

Preface

In sixties, many algorithms for processing of single and two-dimensional signals were theoretically developed. For most of them, being based on Fourier analysis, the practical applications were rather limited by the lack of the fast computing algorithms for the (complex-valued) Fourier coefficients. Due to the pressure of concrete applications, the importance of which was related also to the political situation in the World, and especially the Cold War, theory was enriched in 1965 with the Cooley-Tukey Fast Fourier Transform (FFT), which is an algorithm for efficient, in terms of space and time, computation of the Discrete Fourier Transform (DFT) ¹. Still, in spite of the existence of FFT, the computing power of hardware at the time, was inadequate to completely support related applications. In particular, difference in the price of the operations of addition and multiplication required to compute the complex-valued Fourier spectral coefficients, motivated reformulation of the Cooley-Tukey FFT and raised a variety of FFT algorithms, as for instance, the Good-Thomas FFT, the Winograd FFT algorithm, the Rader-Brenner FFT algorithm, whose numerical instability was improved in the split radix Cooley-Tukey FFT, prime factor FFT, etc. An alternative was to look for a fast to compute transform having the good properties of the Fourier transform on the real line. This motivated the interest in Walsh (Fourier) analysis, since the Walsh functions take just two values, 1 and -1 , and, therefore, computing of the Walsh (Fourier) transform coefficients requires no multiplications. The interest was extended to the computationally even faster Haar transform based on three-valued $(1, 0, -1)$ Haar functions.

In 1969, Henning F. Harmuth, who set the foundations for the applications of Walsh functions in communication and electromagnetic theory, wrote ²

Communication theory was founded on the system of sine-cosine functions. A more general theory has become known more recently; it replaces the sine-cosine by other systems of orthogonal functions, and the concept of frequency by that of sequency. Of these systems, the Walsh functions are of great practical interest since

¹ Cooley, J.W., Tukey, J.W., "An algorithm for the machine calculation of complex Fourier series", *Math. Computation*, Vol. 19, 297-301, 1965.

² Harmuth, H.F., "Applications of Walsh functions in communications", *IEEE Spectrum*, No. 11, 1969, 82-91.

they lead to equipment that is easily implemented by semiconductor technology.

In 1972, Harmuth and his associates stated ³

Coils, capacitors, and optical lenses favor sine-cosine functions. The digital computer has other favorites, such as Walsh, Hadamard, and Haar functions, which have become increasingly significant in communications since the era of semiconductors.

Therefore, the new theory, called the discrete Walsh dyadic analysis, emerged to meet characteristics of the technology at the time with the discrete Walsh functions as the basic concept in the theory, due to their simplicity and compatibility with bistable devices used to realize the corresponding systems. Within this theory, the notion of the derivative, as an operator allowing to estimate the rate of change as well as the direction of the change of a signal, was necessary, especially regarding the applications intended. In such circumstances, due to the interest in Walsh functions and the dyadic analysis, the finite dyadic derivative, initially called the logic derivative, now called the Gibbs derivative, having Walsh functions as its eigenfunctions, was defined in 1967 by James Edmund Gibbs. ⁴

In the forty five years since the concept of Gibbs derivative was first introduced in the restricted context of a finite dyadic group ⁵, the scope of the definition has been greatly extended, many results have been obtained, and areas of application have been established ⁶. In overall, a beautiful mathematical theory of Gibbs dyadic differentiation, with the Butzer-Wagner dyadic derivative as the most prominent concept from the mathematical analysis point of view, was created and is still continuously developing.

The very high interest in Walsh and dyadic analysis in seventieths, was obvious from the organization of international workshops and conferences devoted exclusively to the applications of Walsh functions from 1970 to 1974 sponsored by the Naval Research Laboratory, and held in Washington DC, USA. Moreover, there were even two conferences on that subject in 1973, the other was the Symposium on Theory and Applications of Walsh and Other Non-sinusoidal Functions, Hatfield,

³ Harmuth, H.F., Anrews, H.C., Shibata, K., "Two-dimensional sequency filters", *IEEE Trans. Communications*, Vol. COM-20, No. 3, 1972, 312-331.

⁴ Gibbs, J.E., "Walsh spectrometry a form of spectral analysis well suited to binary digital computation", *NPL DES Repts.*, National Physical Lab., Teddington, Middlesex, England, 1967.

⁵ Gibbs, J.E., Sine waves and Walsh waves in physics, *Proc. Sympos. Applic. Walsh Functions*, Washington, D.C., 1970 Mar. 31-Apr. 3, 260-274, (1970).

Gibbs, J.E., "Functions that are solutions of a logical differential equation", *NPL DES Rept*, No. 4, iv+21 pp. (1970).

Gibbs, J.E., Millard, Margaret J., "Walsh functions as solutions of a logical differential equation", *NPL DES Rept*, No. 1, 9 pp. (1969).

⁶ Butzer, P.L., Wagner, H.J., "Approximation by Walsh polynomials and the concept of a derivative", *Proc. Sympos. Applic. Walsh Functions*, Washington, D.C., 388-392, (1972).

Butzer, P.L., Wagner, H.J., "On a Gibbs-type derivative in Walsh-Fourier analysis with applications", *Proc. Nat. Electron. Conf.*, 27, 393-398, (1972).

UK. At that time, there were expectations that the dyadic calculus and the derivative might have a role in information sciences and computing comparable to that of the Newton-Leibniz derivative in classical mathematical analysis and mechanics.

This interest in Walsh (Fourier) analysis was gradually reduced in the next decades for various reasons, partially due also to the advent of computing power overcoming the restrictions to use complex computations in classical Fourier analysis, and also for the advent of alternative theoretical tools. In these settings, the Walsh and dyadic analysis become a standard classical tool in certain areas, with a well established role and place. The related spectral methods are a standard subject within the scope of several international conferences and journals. It is, however, a feeling that this area has a great potential and is still waiting for wider and more important applications.

The situation in present computing technologies reminds that in former times in the sense that a revolutionary future development is necessary. Further increasing the number of transistors and elementary gates does not provide right solution, since the present technology is approaching to its physical limits in that respect. Improving the computational power by increasing the clock frequency is also limited by high energy dissipation and heating, both features hardly acceptable in practice. Thus, some essentially new computing technologies are needed and nothing like that still appears at the horizon, at least looking from the engineering perspective. Alternative approaches such as, for instance, quantum computing, DNA computing, and similar, are encouraging, but still far from immediate concrete engineering applications.

Multi-core central processing units (CPU) and Graphic processing units (GPU) that are recently open for general purpose computing (GPGPU) provide a short break in the search for new technologies. In these devices, the situation is reverse to that in seventies, since now computing is cheap, and memory latency and memory bandwidth are the critical factors. Therefore, the same rethinking of algorithms as that in devising various FFT algorithms in seventies is required, just in the opposite sense. Instead of reducing the number of operations, we should minimize the amount of memory transfers.

These examples discussed above, illustrate that taking a fresh look into existing theories, as dyadic calculus in general and the dyadic differentiation as a facet of it in particular, may help in borrowing old or devising new ideas that might appear useful presently in various aspects. This is an excuse which we would be glad to offer and will be happy if accepted for bringing to public notice the former work in this area. We are not so enthusiastic to think that dyadic differential calculus will be immediately useful in revolutionizing computing technologies. It however might raise new questions and challenging problems. Most of them would be probably trivial to solve, some more difficult as student exercises, but a couple of them could be really interesting.

Acknowledgments

The four editors of this monograph, in particular Radomir S. Stanković and Paul L. Butzer, express their thanks for its preparation to the authors of the monograph together with their former students. The initiator of dyadic differentiation, James Edmund Gibbs, National Physical Laboratory, Teddington, UK, died much too early, in January 2007, and so could, most unfortunately, not participate in our present project.

Mrs. Merion Gibbs reported that according to her husbands diary, his first definition of his "logical derivative" was written in January 1967. Of his 27 publications on dyadic derivatives six were written together with Dr. Brian Ireland, Bath University, UK, who joined Dr. Gibbs in the work on this subject in 1971.

The joint work of authors towards the present monograph, which took place over a period of almost four years, was very constructive and unbelievably harmonious, and even during the time from 1969 until 1990 when the chief results of dyadic Walsh analysis were developed by them. This is by no means self-evident since the authors come from ten countries, namely Austria, Canada, China, England, Germany, Hungary, Japan, Russia, Serbia, USA, with quite different mathematical/scientific traditions.

The majority of these, together with some of their former students, only met for the first time at the Workshop "Theory and Applications of Gibbs Derivatives", conducted by Radomir S. Stanković at Kupari- Dubrovnik, Yugoslavia, on September 26-28, 1989.

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The Editors

Dyadic Walsh Analysis from 1924 Onwards

Walsh-Gibbs-Butzer Dyadic Differentiation in Science

Volume 2 Extensions and Generalizations

A Monograph Based on Articles of the Founding

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