Contents

1 Chemotaxis 1
- 1.1 Basic Notions 1
- 1.2 Collapse Formation 5
- 1.3 Mean Field Hierarchy 10
- 1.4 Stationary State 13
- 1.5 Localization and Symmetrization 15
- 1.6 Weak Solutions 18
- 1.7 Rescaling 24
- 1.8 Collapse Mass Quantization 27
- 1.9 Blowup in Infinite Time 33
- 1.10 Simple Blowup Points 41
- 1.11 Summary 44

2 Time Relaxization 47
- 2.1 Full System of Chemotaxis 47
- 2.2 Non-local Parabolic Equation 48
- 2.3 Smoluchowski-ODE System 51
- 2.4 Harmonic Heat Flow 60
- 2.5 Normalized Ricci Flow 65
- 2.6 Concentration of Probability Measures 75
- 2.7 Summary 78

3 Toland Duality 81
- 3.1 Full System of Chemotaxis Revisited 81
- 3.2 Lagrangian and Duality 87
- 3.3 Gradient Systems with Duality 98
- 3.4 Zygmund Spaces 104
- 3.5 Summary 110

4 Phenomenology 111
- 4.1 Non-convex Evolution 111
- 4.2 Gradient and Skew-Gradient Systems 120

4.3 Semi-unfolding-Minimality . . . 126
4.4 Kuhn-Tucker Duality . . . 132
4.5 Summary . . . 139

5 Phase Transition . . . 141
5.1 Thermal Equilibrium. . . . 141
5.2 Stefan Problem . . . 147
5.3 Phase Field Model . . . 150
5.4 Summary . . . 157

6 Critical Phenomena of Isolated Systems . . . 159
6.1 Non-equilibrium Thermodynamics . . . 159
6.2 Penrose-Fife Theory . . . 166
6.3 Penrose-Fife Equation. . . . 170
6.4 Coupled Cahn-Hilliard Equation. . . . 174
6.5 Shape Memory Alloys . . . 178
6.6 Summary . . . 201

7 Self-interacting Fluids. . . . 203
7.1 Ideal Fluids . . . 203
7.2 Gas Dynamics . . . 210
7.3 Self-gravitating Fluids. . . . 215
7.4 Plasma Confinements . . . 232
7.5 Related Models . . . 239
7.6 Summary . . . 245

8 Magnetic Fields . . . 247
8.1 Interface Vanishing . . . 247
8.2 Plasma Equilibrium . . . 259
8.3 MHD Fluids . . . 264
8.4 Summary . . . 268

9 Boltzmann-Poisson Equation. . . . 269
9.1 Point Vortex . . . 269
9.2 Boltzmann Relation . . . 274
9.3 Ensemble . . . 276
9.4 Turbulence . . . 280
9.5 Summary . . . 295

10 Particle Kinetics. . . . 297
10.1 Kramers-Moyal Expansion . . . 297
10.2 Kinetic Model . . . 304
10.3 Maximum Entropy Production . . . 311
10.4 Summary . . . 317

11 Parabolic Equations . . . 319
11.1 Semilinear Equations . . . 319
11.2 Degenerate Equations . . . 324

11.3 Blowup Threshold 327
11.4 Structure of the Blowup Set. 330
11.5 Other Properties 337
11.6 Summary 346

12 Gauge Fields 349
12.1 Field Theory 349
12.2 Exponential Nonlinearity Revisited 358
12.3 Scaling Invariance 363
12.4 Liouville-Bandle Theory 365
12.5 Alexandroff-Bol's Inequality 373
12.6 Pre-scaled Analysis 386
12.7 Entire Solution. 393
12.8 Blowup Analysis 398
12.9 Summary 409

13 Higher-Dimensional Blowup 411
13.1 Method of Duality 411
13.2 Higher-Dimensional Quantization 416
13.3 Dimension Control of the Blowup Set 419
13.4 Summary 422

Bibliography 423

Index 441

http://www.springer.com/978-94-6239-153-6

Mean Field Theories and Dual Variation - Mathematical Structures of the Mesoscopic Model

Suzuki, T.

2015, XIII, 444 p., Hardcover
ISBN: 978-94-6239-153-6
A product of Atlantis Press