

Contents

Lecture 1: Elements of the History of Quantum Mechanics I	1
1 Introduction	1
2 Birth of Quantum Mechanics. The Early Years	5
3 Birth of Quantum Mechanics 1. The Work of de Broglie	10
4 Birth of Quantum Mechanics 2. Schrödinger's Formalism	12
References	15
 Lecture 2: Elements of the History of Quantum Mechanics II	 17
1 Birth of Quantum Mechanics 3: Born, Heisenberg, Jordan	17
2 Birth of Quantum Mechanics 4. Heisenberg and the Algebra of Matrices	21
3 Birth of Quantum Mechanics 5. Born's Postulate	24
4 Birth of Quantum Mechanics 6. Pauli; Spin, Statistics	25
5 Further Developments: Dirac, Heisenberg, Pauli, Jordan, von Neumann	28
6 Abstract Formulation	29
7 Quantum Field Theory	30
8 Anticommutation Relations	32
9 Algebraic Structures of Hamiltonian and Quantum Mechanics. Pauli's Analysis of the Spectrum of the Hydrogen Atom	33
10 Dirac's Theorem	36
References	37
 Lecture 3: Axioms, States, Observables, Measurement, Difficulties. . . .	 39
1 Introduction	39
2 The Axioms of Quantum Mechanics	40
3 States and Observables	42
4 Schrödinger's Quantum Mechanics	43
5 The Quantization Problem	44
6 Heisenberg's Quantum Mechanics	46
7 On the Equivalence	46

8	The Axioms	47
9	Conceptual Problems	52
10	Information-Theoretical Analysis of Born's Rule	56
	References	58

Lecture 4: Entanglement, Decoherence, Bell's Inequalities,

	Alternative Theories	59
1	Decoherence. I	59
2	Decoherence. II	62
3	Experiments	64
4	Bell's Inequalities	66
5	Alternative Theories	69
	References	75

Lecture 5: Automorphisms; Quantum Dynamics; Theorems

	of Wigner, Kadison, Segal; Continuity and Generators	77
1	Short Summary of Hamiltonian Mechanics	77
2	Quantum Dynamics	79
3	Automorphisms of States and Observables	80
4	Proof of Wigner's Theorem.	82
5	Proof of Kadison's and Segal's Theorems	85
6	Time Evolution, Continuity, Unitary Evolution	87
7	Time Evolution: Structural Analogies with Classical Mechanics.	94
8	Evolution in Quantum Mechanics and Symplectic Transformations	97
9	Relative Merits of Heisenberg and Schrödinger Representations.	99
	References	101

Lecture 6: Operators on Hilbert Spaces I; Basic Elements

1	Characterization of the Self-adjoint Operators	107
2	Defect Spaces	110
3	Spectral Theorem, Bounded Case.	112
4	Extension to Normal and Unbounded Self-adjoint Operators	117
5	Stone's Theorem	118
6	Convergence of a Sequence of Operators	120
7	Ruelle's Theorem.	122
	References	124

Lecture 7: Quadratic Forms

1	Relation Between Self-adjoint Operators and Quadratic Forms.	126
2	Quadratic Forms, Semi-qualitative Considerations	127
3	Further Analysis of Quadratic Forms	131
4	The KLMN Theorem; Friedrichs Extension.	133
5	Form Sums of Operators.	137

6	The Case of Dirichlet Forms	138
7	The Case of $-\Delta + \lambda x ^{-\alpha}$, $x \in R^3$	140
8	The Case of a Generic Dimension d	143
9	Quadratic Forms and Extensions of Operators	145
10	A Simple Example.	146
	References	148

Lecture 8: Properties of Free Motion, Anholonomy,

Geometric Phase	151
1 Space-Time Inequalities (Strichartz Inequalities).	153
2 Asymptotic Analysis of the Solution of the Free Schrödinger Equation	154
3 Asymptotic Analysis of the Solution of the Schrödinger Equation with Potential V	156
4 Duhamel Formula	158
5 The Role of the Resolvent.	159
6 Harmonic Oscillator	160
7 Parallel Transport. Geometric Phase	161
8 Anholonomy and Geometric Phase in Quantum Mechanics	163
9 A Two-Dimensional Quantum System	164
10 Formal Analysis of the General Case	165
11 Adiabatic Approximation	166
12 Rigorous Approach	168
References	172

Lecture 9: Elements of C^* -algebras, GNS Representation,

Automorphisms and Dynamical Systems.	173
1 Elements of the Theory of C^* -algebras	173
2 Topologies	177
3 Representations	180
4 The Gel'fand-Neumark-Segal Construction	181
5 von Neumann Algebras.	183
6 von Neumann Density and Double Commutant Theorems. Factors, Weights	184
7 Density Theorems, Spectral Projection, Essential Support	186
8 Automorphisms of a C^* -algebra. C^* -dynamical Systems	188
9 Non-commutative Radon-Nikodim Derivative	192
References	194

Lecture 10: Derivations and Generators. K.M.S. Condition.

Elements of Modular Structure. Standard Form	195
1 Derivations	195
2 Derivations and Groups of Automorphisms	198
3 Analytic Elements	201

4	Two Examples from Quantum Statistical Mechanics and Quantum Field Theory on a Lattice	202
4.1	Example 1	202
4.2	Example 2	205
5	K.M.S. Condition	205
6	Modular Structure	210
7	Standard Cones	212
8	Standard Representation (Standard Form)	213
9	Standard Liouvillian	214
	References	216

Lecture 11: Semigroups and Dissipations. Markov Approximation.

	Quantum Dynamical Semigroups I	217
1	Semigroups on Banach Spaces: Generalities	218
2	Contraction Semigroups	220
3	Markov Approximation in Quantum Mechanics	228
4	Quantum Dynamical Semigroups I	232
5	Dilation of Contraction Semigroups	233
	References	237

Lecture 12: Positivity Preserving Contraction Semigroups

	on C^*-algebras. Conditional Expectations. Complete Dissipations	239
1	Complete Positivity. Dissipations	239
2	Completely Positive Semigroups. Conditional Expectations	243
3	Steinspring Representation. Bures Distance	247
4	Properties of Dissipations	250
5	Complete Dissipations	255
6	General Form of Completely Dissipative Generators	257
	References	259

Lecture 13: Weyl System, Weyl Algebra, Lifting Symplectic

	Maps. Magnetic Weyl Algebra	261
1	Canonical Commutation Relations	261
2	Weyl System	264
3	Weyl Algebra. Moyal Product	265
4	Weyl Quantization	267
5	Construction of the Representations	269
6	Lifting Symplectic Maps. Second Quantization	270
7	The Magnetic Weyl Algebra	274
8	Magnetic Translations in the Magnetic Weyl Algebra	276
	References	281

Lecture 14: A Theorem of Segal. Representations of Bargmann, Segal, Fock. Second Quantization. Other Quantizations

(Deformation, Geometric).	283
1 Fock Space	285
2 Complex Bargmann-Segal Representation	288
3 Berezin-Fock Representation	291
4 Toeplitz Operators	292
5 Landau Hamiltonian Constant Magnetic Field in R^3	293
6 Non-constant Magnetic Field	295
7 Real Bargmann-Segal Representation	297
8 Conditions for Equivalence of Representations Under Linear Maps	299
9 Second Quantization	300
10 The Formalism of Quantization	302
11 Poisson Algebras	302
12 Quantization of a Poisson Algebra	302
13 Deformation Quantization, $*$ -product	304
14 Strict Deformation Quantization	306
15 Berezin-Toeplitz $*$ -product	306
16 “Dequantization”	307
17 Geometric Quantization	309
18 Bohr-Sommerfeld Quantization	311
References	312

Lecture 15: Semiclassical Limit; Coherent States;

Metaplectic Group.	313
1 States Represented by Wave Functions of Class A	315
2 Qualitative Outline of the Proof of (1), (2), (3), (4)	317
3 Tangent Flow, Quadratic Hamiltonians	318
4 Coherent States	319
5 Quadratic Hamiltonians. Metaplectic Algebra.	321
6 Semiclassical Limit Through Coherent States: One-Dimensional Case	322
7 Semiclassical Approximation Theorems	323
8 N Degrees of Freedom. Bogolyubov Operators	327
9 Linear Maps and Metaplectic Group. Maslov Index	330
References	334

Lecture 16: Semiclassical Approximation for Fast Oscillating Phases. Stationary Phase. W.K.B. Method.

Semiclassical Quantization Rules	335
1 Free Schrödinger Equation	335
2 The Non-stationary Phase Theorem	336
3 The Stationary Phase Theorem.	337

4	An Example	340
5	Transport and Hamilton-Jacobi Equations	343
6	The Stationary Case	346
7	Geometric Intepretation.	348
8	Semiclassical Quantization Rules	350
8.1	One Point of Inversion	352
8.2	Two Points of Inversion	353
9	Approximation Through the Resolvent	354
	References	356

Lecture 17: Kato-Rellich Comparison Theorem. Rollnik

	and Stummel Classes. Essential Spectrum.	357
1	Comparison Results	357
2	Rollnik Class Potentials	363
3	Stummel Class Potentials	367
4	Operators with Positivity Preserving Kernels	370
5	Essential Spectrum and Weyl's Comparison Theorems	373
6	Sch'nol Theorem	379
	References	382

Lecture 18: Weyl's Criterium, Hydrogen and Helium Atoms.

1	Weyl's Criterium	383
2	Coulomb-Like Potentials. Spectrum of the Self-adjoint Operator	388
3	The Hydrogen Atom. Group Theoretical Analysis	390
4	Essential Spectrum.	395
5	Pauli Exclusion Principle, Spin and Fermi-Dirac Statistics.	396
5.1	Spin	396
5.2	Statistics	396
5.3	Pauli Exclusion Principle.	397
6	Helium-Like Atoms	398
7	Point Spectrum	401
8	Two-Dimensional Hydrogen Atom.	404
9	One-Dimensional Hydrogen Atom	405
10	Capacity	407
	References	408

Lecture 19: Estimates of the Number of Bound States.

	The Feshbach Method	409
1	Comparison Theorems	409
2	Estimates Depending on Banach Norms	416
3	Estimates for Central Potentials	419
4	Semiclassical Estimates.	420

5	Feshbach Method	424
5.1	The Physical Problem	424
5.2	Abstract Setting	425
	References	428

Lecture 20: Self-adjoint Extensions. Relation with Quadratic Forms. Laplacian on Metric Graphs. Boundary Triples.

	Point Interaction	429
1	Self-adjoint Operators: Criteria and Extensions	429
2	von Neumann Theorem; Krein's Parametrization	432
3	The Case of a Symmetric Operator Bounded Below	436
4	Relation with the Theory of Quadratic Forms	437
5	Special Cases: Periodic, Dirichlet and Neumann Boundary Conditions.	440
6	Self-adjoint Extensions of the Laplacian on a Locally Finite Metric Graph	441
7	Point Interactions on the Real Line.	444
8	Laplacians with Boundary Conditions at Smooth Boundaries in R^3	446
9	The Trace Operator	447
10	Boundary Triples	449
11	Weyl Function.	451
12	Interaction Localized in N Points	452
	References	454
	Index	455

Lectures on the Mathematics of Quantum Mechanics I

DellAntonio, G.

2015, XXI, 459 p., Hardcover

ISBN: 978-94-6239-117-8

A product of Atlantis Press