

# Contents

|          |                                                                                    |          |
|----------|------------------------------------------------------------------------------------|----------|
| <b>1</b> | <b>Early Work in Gibbs Differentiation in China</b>                                | <b>1</b> |
|          | Weiyl Su                                                                           |          |
| 1.1      | Research in Butzer-Gibbs Calculus in China                                         |          |
|          | - A glance of $p$ -type calculus                                                   | 2        |
|          | References                                                                         | 3        |
| 1.2      | The Generalized Logical Derivative and Its Applications                            | 5        |
| 1.2.1    | Introduction                                                                       | 5        |
| 1.2.2    | The $p$ -adic Logical Derivatives                                                  | 7        |
| 1.2.3    | Bernstein Type Inequality and Bernstein Type Theorem                               | 11       |
|          | References                                                                         | 14       |
| 1.3      | The Generalized Walsh Transform and An Extremum Problem                            | 15       |
| 1.3.1    | Introduction                                                                       | 15       |
| 1.3.2    | Properties of $W_N$ -type functions                                                | 16       |
| 1.3.3    | The Descent Order of $W_N$ -type Functions                                         | 18       |
| 1.3.4    | $p$ -adic Logic Derivatives                                                        | 23       |
| 1.3.5    | Logic derivative                                                                   | 24       |
| 1.3.6    | Adjoint logic derivative                                                           | 25       |
| 1.3.7    | A Class of Extremum Problems                                                       | 27       |
|          | References                                                                         | 32       |
| 1.4      | The Logical Derivatives and Integrals-Zheng Weixing,<br>Su Weiyl                   | 33       |
| 1.4.1    | Introduction                                                                       | 33       |
| 1.4.2    | Notation and important results                                                     | 34       |
| 1.4.3    | $p$ -adic derivatives and integrals of functions in $L_G^q$ ,<br>$1 \leq q \leq 2$ | 36       |
|          | References                                                                         | 48       |
| 1.5      | The Logical Derivatives and Integrals Second Part                                  | 49       |

|          |                                                                                |            |
|----------|--------------------------------------------------------------------------------|------------|
| <b>2</b> | <b>Gibbs-Butzer Calculus and Pseudo-differential Operators on Local Fields</b> | <b>65</b>  |
|          | Weiyl Su, Qiu Hua                                                              |            |
| 2.1      | Introduction                                                                   | 65         |
| 2.2      | Notations                                                                      | 66         |
| 2.3      | G-B type derivative and G-B type integrals                                     | 67         |
| 2.4      | Function Spaces on Local Fields                                                | 75         |
| 2.5      | A Comparison of Newton calculus with G-B calculus                              | 79         |
| 2.6      | Applications of Gibbs-Butzer-type Calculus                                     | 82         |
| 2.6.1    | Application to Approximation Theory on Local Fields                            | 82         |
| 2.6.2    | Application to determine function space for which a function belongs to it     | 83         |
| 2.6.3    | Application to partial differential equations with fractal boundaries          | 84         |
| 2.6.4    | Application to medical science                                                 | 89         |
|          | References                                                                     | 90         |
| <b>3</b> | <b>Early Work in Dyadic Differentiation in Soviet Union - SSSR</b>             | <b>93</b>  |
|          | Radomir S. Stanković, Paul L. Butzer                                           |            |
| 3.1      | Interest in Dyadic Differentiation in Soviet Union                             | 93         |
| <b>4</b> | <b>Research in Dyadic Differentiation in Russia</b>                            | <b>125</b> |
|          | Boris I. Golubov, Sergei S. Volosivets                                         |            |
|          | References                                                                     | 128        |
| <b>5</b> | <b>Generalized Derivatives and Integrals on Vilenkin Groups</b>                | <b>131</b> |
|          | Boris I. Golubov, Sergei S. Volosivets                                         |            |
| 5.1      | A. The case of compact groups                                                  | 131        |
| 5.2      | B. The case of locally compact groups                                          | 137        |
|          | References                                                                     | 141        |
| <b>6</b> | <b>Calculus on Walsh and Vilenkin Groups</b>                                   | <b>145</b> |
|          | György Gát, Rodolfo Toledo                                                     |            |
| 6.1      | Introduction                                                                   | 145        |
| 6.2      | Results related to Vilenkin systems                                            | 148        |
| 6.3      | Results related to representative product systems                              | 151        |
| 6.4      | Results related to Walsh-Kaczmarz systems                                      | 152        |
| 6.5      | Results related to two-dimensional Walsh-Paley systems                         | 153        |
|          | References                                                                     | 155        |
| <b>7</b> | <b>Gibbs Derivatives on Groups</b>                                             | <b>157</b> |
|          | Radomir S. Stanković                                                           |            |
| 7.1      | Introduction                                                                   | 157        |
| 7.2      | Basic Concepts                                                                 | 159        |
| 7.3      | Gibbs Derivatives on Finite Non-Abelian Groups                                 | 161        |
| 7.4      | Summary and Recent Work                                                        | 165        |
|          | References                                                                     | 166        |

|           |                                                                            |     |
|-----------|----------------------------------------------------------------------------|-----|
| <b>8</b>  | <b>My Research in Gibbs Derivatives on Finite Groups</b>                   | 169 |
|           | Radomir S. Stanković                                                       |     |
|           | References                                                                 | 171 |
| <b>9</b>  | <b>Efficient Computation of Gibbs Derivatives on Finite Abelian Groups</b> | 211 |
|           | Radomir S. Stanković, Dušan Gajić                                          |     |
| 9.1       | Introduction                                                               | 211 |
| 9.2       | Definitions of Gibbs Derivatives on Finite Abelian Groups                  | 212 |
| 9.3       | Generalizations                                                            | 214 |
| 9.4       | Fast Fourier Transform on Finite Abelian Groups                            | 215 |
| 9.5       | Gibbs Derivatives on Finite Abelian Groups                                 | 217 |
| 9.6       | Computing the Gibbs Derivatives on Finite Abelian Groups                   | 219 |
|           | 9.6.1 Convolution-like algorithm                                           | 219 |
|           | 9.6.2 Computing in terms of partial Gibbs derivatives                      | 219 |
| 9.7       | GPGPU Computing Platform                                                   | 220 |
| 9.8       | Mapping the Algorithms to GPGPU Architecture                               | 222 |
| 9.9       | Experimental Results                                                       | 223 |
|           | References                                                                 | 227 |
| <b>10</b> | <b>Gibbs Derivative and Walsh Harmonizable DSP</b>                         | 277 |
|           | Yasushi Endow                                                              |     |
| 10.1      | Introduction                                                               | 277 |
| 10.2      | Q-loop and dyadic time                                                     | 277 |
| 10.3      | Gibbs derivative and Walsh functions                                       | 279 |
|           | 10.3.1 Walsh functions                                                     | 279 |
|           | 10.3.2 Gibbs derivative                                                    | 280 |
| 10.4      | Dyadic system and dyadic flow                                              | 281 |
| 10.5      | Dyadic stationary process                                                  | 282 |
|           | 10.5.1 DSP in strict sense                                                 | 282 |
|           | 10.5.2 DSP in wide sense and its W-harmonizability                         | 283 |
|           | 10.5.3 Walsh harmonizability of DSP                                        | 284 |
|           | 10.5.4 DSP and LDP                                                         | 285 |
|           | 10.5.5 Walsh Series and Approximation of DSP                               | 286 |
| 10.6      | Gibbs Differentiation of DSP                                               | 286 |
|           | 10.6.1 Gibbs Differentiability Conditions                                  | 287 |
|           | 10.6.2 A Linear Gibbs Differential Equation                                | 287 |
| 10.7      | Extensions and Comments                                                    | 288 |
|           | 10.7.1 Chrestenson functions                                               | 289 |
|           | 10.7.2 A $p$ -adic SP and C-harmonizability                                | 292 |
|           | 10.7.3 W-Harmonizable and C-Harmonizable DSPs                              | 293 |
|           | 10.7.4 Open Problems                                                       | 293 |
|           | References                                                                 | 294 |

|                                                                                  |     |
|----------------------------------------------------------------------------------|-----|
| <b>11 My Involvement in Gibbs Derivatives and Walsh Harmonizable Processes</b>   | 297 |
| Yasushi Endow                                                                    |     |
| 11.0.1 Introduction                                                              | 297 |
| 11.0.2 Walsh Harmonizable DSP                                                    | 298 |
| 11.0.3 DSP and LDP                                                               | 298 |
| 11.0.4 Walsh Series and Approximation of DSP                                     | 299 |
| 11.0.5 Gibbs Differentiability of DSP                                            | 300 |
| 11.0.6 A Linear Gibbs Differential Equation                                      | 301 |
| 11.0.7 Extensions and Comments                                                   | 302 |
| References                                                                       | 302 |
| <b>12 Open Problems in Theory and Applications of Dyadic Derivatives</b>         | 325 |
| References                                                                       | 326 |
| References                                                                       | 327 |
| References                                                                       | 328 |
| 12.1 Hardy Spaces                                                                | 329 |
| 12.2 The one-dimensional dyadic derivative                                       | 330 |
| References                                                                       | 332 |
| <b>List of Publications on Dyadic Differentiation by the Authors of the Book</b> | 333 |
| <b>Index</b>                                                                     | 347 |
| <b>Biographies of Authors</b>                                                    | 351 |

Dyadic Walsh Analysis from 1924 Onwards

Walsh-Gibbs-Butzer Dyadic Differentiation in Science

Volume 2 Extensions and Generalizations

A Monograph Based on Articles of the Founding

Authors, Reproduced in Full

Stankovic, R.S.; L. Butzer, P.; Schipp, F.; Wade, W.R.; Su,

W.; Endow, Y.; Fridli, S.; Golubov, B.I.; Pichler, F.

2015, XVIII, 360 p. 9 illus., Hardcover

ISBN: 978-94-6239-162-8

A product of Atlantis Press