic known from the literature, deontic operators are considered as unary sentence-generating functors defined on sentences. In semantic stylizations, these formulations distinguish permitted, forbidden, and obligatory *states of affairs*.

In this book, a formulation of deontic logic is presented, according to which the deontic operators belong to quite a different category: they are defined on actions, and not on states of affairs. Thus, in this formulation, these are actions that are permitted, forbidden, or obligatory entities, and not sets of states. It leads to two simple formalized systems of deontic logic, whose semantics is founded on elementary action systems. The difference between the systems consists in the fact that the first one validates the so-called closure principle, while the other rejects it. The closure principle says that every action which is not forbidden is permitted. These systems are free from deontic paradoxes. Completeness theorems for these systems are proved. Deontic models of compound actions are also considered.

The book presents a new approach to norms. Norms in the broad sense are viewed as certain rules of action. In the simplest case they are instructions which, under given circumstances, permit, forbid, or order the performance of an action.

G. Relations with the Theory of Algorithms and Programming

The theory of algorithmizable actions is a vital part of action theory. Here, algorithmizable actions are set against actions that are creative, single, and unique in their nature.

There is no satisfactory definition of algorithmizable actions. According to an informal definition, an algorithm is a set of rules that precisely defines sequence of actions. Instances of algorithmizable actions are regular or context-free compound actions. According to the above linguistic approach, regular (respectively, context-free) compound actions are defined as regular (context-free) languages over the alphabet consisting of atomic actions.

A part of the book establishes certain results on algorithmizable actions referring to the notion of an action program. The prototype here is the meaning of the term “program”, with which it is invested by computer science. The above-mentioned problem area displays relations with algorithmic logic in the sense of Salwicki and the theory of algorithms; yet, it is not identical with them.

H. Non-monotonic Reasonings and Action

An approach to non-monotonic reasonings which links them with the theory of action is outlined. A general *semantic* scheme of defining non-monotonic reasonings in terms of frames is presented. Each frame F is a set of states W endowed with a family R of relations of a certain kind. If S is a propositional language, then each frame determines in a natural way a consequence-like operation on S . The latter does not generally exhibit all properties of consequence operations as, e.g., monotonicity. Such operations exemplify non-monotonic patterns of reasoning. The class of resulting structures encompasses preferential model structures and supra-classical reasonings.

I. The Existing State of Knowledge in the Scope of the Study Area

Reflection on the rational and irrational actions of human beings is not alien to the Polish logical tradition. Books by Tadeusz Kotarbiński took the lead. The works of Kazimierz Ajdukiewicz, directed toward practical application of logic, and the activity of Kotarbiński's disciples (Nowakowska, Stonert, Konieczny, Gasparski) testify to this only too well. It seems research in this area needs new impulses so as to permit it ultimately to penetrate to a broader extent and more profoundly into the scientific and technical achievements of recent decades, especially as regards dynamic logic, theory of automata, and programming.

The present book puts the emphasis on the mathematical and formal-logical aspects of an action. Despite being firmly grounded in the tradition of Lvov-Warsaw School, to a broad extent it takes into account the output of many schools, including the Scandinavian, Dutch or Pittsburgh ones, to mention a few. It also takes into consideration the author's own modest contributions. In the years 2001–2003, the author was in charge of the research project “Logika i działanie (Logic and Action)”, signature 1 H01A 011 18, financed within a generous grant obtained from the then *Komitet Badań Naukowych* (*State Committee for Scientific Research*). The support from the Committee resulted in the first drafts of this book.

Fragments of the book have been presented by the author over many years at different conferences, both at home and abroad. The first presentation took place at the University of Konstanz in October 1992, and the next during a meeting at Umeå University (Sweden) in September 1993. Among other conferences the following must be noted: *Logic and its Applications in Philosophy and the Foundations of Mathematics*, organized annually since 1996 in Karpacz or Szklarska Poreba (The Giant Mountains, Poland), where the author presented his papers relating to various aspects of action a number of times. Throughout all this time the author has received valuable advice and criticism from many people whose individual names shall not be mentioned, but who are all cordially thanked.

It remains to hope that the book will (modestly) contribute to the formulation of a generally accepted paradigm of action theory.

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