

Preface

This volume originated from the lectures on the local and global inverse problem of the calculus of variations delivered at the 18th International Summer School on Global Analysis held during August 12–17, 2013, in Levoča, Slovakia. The aim is to present a systematic exposition of some of the contemporary trends in this field.

The first two chapters are devoted to the discussion of the inverse problem for systems of *second-order ordinary differential equations*. New ideas are introduced for possible applications of the inverse problem and the Helmholtz conditions to *control and feedback stabilization* of mechanical systems (Bloch, Krupka, Zenkov). The inverse problem due to Sonin and Douglas is explained by means of the elementary integrability conditions of the Frobenius type (Krupka). Next, the variational methods are applied to the higher-order mechanics and certain third-order equations with symmetry are analyzed (Matsyuk). The geometric structure of *parameter-invariant variational principles* for curves and the corresponding problems for submanifolds are studied by Urban. Further applications of the general inverse problem ideas appear for systems of (not necessarily variational) partial differential equations. One assigns to a non-variational system a variational one by means of the so-called *variational extension*. However, the resulting equations are not necessarily equivalent to the initial (non-variational) equations (Voicu). The last chapter is devoted to the variational sequence theory, providing the tool for studying *global characteristics* of the inverse problem on smooth manifolds. The first-order sequence is considered, explaining, however, all basic examples of the second-order partial differential equations in field theory (Urban, Volna).

As the chapters do not cover the entire variety of themes of the general inverse problem theory and its applications, we shall mention here at least the theory of variational sprays based on the Douglas problem and possible extensions to partial differential equations, the inverse problem for equations with symmetry, and the

general inverse problem for homogeneous partial differential equations and for submanifolds. On the other hand, the chapters open new topics and suggest areas for new research directions.

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