Chapter 2
Sensory Task-Space Setpoint Control

Increasing demand for robotic systems operating in unstructured environments has led to the development of sensory feedback control methods. This chapter introduces the fundamental concepts for design and analysis of task-space sensory feedback control of robotic systems with uncertainty. The joint-space method for control of robots is first reviewed, and several task-space sensory feedback control schemes for setpoint control or point-to-point control of a robot manipulator are introduced. Such control schemes were inspired by human visually guided reaching movements, which do not require an accurate knowledge of kinematics and dynamics of the arm. Using visual sensory feedback information, a human being is able to interact with the environment by picking up a new or unknown tool and manipulating it easily and skillfully to accomplish a task. Such basic ability of sensing and responding to uncertainty and changes without an accurate knowledge of sensory-to-motor transformation gives human beings a high degree of flexibility in dealing with unforeseen changes in the real world.

2.1 Joint-Space Setpoint Control

In this section, a standard joint-space setpoint control design problem based on the Lyapunov method is first reviewed. In the joint-space control methodology, a desired position of the end effector is first specified in Cartesian space and then converted to a corresponding desired joint configuration by solving an inverse kinematic problem, as illustrated in Fig. 2.1. The robot kinematics model is assumed to be known exactly in the joint-space control method. The setpoint control law is then designed so that the robot joints follow the desired joint positions.

To illustrate the use of Lyapunov-based techniques for robot joint-space control design, a single-link robot, as shown in Fig. 2.2, is first considered. The equation of motion of the robot is described by

$$m\ddot{q} + b\dot{q} = \tau, \tag{2.1}$$

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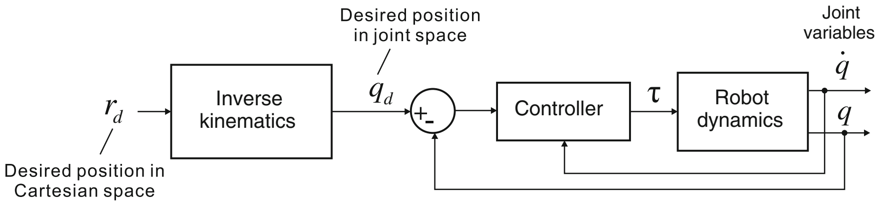
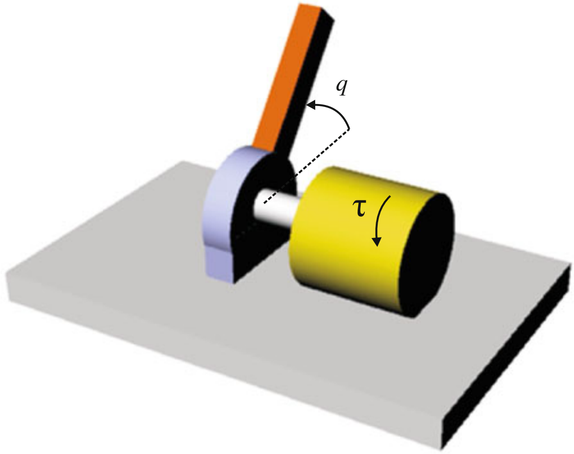


Fig. 2.1 Joint-space control

Fig. 2.2 A single-link robot



where m is the inertia, b represents the damping coefficient, q represents the joint angle, and τ is the control input torque exerted on the joint.

Consider the following proportional-derivative (PD) controller:

$$\tau = -k_v\dot{q} - k_p\Delta q, \tag{2.2}$$

where q_d is a constant desired position of the joint, $\Delta q = q - q_d$ denotes the position error, and k_v and k_p are positive control gains for the proportional and derivative errors respectively. The PD controller in Eq. (2.2) ensures the stability of the closed-loop system. To prove this, substituting the PD controller into the dynamic model described by Eq. (2.1) yields the following closed-loop equation:

$$m\ddot{q} + (b + k_v)\dot{q} + k_p\Delta q = 0. \tag{2.3}$$

Next, multiplying Eq. (2.3) by an output specified as $y = \dot{q}$ yields

$$m\ddot{q}\dot{q} + (b + k_v)\dot{q}^2 + k_p\Delta q\dot{q} = 0. \tag{2.4}$$

The above equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{1}{2}m\dot{q}^2 + \tfrac{1}{2}k_p\Delta q^2\right) + (b + k_v)\dot{q}^2 = 0, \tag{2.5}$$

where $\frac{1}{2}m\dot{q}^2$ is the kinetic energy of the robot system, $\frac{1}{2}k_p\Delta q^2$ is the artificial potential energy associated with the springlike proportional control term, $b\dot{q}^2$ is the amount of energy dissipated due to joint friction, and $k_v\dot{q}^2$ is the amount of energy loss due to the damping or friction injected into the system by the frictionlike derivative control term.

Let

$$V(\Delta q, \dot{q}) = \tfrac{1}{2}m\dot{q}^2 + \tfrac{1}{2}k_p\Delta q^2, \tag{2.6}$$

and

$$W(\dot{q}) = (b + k_v)\dot{q}^2, \tag{2.7}$$

and represent Eq. (2.5) as

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\Delta q, \dot{q}) = -W(\dot{q}). \tag{2.8}$$

The function $V(\Delta q, \dot{q})$ represents the sum of the kinetic energy and the artificial potential energy, and its time derivative is equal to $-W(\dot{q})$. Since $-W(\dot{q})$ is negative unless $\dot{q} = 0$, energy is dissipated by the frictional terms, and the system eventually stops at an equilibrium point.

In control theory, $V(\Delta q, \dot{q})$ is called the *Lyapunov function*, which is positive definite in the state variables Δq and $\dot{q}$, and $-W(\dot{q})$ denotes its time derivative, which is negative definite in $\dot{q}$ only. Thus $V(\Delta q, \dot{q})$ is positive definite, and its derivative is negative semidefinite, and the stability of the closed-loop system can be concluded using the Lyapunov method. That is, both Δq and $\dot{q}$ are bounded. To show the asymptotic stability of the equilibrium state, *LaSalle's Invariance Theorem* is used. Since $W(\dot{q}) = 0$ implies $\dot{q} = 0$, Eq. (2.3) reduces to $k_p\Delta q = 0$, and thus the position error Δq is zero. Therefore, the maximum invariant set is the origin, and the equilibrium at the origin is asymptotic stable.

We now consider the PD control problem of a robotic manipulator with n degrees of freedom. From Eq. (1.5) in Chap. 1, the nonlinear robot dynamics is described as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}. \tag{2.9}$$

Note that when the manipulator stops at an equilibrium point, the only torque exerted on the system is the gravitational term $\boldsymbol{g}(\boldsymbol{q})$. Consider the following PD controller with gravity compensation:

$$\boldsymbol{\tau} = -\boldsymbol{K}_p\Delta\boldsymbol{q} - \boldsymbol{K}_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}), \tag{2.10}$$

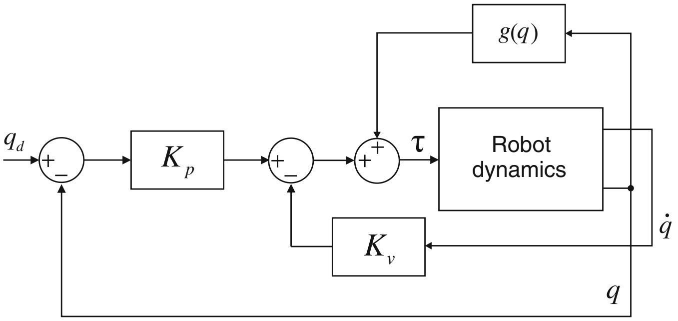


Fig. 2.3 Block diagram of a PD controller with gravity compensation

where $\dot{\boldsymbol{q}} \in \Re^n$ is the joint-space velocity, $\Delta\boldsymbol{q} = \boldsymbol{q} - \boldsymbol{q}_d \in \Re^n$ denotes the joint-space error, where $\boldsymbol{q}_d$ is the desired joint configuration, and $\boldsymbol{K}_p \in \Re^{n\times n}$ and $\boldsymbol{K}_v \in \Re^{n\times n}$ are diagonal and positive definite matrices. A block diagram of the controller described by Eq. (2.10) is illustrated in Fig. 2.3.

The closed-loop equation is obtained by substituting Eq. (2.10) into Eq. (2.9):

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \boldsymbol{K}_p\Delta\boldsymbol{q} + \boldsymbol{K}_v\dot{\boldsymbol{q}} = \boldsymbol{0}, \tag{2.11}$$

where we note that with the compensation of the gravitational term, $\Delta\boldsymbol{q}$ is zero when the manipulator stops.

Next, the inner product of Eq. (2.11) with an output vector $\boldsymbol{y} = \dot{\boldsymbol{q}}$ yields

$$\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{K}_p\Delta\boldsymbol{q} + \dot{\boldsymbol{q}}^T\boldsymbol{K}_v\dot{\boldsymbol{q}} = 0, \tag{2.12}$$

which can be written as

$$\tfrac{\mathrm{d}}{\mathrm{d}t}V(\Delta\boldsymbol{q},\dot{\boldsymbol{q}}) = -W(\dot{\boldsymbol{q}}), \tag{2.13}$$

where

$$V(\Delta\boldsymbol{q},\dot{\boldsymbol{q}}) = \tfrac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \tfrac{1}{2}\Delta\boldsymbol{q}^T\boldsymbol{K}_p\Delta\boldsymbol{q}, \tag{2.14}$$

and

$$W(\dot{\boldsymbol{q}}) = \dot{\boldsymbol{q}}^T\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{K}_v\dot{\boldsymbol{q}}. \tag{2.15}$$

From *Property* 1.1 in Chap. 1, the inertia matrix $\boldsymbol{M}(\boldsymbol{q})$ is positive definite, and hence the function $V(\Delta\boldsymbol{q},\dot{\boldsymbol{q}})$ is a Lyapunov function that is positive definite in $\dot{\boldsymbol{q}}$ and $\Delta\boldsymbol{q}$. From *Property* 1.2, the term $\dot{\boldsymbol{q}}^T\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ in Eq. (2.15) reduces to zero, and hence $W(\dot{\boldsymbol{q}})$ is positive definite in $\dot{\boldsymbol{q}}$. Therefore, the system is stable in the sense that $\dot{\boldsymbol{q}}$ and $\Delta\boldsymbol{q}$ are bounded. To show the global asymptotic stability of the origin, LaSalle's

Invariance Theorem is used. Since $W(\dot{\boldsymbol{q}}) = 0$ implies $\dot{\boldsymbol{q}} = \boldsymbol{0}$, and both $\boldsymbol{M}(\boldsymbol{q})$ and $\boldsymbol{K}_p$ are positive definite matrices, Eq. (2.11) satisfies $\Delta\boldsymbol{q} = \boldsymbol{0}$, and the maximum invariant set is the origin. Therefore, $\dot{\boldsymbol{q}} \to \boldsymbol{0}$ as $t \to \infty$, which also implies $\boldsymbol{q} \to \boldsymbol{q}_d$ from Eq. (2.11).

2.2 Cartesian-Space Setpoint Control

To describe a task for a robotic manipulator, it is convenient to specify the position and orientation of the end effector in Cartesian space. In joint-space control methods, the inverse kinematics problem is used to obtain a desired joint configuration that corresponds to the desired position and orientation of the end effector. To eliminate the requirement of solving the inverse kinematics problem, Cartesian-space control methods can be formulated using feedback information directly in Cartesian space. A coordination transformation in the form of a Jacobian matrix is used in the feedback control law to transform the Cartesian-space feedback information into joint space.

The Cartesian-space setpoint controller is designed as

$$\boldsymbol{\tau} = -\boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_p\Delta\boldsymbol{r} - \boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_v\dot{\boldsymbol{r}} + \boldsymbol{g}(\boldsymbol{q}), \tag{2.16}$$

where $\Delta\boldsymbol{r} = \boldsymbol{r} - \boldsymbol{r}_d \in \Re^n$ and $\boldsymbol{r}_d \in \Re^n$ is a desired position in Cartesian space, $\dot{\boldsymbol{r}} \in \Re^n$ is the task-space velocity of the end effector, and $\boldsymbol{K}_p \in \Re^{n\times n}$ and $\boldsymbol{K}_v \in \Re^{n\times n}$ are diagonal and positive definite matrices. In the above Cartesian-space controller, the Cartesian-space feedback quantities $\boldsymbol{r}$ and $\dot{\boldsymbol{r}}$ are usually computed from the forward kinematics equations. The matrix $\boldsymbol{J}_m(\boldsymbol{q}) \in \Re^{n\times n}$ denotes the Jacobian matrix from joint space to Cartesian space. Using the Jacobian matrix, the feedback information in Cartesian space is directly transformed into the torque input of the robot in joint space. A block diagram of the controller in Eq. (2.16) is illustrated in Fig. 2.4.

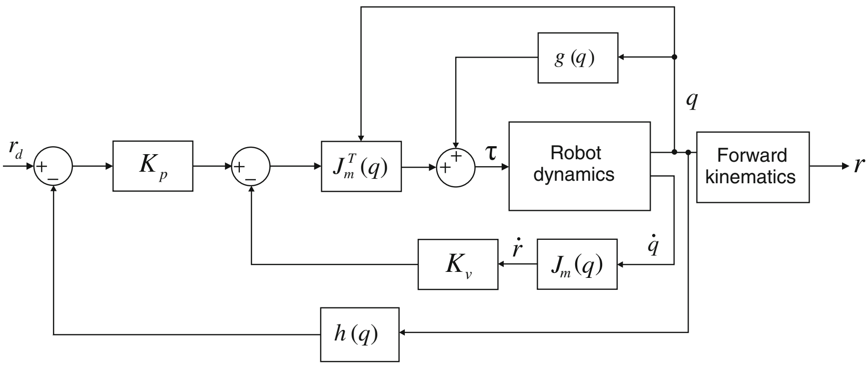
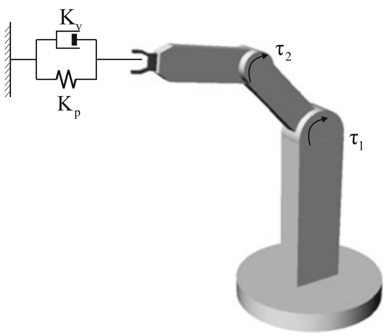


Fig. 2.4 Block diagram of a Cartesian-space setpoint control

Fig. 2.5 Illustration of virtual forces exerted on the end effector



In the setpoint controller, a virtual restoring spring force $\boldsymbol{F}_s = -\boldsymbol{K}_p \Delta \boldsymbol{r}$ and a damping force $\boldsymbol{F}_d = -\boldsymbol{K}_v \dot{\boldsymbol{r}}$ are generated in Cartesian space, as illustrated in Fig. 2.5.

The closed-loop equation is obtained by substituting the controller in Eq. (2.16) into Eq. (2.9) to yield

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_p \Delta\boldsymbol{r} + \boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_v \dot{\boldsymbol{r}} = \boldsymbol{0}. \quad (2.17)$$

with the compensation of the gravitational term, $\boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_p \Delta\boldsymbol{r}$ is zero when the manipulator stops.

Next, the inner product of Eq. (2.17) and the output $\boldsymbol{y} = \dot{\boldsymbol{q}}$ yields

$$\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_p \Delta\boldsymbol{r} + \dot{\boldsymbol{q}}^T\boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_v \dot{\boldsymbol{r}} = 0. \quad (2.18)$$

That is,

$$\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \dot{\boldsymbol{r}}^T\boldsymbol{K}_p \Delta\boldsymbol{r} + \dot{\boldsymbol{r}}^T\boldsymbol{K}_v \dot{\boldsymbol{r}} = 0, \quad (2.19)$$

which can be written as

$$\tfrac{\mathrm{d}}{\mathrm{d}t} V(\Delta\boldsymbol{r},\dot{\boldsymbol{q}}) = -W(\dot{\boldsymbol{r}}), \quad (2.20)$$

where

$$V(\Delta\boldsymbol{r},\dot{\boldsymbol{q}}) = \tfrac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \tfrac{1}{2}\Delta\boldsymbol{r}^T\boldsymbol{K}_p\Delta\boldsymbol{r} \quad (2.21)$$

and

$$W(\dot{\boldsymbol{r}}) = \dot{\boldsymbol{q}}^T\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \dot{\boldsymbol{r}}^T\boldsymbol{K}_v\dot{\boldsymbol{r}} \leq 0. \quad (2.22)$$

From *Property* 1.1 in Chap. 1, $\boldsymbol{M}(\boldsymbol{q})$ is positive definite, and hence the function $V(\Delta\boldsymbol{r},\dot{\boldsymbol{q}})$ is positive definite in $\dot{\boldsymbol{q}}$ and $\Delta\boldsymbol{r}$. From *Property* 1.2, the term $\dot{\boldsymbol{q}}^T\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ in Eq. (2.22) reduces to zero, and hence $W(\dot{\boldsymbol{r}})$ is positive definite in $\dot{\boldsymbol{q}}$. Therefore, the variables $\dot{\boldsymbol{q}}$ and $\Delta \boldsymbol{r}$ are bounded. From LaSalle's Invariance Theorem, $W(\dot{\boldsymbol{r}}) = 0$ implies that $\dot{\boldsymbol{r}} = \mathbf{0}$, which also indicates that $\dot{\boldsymbol{q}} = \mathbf{0}$, since it is assumed that $\boldsymbol{J}_m(\boldsymbol{q})$ is nonsingular in a finite workspace such that $\boldsymbol{J}_m^{-1}(\boldsymbol{q})$ exists. Therefore, Eq. (2.17) satisfies $\boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{K}_p\Delta\boldsymbol{r} = 0$, that is, the position of the end effector in task space converges to the desired steady-state position in task space, in a finite task space where singularities are avoided.

In general, the positive definiteness of $V(\Delta\boldsymbol{r}, \dot{\boldsymbol{q}})$ in $\Delta\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ cannot be concluded even if $\boldsymbol{q}_d$ satisfies $\boldsymbol{r}_d = \boldsymbol{h}(\boldsymbol{q}_d)$, because $\boldsymbol{h}(\cdot)$ is periodic in $\boldsymbol{q}$. However, if the manipulator is working in a finite workspace such that the kinematics mapping between Cartesian space and joint space is one-to-one and the Jacobian matrix $\boldsymbol{J}_m(\boldsymbol{q})$ is nonsingular, then asymptotic stability can be concluded.

2.3 Sensory-Space Setpoint Control

In joint-space control methods, exact knowledge of the robot kinematics is required to solve an inverse kinematics problem to generate a desired position in joint space. In the presence of kinematic uncertainty, it is impossible to derive the desired joint angles from the desired end effector's position. Therefore, the assumption of an exact kinematic model means that the robot is unable to react to changes and uncertainties in its kinematics. For example, when the robot picks up objects of different dimensions or with unknown orientations or gripping points, as illustrated in Fig. 2.6, the overall kinematics from the robot base to the tip of the object changes and is therefore difficult to derive exactly. When the control problem is formulated directly in Cartesian space, the inverse kinematics problem is replaced by the transposed Jacobian matrix from joint space to task space in the control law. However, exact parameter values of the kinematics are still required to construct the Jacobian matrix.

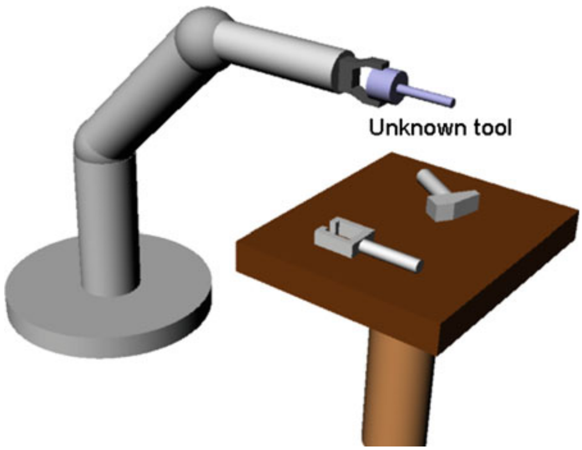


Fig. 2.6 The kinematics model is uncertain when the robot grasps and manipulates an unknown tool

In many modern control applications of robot manipulators, task-space sensory feedback information such as visual feedback is used to monitor the position of the end effector. Using sensory feedback of the position error, the control systems are robust to kinematics uncertainty. Though the position error of the end effector can be measured by task-space sensors, the uncertainty of the Jacobian matrix due to kinematic uncertainty and camera/sensor calibration error poses a challenging dynamic control problem.

2.3.1 Sensory Feedback Control of a Mass-Damper System

To illustrate the concept of sensory feedback control, we first consider a mass–damper system as illustrated in Fig. 2.7. The dynamics equation of the system is obtained from Newton's law as

$$m\ddot{\boldsymbol{r}} + \boldsymbol{B}\dot{\boldsymbol{r}} = \boldsymbol{u}, \tag{2.23}$$

where $\boldsymbol{r} = [r_1, r_2]^T \in \Re^2$ denotes the position of the block in two-dimensional Cartesian space, m denotes the mass of the block, $\boldsymbol{B} = \mathrm{diag}\{b_1, b_2\} \in \Re^{2\times 2}$ denotes the damping matrix, and $\boldsymbol{u} = [u_1, u_2]^T \in \Re^2$ is a vector that represents the control input forces exerted on the system in Cartesian space.

When a sensor is employed to monitor the position or position error of the block for feedback control, the task coordinates are defined in sensory space. When there are rotation and scaling transformations between the sensory space and Cartesian space, the relationship between the Cartesian space and the sensory space is described as

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos(\delta) & \sin(\delta) \\ -\sin(\delta) & \cos(\delta) \end{bmatrix} \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} - \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \tag{2.24}$$

where $\boldsymbol{x} = [x_1, x_2]^T \in \Re^2$ is the position in sensory space, δ is the rotation angle between the Cartesian-space coordinates and the sensory-space coordinates, s_1 and s_2 represent the scaling factors in the two coordinates, and $\boldsymbol{d} = [d_1, d_2]^T \in \Re^2$ denotes the offset between the Cartesian coordinates and the sensory coordinates. An example of a sensor that can be used to measure the position is a camera, as

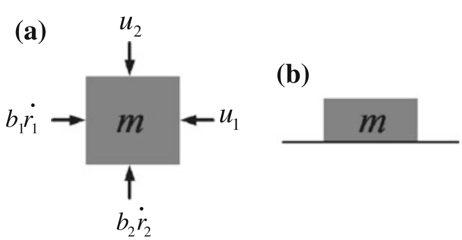


Fig. 2.7 **a** *Top view* of the mass-dmaper system. **b** *Side view* of the mass-damper system

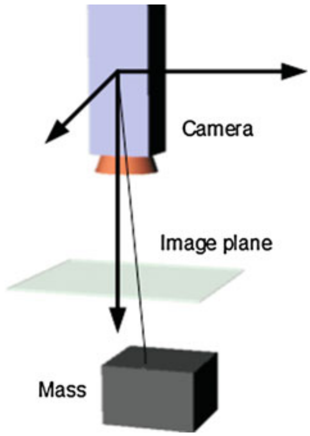


Fig. 2.8 Mass-damper system with an uncalibrated camera

illustrated in Fig. 2.8. The position of the block in Cartesian space is projected onto an image plane, and therefore the sensory space is the image space in pixels. When the camera is not mounted properly, with a rotation error, the relationship between the position in image space and the position in Cartesian space is described by Eq. (2.24).

Differentiating Eq. (2.24) with respect to time, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \cos(\delta) & \sin(\delta) \\ -\sin(\delta) & \cos(\delta) \end{bmatrix} \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} \dot{r}_1 \\ \dot{r}_2 \end{bmatrix} = \boldsymbol{R} \cdot \boldsymbol{S} \begin{bmatrix} \dot{r}_1 \\ \dot{r}_2 \end{bmatrix}, \tag{2.25}$$

where $\boldsymbol{R} \in \Re^{2\times 2}$ and $\boldsymbol{S} \in \Re^{2\times 2}$ are the rotation matrix and the scaling matrix. Let

$$\boldsymbol{J}_s = \boldsymbol{R} \cdot \boldsymbol{S}. \tag{2.26}$$

Then $\boldsymbol{J}_s \in \Re^{2\times 2}$ denotes the sensory Jacobian matrix from Cartesian space to sensory space. The relationship between Cartesian space and sensory space is shown in Fig. 2.9.

In the presence of estimation or modeling errors in the rotation and scaling transformation, the sensory Jacobian matrix $\boldsymbol{J}_s$ is uncertain and is estimated by an approximate Jacobian matrix as $\hat{\boldsymbol{J}}_s$. Using the approximate Jacobian matrix, the sensory-space feedback controller is designed as

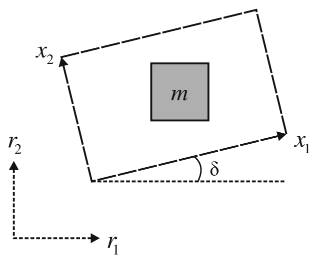


Fig. 2.9 Relationship between Cartesian-space coordinates and sensory-space coordinates

$$u = -\hat{J}_s^T K_v \dot{x} - \hat{J}_s^T K_p \Delta x, \tag{2.27}$$

where $\dot{\boldsymbol{x}} \in \Re^2$ represents the sensory-space velocity, $\Delta \boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_d \in \Re^2$ is the position error, $\boldsymbol{x}_d \in \Re^2$ is the desired position in sensory space, and $\boldsymbol{K}_p \in \Re^{2\times 2}$ and $\boldsymbol{K}_v \in \Re^{2\times 2}$ are diagonal and positive definite matrices. Note that $\Delta \boldsymbol{x}$ represents the relative distance between the actual position and the desired position in sensory space, which can be directly measured by a sensor such as a camera. A virtual restoring spring force $-\boldsymbol{K}_p \Delta \boldsymbol{x}$ and a damping force $-\boldsymbol{K}_v \dot{\boldsymbol{x}}$ are generated in sensory space using the sensory feedback information. The feedback information in sensor space is directly transformed into the input forces in Cartesian space using the transpose of the approximate sensory Jacobian matrix. Since the relative position error is used, the sensory feedback controller in Eq. (2.27) is robust to the modeling and calibration errors. However, the stability of the system may not be ensured in the presence of uncertainty in the sensory Jacobian matrix. To illustrate this, the following example is considered.

Example 2.1 Consider the mass–damper system described by Eq. (2.23) with a rotation angle θ of $\pi/3$ rad. First, a simulation study was performed on the sensory feedback control system with the exact knowledge of the Jacobian matrix. The dynamic parameters of the mass–damper system were set as $m = 1$ kg and $\boldsymbol{B} = \text{diag}\{0.1, 0.1\}$ kg/s, and the parameters of the Jacobian matrix in Eq. (2.24) were set as $s_1 = 0.7$, $s_2 = 0.8$, $d_1 = 0.1$, and $d_2 = 0.2$. The control parameters in Eq. (2.27) were set as $\boldsymbol{K}_p = \text{diag}\{1, 1\}$, and $\boldsymbol{K}_v = \text{diag}\{1.3, 1.3\}$. The initial position was set as $\boldsymbol{x}(0) = [2.1926, 5.0858]^T$, and the desired position was set as $\boldsymbol{x}_d = [0, 0]^T$. Assuming that the exact Jacobian matrix is known, the path of the block and the position error are shown in Fig. 2.10, where the position error converges to zero at steady state.

Next, the rotation angle was estimated as zero, which results in an approximate or uncertain Jacobian matrix. All the other parameters remained unchanged, and the

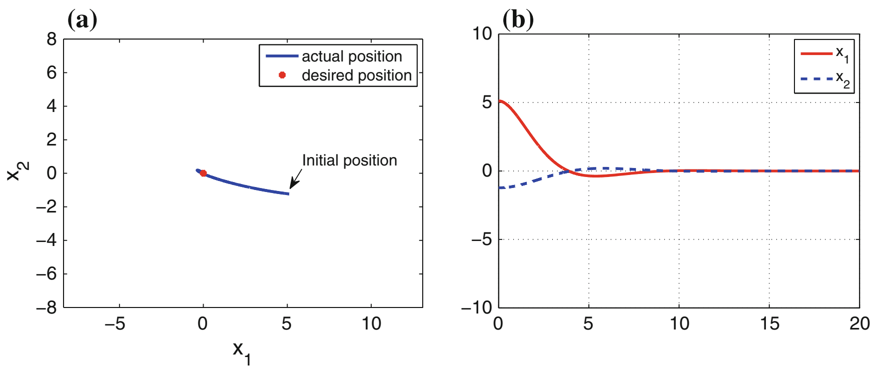


Fig. 2.10 A stable response when the Jacobian matrix is known exactly. **a** Path of block. **b** Position error

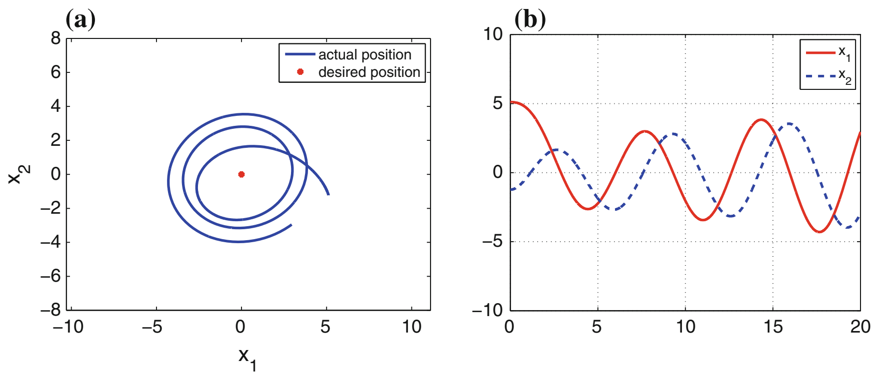


Fig. 2.11 An unstable response when the Jacobian matrix is uncertain. **a** Path of block. **b** Position error

simulation results in Fig. 2.11 show the path of the block and the position error. The closed-loop system is now unstable. □

To analyze the stability of a mass–spring–damper system with uncertain Jacobian or transformation from sensory space to Cartesian space, we substitute Eq. (2.27) into Eq. (2.23) to obtain the following closed-loop equation:

$$\boldsymbol{M}\ddot{\boldsymbol{r}} + \boldsymbol{B}\dot{\boldsymbol{r}} + \hat{\boldsymbol{J}}_s^T \boldsymbol{K}_v \dot{\boldsymbol{x}} + \hat{\boldsymbol{J}}_s^T \boldsymbol{K}_p \Delta \boldsymbol{x} = \boldsymbol{0}. \tag{2.28}$$

The above closed-loop equation describes the motion of the system in Cartesian space with feedback control information obtained in sensory space.

Next, a new output vector is defined as

$$\boldsymbol{y} = \dot{\boldsymbol{r}} + \alpha \hat{\boldsymbol{J}}_s^{-1} \Delta \boldsymbol{x}, \tag{2.29}$$

where α is a positive constant and $\hat{\boldsymbol{J}}_s^{-1}$ is the inverse of the approximate Jacobian matrix. The above output consists of a combination of the velocity in Cartesian space and the position error in sensory space that is being transformed to Cartesian space using the inverse approximate Jacobian matrix. The inner product od this output and the closed-loop equation will eventually lead to a Lyapunov-like function that is positive definite in the task-space error and velocity, and to a derivative of the Lyapunov-like function that is negative definite in the task-space error and velocity. Since the output now contains sensory feedback position error with approximate Jacobian, it can also be used in designing PID setpoint controllers and adaptive setpoint controllers with approximate Jacobian matrix.

The inner product of Eq. (2.28) with the output defined in Eq. (2.29) now yields

$$(\dot{\boldsymbol{r}} + \alpha \hat{\boldsymbol{J}}_s^{-1} \Delta \boldsymbol{x})^T (\boldsymbol{M}\ddot{\boldsymbol{r}} + \boldsymbol{B}\dot{\boldsymbol{r}} + \hat{\boldsymbol{J}}_s^T \boldsymbol{K}_v \dot{\boldsymbol{x}} + \hat{\boldsymbol{J}}_s^T \boldsymbol{K}_p \Delta \boldsymbol{x}) = 0. \tag{2.30}$$

That is,

$$\begin{aligned}&\dot{\boldsymbol{r}}^T\boldsymbol{M}\ddot{\boldsymbol{r}}+\dot{\boldsymbol{r}}^T\boldsymbol{B}\dot{\boldsymbol{r}}+\dot{\boldsymbol{r}}^T\hat{\boldsymbol{J}}_s^T\boldsymbol{K}_v\dot{\boldsymbol{x}}+\dot{\boldsymbol{r}}^T\hat{\boldsymbol{J}}_s^T\boldsymbol{K}_p\Delta\boldsymbol{x}+\alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\ddot{\boldsymbol{r}}\\&+\alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{B}\dot{\boldsymbol{r}}+\alpha\Delta\boldsymbol{x}^T\boldsymbol{K}_v\dot{\boldsymbol{x}}+\alpha\Delta\boldsymbol{x}^T\boldsymbol{K}_p\Delta\boldsymbol{x}=0,\end{aligned}\tag{2.31}$$

where $\hat{\boldsymbol{J}}_s^{-T}$ represents the transpose of $\hat{\boldsymbol{J}}_s^{-1}$. Equation (2.31) can be written as

$$\begin{aligned}&\tfrac{\mathrm{d}}{\mathrm{d}t}\left\{\tfrac{1}{2}\dot{\boldsymbol{r}}^T\boldsymbol{M}\dot{\boldsymbol{r}}+\alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\dot{\boldsymbol{r}}+\tfrac{1}{2}\Delta\boldsymbol{x}^T(\boldsymbol{K}_p+\alpha\boldsymbol{K}_v)\Delta\boldsymbol{x}\right\}\\&+\dot{\boldsymbol{r}}^T\boldsymbol{B}\dot{\boldsymbol{r}}+\dot{\boldsymbol{r}}^T\hat{\boldsymbol{J}}_s^T\boldsymbol{K}_v\dot{\boldsymbol{x}}+\dot{\boldsymbol{r}}^T\hat{\boldsymbol{J}}_s^T\boldsymbol{K}_p\Delta\boldsymbol{x}-\alpha\dot{\boldsymbol{x}}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\dot{\boldsymbol{r}}\\&-\dot{\boldsymbol{r}}^T\boldsymbol{J}_s^T\boldsymbol{K}_p\Delta\boldsymbol{x}+\alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{B}\dot{\boldsymbol{r}}+\alpha\Delta\boldsymbol{x}^T\boldsymbol{K}_p\Delta\boldsymbol{x}=0.\end{aligned}\tag{2.32}$$

Therefore, we have

$$\tfrac{\mathrm{d}}{\mathrm{d}t}V(\Delta\boldsymbol{x},\dot{\boldsymbol{r}})=-W(\Delta\boldsymbol{x},\dot{\boldsymbol{r}}),\tag{2.33}$$

where

$$V(\Delta\boldsymbol{x},\dot{\boldsymbol{r}})=\tfrac{1}{2}\dot{\boldsymbol{r}}^T\boldsymbol{M}\dot{\boldsymbol{r}}+\alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\dot{\boldsymbol{r}}+\tfrac{1}{2}\Delta\boldsymbol{x}^T(\boldsymbol{K}_p+\alpha\boldsymbol{K}_v)\Delta\boldsymbol{x},\tag{2.34}$$

and

$$\begin{aligned}W(\Delta\boldsymbol{x},\dot{\boldsymbol{r}})=&\,\dot{\boldsymbol{r}}^T\boldsymbol{B}\dot{\boldsymbol{r}}+\dot{\boldsymbol{r}}^T\hat{\boldsymbol{J}}_s^T\boldsymbol{K}_v\dot{\boldsymbol{x}}+\dot{\boldsymbol{r}}^T\hat{\boldsymbol{J}}_s^T\boldsymbol{K}_p\Delta\boldsymbol{x}-\alpha\dot{\boldsymbol{x}}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\dot{\boldsymbol{r}}\\&-\dot{\boldsymbol{r}}^T\boldsymbol{J}_s^T\boldsymbol{K}_p\Delta\boldsymbol{x}+\alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{B}\dot{\boldsymbol{r}}+\alpha\Delta\boldsymbol{x}^T\boldsymbol{K}_p\Delta\boldsymbol{x}.\end{aligned}\tag{2.35}$$

With output that consists of a combination of the velocity and the position error, the scalar function $V(\Delta\boldsymbol{x},\dot{\boldsymbol{r}})$ in Eq. (2.34) now contains a coupling term between the velocity and position error, in addition to the kinematic energy in Cartesian space and virtual potential energy in sensory space. To show that $V(\Delta\boldsymbol{x},\dot{\boldsymbol{r}})$ is positive definite in $\dot{\boldsymbol{r}}$ and $\Delta\boldsymbol{x}$, we express Eq. (2.34) as

$$\begin{aligned}V(\Delta\boldsymbol{x},\dot{\boldsymbol{x}})&=\tfrac{1}{2}\dot{\boldsymbol{x}}^T\boldsymbol{J}_s^{-T}\boldsymbol{M}\boldsymbol{J}_s^{-1}\dot{\boldsymbol{x}}+\alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\boldsymbol{J}_s^{-1}\dot{\boldsymbol{x}}+\tfrac{1}{2}\Delta\boldsymbol{x}^T(\boldsymbol{K}_p+\alpha\boldsymbol{K}_v)\Delta\boldsymbol{x}\\&=\tfrac{1}{4}\dot{\boldsymbol{x}}^T\boldsymbol{M}_x\dot{\boldsymbol{x}}+\tfrac{1}{4}(\dot{\boldsymbol{x}}+2\alpha\boldsymbol{J}_s\hat{\boldsymbol{J}}_s^{-1}\Delta\boldsymbol{x})^T\boldsymbol{M}_x(\dot{\boldsymbol{x}}+2\alpha\boldsymbol{J}_s\hat{\boldsymbol{J}}_s^{-1}\Delta\boldsymbol{x})\\&\quad+\tfrac{1}{2}\Delta\boldsymbol{x}^T(\boldsymbol{K}_p+\alpha\boldsymbol{K}_v-2\alpha^2\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\hat{\boldsymbol{J}}_s^{-1})\Delta\boldsymbol{x},\end{aligned}\tag{2.36}$$

where $\boldsymbol{M}_x\triangleq\boldsymbol{J}_s^{-T}\boldsymbol{M}\boldsymbol{J}_s^{-1}$ and $\dot{\boldsymbol{r}}=\boldsymbol{J}_s^{-1}\dot{\boldsymbol{x}}$, as seen from Eq. (2.25). Let $\lambda_m\triangleq\lambda_{\max}[\hat{\boldsymbol{J}}_s^{-T}\boldsymbol{M}\hat{\boldsymbol{J}}_s^{-1}]$, and let $\lambda_{\max}[\cdot]$ denote the maximum eigenvalue of the matrix. Then $V(\Delta\boldsymbol{x},\dot{\boldsymbol{x}})$ is positive definite in $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$ when

$$k_{vi}-2\alpha\lambda_m>0,\tag{2.37}$$

where k_{vi} denotes the ith diagonal element of $\boldsymbol{K}_v$.

The output also leads to a derivative of $V(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}})$ that contains various terms that are dependent on $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$, as seen from Eq. (2.35). Next, we proceed to show that the derivative of $V(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}})$ is negative definite in $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$ with proper choices of control gains. Clearly, this is equivalent to showing that $W(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}})$ is positive definite in $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$.

Equation (2.35) is first expressed as

$$\begin{aligned} W(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}}) = {} & \dot{\boldsymbol{x}}^T \boldsymbol{J}_s^{-T} \boldsymbol{B} \boldsymbol{J}_s^{-1} \dot{\boldsymbol{x}} + \dot{\boldsymbol{x}}^T \boldsymbol{J}_s^{-T} \hat{\boldsymbol{J}}_s^T \boldsymbol{K}_v \dot{\boldsymbol{x}} + \dot{\boldsymbol{x}}^T \boldsymbol{J}_s^{-T} \hat{\boldsymbol{J}}_s^T \boldsymbol{K}_p \Delta\boldsymbol{x} \\ & - \alpha \dot{\boldsymbol{x}}^T \hat{\boldsymbol{J}}_s^{-T} \boldsymbol{M} \boldsymbol{J}_s^{-1} \dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}^T \boldsymbol{K}_p \Delta\boldsymbol{x} + \alpha \Delta\boldsymbol{x}^T \hat{\boldsymbol{J}}_s^{-T} \boldsymbol{B} \boldsymbol{J}_s^{-1} \dot{\boldsymbol{x}} \\ & + \alpha \Delta\boldsymbol{x}^T \boldsymbol{K}_p \Delta\boldsymbol{x}. \end{aligned} \tag{2.38}$$

To analyze the effects of an uncertain Jacobian on the stability of the system, we define a matrix Δ_J to represent the deviation or estimation error between the actual Jacobian matrix and estimated Jacobian matrix as follows:

$$\Delta_J = \boldsymbol{I}_2 - \boldsymbol{J}_s^{-T} \hat{\boldsymbol{J}}_s^T, \tag{2.39}$$

where $\boldsymbol{I}_2 \in \Re^{2\times 2}$ is the 2×2 identity matrix. Using Δ_J and letting $\boldsymbol{B}_x \triangleq \boldsymbol{J}_s^{-T} \boldsymbol{B} \boldsymbol{J}_s^{-1}$, Eq. (2.38) can be written as

$$\begin{aligned} W(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}}) = {} & \dot{\boldsymbol{x}}^T \boldsymbol{B}_x \dot{\boldsymbol{x}} + \dot{\boldsymbol{x}}^T (\boldsymbol{I}_2 - \Delta_J) \boldsymbol{K}_v \dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}^T \Delta_J \boldsymbol{K}_p \Delta\boldsymbol{x} \\ & + \alpha \Delta\boldsymbol{x}^T \boldsymbol{K}_p \Delta\boldsymbol{x} - \alpha h(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}}), \end{aligned} \tag{2.40}$$

where $h(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}}) = \dot{\boldsymbol{x}}^T \hat{\boldsymbol{J}}_s^{-T} \boldsymbol{M} \boldsymbol{J}_s^{-1} \dot{\boldsymbol{x}} - \Delta\boldsymbol{x}^T \hat{\boldsymbol{J}}_s^{-T} \boldsymbol{B} \boldsymbol{J}_s^{-1} \dot{\boldsymbol{x}}$ contains the terms that are independent of the proportional control gain $\boldsymbol{K}_p$ and the derivative control gain $\boldsymbol{K}_v$. Since $h(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}})$ is dependent on $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$, there exist positive constants c_0, c_1 such that

$$-\alpha h(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}}) \geq -\alpha c_0 ||\dot{\boldsymbol{x}}||^2 - \alpha c_1 ||\Delta\boldsymbol{x}||^2. \tag{2.41}$$

Substituting the above inequality into Eq. (2.40) yields

$$\begin{aligned} W(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}}) \geq {} & \dot{\boldsymbol{x}}^T (\boldsymbol{B}_x + (\boldsymbol{I}_2 - \Delta_J) \boldsymbol{K}_v - \alpha c_0 \boldsymbol{I}_2) \dot{\boldsymbol{x}} \\ & - \dot{\boldsymbol{x}}^T \Delta_J \boldsymbol{K}_p \Delta\boldsymbol{x} + \alpha \Delta\boldsymbol{x}^T (\boldsymbol{K}_p - c_1 \boldsymbol{I}_2) \Delta\boldsymbol{x}. \end{aligned} \tag{2.42}$$

Let $\hat{\boldsymbol{J}}_s$ be chosen such that $||\Delta_J|| = ||\boldsymbol{I}_2 - \boldsymbol{J}_s^{-T} \hat{\boldsymbol{J}}_s^T|| \leq p$, where p is a positive constant. We proceed to derive a sufficient condition to guarantee that $W(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}})$ is positive definite in $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$ or that $-W(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}})$ is negative definite in $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$. From Eq. (2.42), we obtain

$$\begin{aligned} W(\Delta\boldsymbol{x}, \dot{\boldsymbol{x}}) \geq {} & (\lambda_{\min}[\boldsymbol{K}_v] - p\lambda_{\max}[\boldsymbol{K}_v] - \alpha c_0) \|\dot{\boldsymbol{x}}\|^2 - p\lambda_{\max}[\boldsymbol{K}_p] ||\Delta\boldsymbol{x}|| \cdot ||\dot{\boldsymbol{x}}|| \\ & + \alpha(\lambda_{\min}[\boldsymbol{K}_p] - c_1) ||\Delta\boldsymbol{x}||^2. \end{aligned} \tag{2.43}$$

Note that $-p\lambda_{\max}[\boldsymbol{K}_p]||\Delta\boldsymbol{x}||\cdot||\dot{\boldsymbol{x}}|| \geq -\frac{1}{2}p\lambda_{\max}[\boldsymbol{K}_p](||\Delta\boldsymbol{x}||^2 + ||\dot{\boldsymbol{x}}||^2)$. Hence,

$$W(\Delta\boldsymbol{x},\dot{\boldsymbol{x}}) \geq \lambda_{\max}[\boldsymbol{K}_v]l_1\|\dot{\boldsymbol{x}}\|^2 + a\lambda_{\max}[\boldsymbol{K}_v]l_2||\Delta\boldsymbol{x}||^2, \tag{2.44}$$

where $l_1 = \frac{\lambda_{\min}[\boldsymbol{K}_v]}{\lambda_{\max}[\boldsymbol{K}_v]} - \frac{\alpha c_0}{\lambda_{\max}[\boldsymbol{K}_v]} - p(1+\frac{a}{2})$, $l_2 = \alpha\frac{\lambda_{\min}[\boldsymbol{K}_p]}{\lambda_{\max}[\boldsymbol{K}_p]} - \frac{\alpha c_1}{\lambda_{\max}[\boldsymbol{K}_p]} - \frac{p}{2}$, and $a = \frac{\lambda_{\max}[\boldsymbol{K}_p]}{\lambda_{\max}[\boldsymbol{K}_v]}$.

When the control parameters are chosen such that

$$\min\left\{\frac{2}{2+a}\left(\frac{\lambda_{\min}[\boldsymbol{K}_v]}{\lambda_{\max}[\boldsymbol{K}_v]} - \frac{\alpha c_0}{\lambda_{\max}[\boldsymbol{K}_v]}\right), 2\alpha\left(\frac{\lambda_{\min}[\boldsymbol{K}_p]}{\lambda_{\max}[\boldsymbol{K}_p]} - \frac{c_1}{\lambda_{\max}[\boldsymbol{K}_p]}\right)\right\} > p, \tag{2.45}$$

and $\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are also chosen large enough that

$$\begin{aligned}\frac{\lambda_{\min}[\boldsymbol{K}_v]}{\lambda_{\max}[\boldsymbol{K}_v]} - \frac{\alpha c_0}{\lambda_{\max}[\boldsymbol{K}_v]} > 0,\\ \frac{\lambda_{\min}[\boldsymbol{K}_p]}{\lambda_{\max}[\boldsymbol{K}_p]} - \frac{c_1}{\lambda_{\max}[\boldsymbol{K}_p]} > 0,\end{aligned} \tag{2.46}$$

then $W(\Delta\boldsymbol{x},\dot{\boldsymbol{x}}) > 0$. We can now state the following Theorem:

Theorem 2.1 *The task-space sensory feedback controller* (2.27) *for the mass–damper system* (2.23) *gives rise to the convergence of the task-space error and velocity such that* $\Delta\boldsymbol{x} \to \boldsymbol{0}$ *and* $\dot{\boldsymbol{x}} \to \boldsymbol{0}$ *as* $t \to \infty$ *if the feedback gains* $\boldsymbol{K}_p$ *and* $\boldsymbol{K}_v$ *are chosen to satisfy conditions* (2.37), (2.45)*, and* (2.46)*, and* $\hat{\boldsymbol{J}}_s$ *is chosen such that*

$$\|\boldsymbol{I}_2 - \boldsymbol{J}_s^{-T}\hat{\boldsymbol{J}}_s^T\| \leq p. \tag{2.47}$$

Proof When conditions (2.37), (2.45), and (2.46) are satisfied and $\|\boldsymbol{I}_2 - \boldsymbol{J}_s^{-T}\hat{\boldsymbol{J}}_s^T\| \leq p$, we have that $V(\Delta\boldsymbol{x},\dot{\boldsymbol{x}})$ and $W(\Delta\boldsymbol{x},\dot{\boldsymbol{x}})$ are both positive definite in $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$. Therefore, $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$ are bounded. The boundedness of $\dot{\boldsymbol{x}}$ ensures the boundedness of $\dot{\boldsymbol{r}}$, since it is assumed that $\boldsymbol{J}_S^{-1}$ exists. From Eq. (2.28), it is seen that $\ddot{\boldsymbol{r}}$ is bounded, which also implies that $\ddot{\boldsymbol{x}}$ is bounded, since $\ddot{\boldsymbol{x}} = \boldsymbol{J}_s\ddot{\boldsymbol{r}}$. Since $\ddot{\boldsymbol{x}}$ and $\dot{\boldsymbol{x}}$ are bounded, both $\dot{\boldsymbol{x}}$ and $\Delta\boldsymbol{x}$ are uniformly continuous. From Eqs. (2.33) and (2.44), it is seen that $\dot{\boldsymbol{x}}, \Delta\boldsymbol{x} \in L_2(0,\infty)$. Therefore, $\dot{\boldsymbol{x}} \to \boldsymbol{0}$ and $\Delta\boldsymbol{x} \to \boldsymbol{0}$ as $t \to \infty$. △△△

From Eq. (2.24), note that

$$\Delta\boldsymbol{x} = \boldsymbol{J}_s\Delta\boldsymbol{r}, \tag{2.48}$$

where $\Delta\boldsymbol{r} = \boldsymbol{r} - \boldsymbol{r}_d$, and $\boldsymbol{r}_d$ is the desired position in Cartesian space that corresponds to the desired position x_d in sensory space. The desired position $\boldsymbol{r}_d$ is unknown, since the sensory Jacobian matrix is uncertain. Similarly, the relationship between $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{r}}$ is described in Eq. (2.25) as

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_s\dot{\boldsymbol{r}}. \tag{2.49}$$

Since $\boldsymbol{J}_s$ as defined in Eq. (2.25) is always of full rank, $V(\Delta\boldsymbol{x}, \dot{\boldsymbol{r}})$ and $W(\Delta\boldsymbol{x}, \dot{\boldsymbol{r}})$ are also positive definite in $\dot{\boldsymbol{r}}$ and $\Delta\boldsymbol{r}$, and thus asymptotic stability can also be concluded using the Lyapunov method directly.

To gain further insight into the condition (2.45), we let $\boldsymbol{K}_v = k_v\boldsymbol{I}_2$, $\boldsymbol{K}_p = ak_v\boldsymbol{I}_2$. Then condition (2.45) becomes

$$p < \min\left\{\tfrac{2}{a+2}\left(1 - \tfrac{\alpha c_0}{k_v}\right), 2\alpha\left(1 - \tfrac{c_1}{ak_v}\right)\right\}. \tag{2.50}$$

When k_v is so large that $\frac{\alpha c_0}{k_v}$ and $\frac{c_1}{ak_v}$ are negligible, condition (2.50) reduces to

$$p < \min\left\{\tfrac{2}{a+2}, 2\alpha\right\}. \tag{2.51}$$

From condition (2.51), when $\alpha \geq \frac{1}{2}$, we have

$$p < \tfrac{2}{a+2} < 1. \tag{2.52}$$

This implies that the sufficient allowable bound for the Jacobian uncertainty is less than 1 and that a can be chosen small enough to guarantee the stability of the system if p is large. If p is small, then from condition (2.51), α can be small, because

$$p < 2\alpha, \qquad 0 < a \leq \bar{a}, \tag{2.53}$$
$$p < \tfrac{2}{a+2}, \qquad \bar{a} < a < \infty, \tag{2.54}$$

where $\alpha < \frac{1}{2}$ and $\bar{a} = \frac{1}{\alpha} - 2$. Hence, a larger range of a can be chosen for a smaller p, as illustrated in Fig. 2.12.

Therefore, condition (2.51) implies that if a is increased, then the allowable bound p of the Jacobian uncertainty $\Delta_J = \boldsymbol{I}_2 - \boldsymbol{J}_s^{-T}\hat{\boldsymbol{J}}_s^T$ is reduced. Hence, a should be kept small so that the allowable bound of the Jacobian uncertainty is larger. This can be done easily by increasing $\boldsymbol{K}_v$.

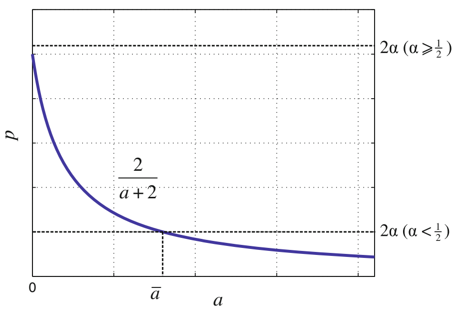


Fig. 2.12 Variation of p with a (k_v is very large)

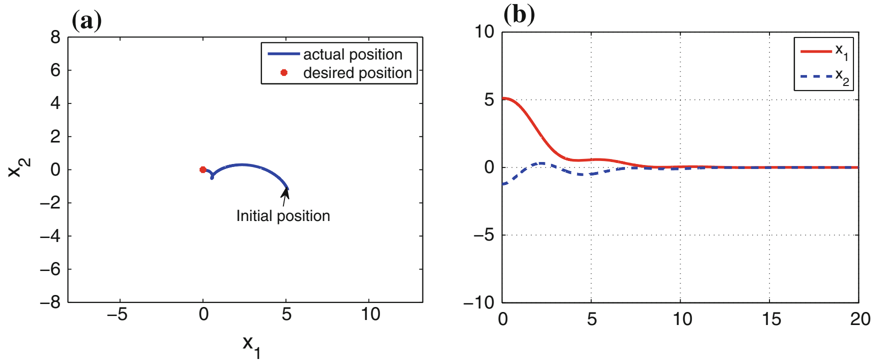


Fig. 2.13 The closed-loop system is stable after the control gains are adjusted. **a** Path of block. **b** Position error

Using the results illustrated in Fig. 2.12, the control parameter was adjusted to reduce a by increasing from $\boldsymbol{K}_v = \text{diag}\{1.3, 1.3\}$ to $\boldsymbol{K}_v = \text{diag}\{2.4, 2.4\}$. The system was stabilized as shown in Fig. 2.13.

2.3.2 Sensory Feedback Control of a Robot Manipulator

Consider now a robot manipulator with n degrees of freedom for which the position or position error of the end effector is measured by a sensor. Let $\boldsymbol{r} \in \Re^p$ represent the position of the end effector in Cartesian space, and let $\boldsymbol{x} \in \Re^m$ represent the position obtained by the sensor in sensory space. The velocity vector $\dot{\boldsymbol{x}}$ in sensory space is related to $\dot{\boldsymbol{r}}$ in Cartesian space by

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_s(\boldsymbol{r})\dot{\boldsymbol{r}}, \tag{2.55}$$

where $\boldsymbol{J}_s(\boldsymbol{r}) \in \Re^{m \times p}$ is the sensory Jacobian matrix, as illustrated in Fig. 2.14. From Eq. (1.24) in Chap. 1, the velocity vector $\dot{\boldsymbol{r}}$ in Cartesian space is related to $\dot{\boldsymbol{q}}$ in joint space by

$$\dot{\boldsymbol{r}} = \boldsymbol{J}_m(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{2.56}$$

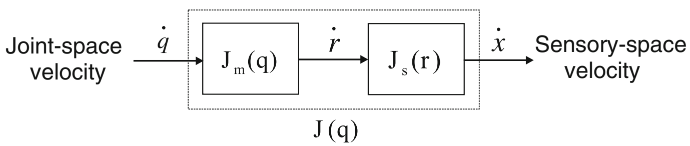


Fig. 2.14 The sensory Jacobian matrix is denoted by $\boldsymbol{J}_s(\boldsymbol{r})$, and the manipulator Jacobian matrix is denoted by $\boldsymbol{J}_m(\boldsymbol{q})$

where $\boldsymbol{J}_m(\boldsymbol{q}) \in \Re^{p \times n}$ is the manipulator Jacobian matrix of the mapping from joint space to Cartesian space.

The overall relationship between the velocity vector $\dot{\boldsymbol{x}}$ in sensory space and $\dot{\boldsymbol{q}}$ in joint space is therefore given by

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_s(\boldsymbol{r})\dot{\boldsymbol{r}} = \boldsymbol{J}_s(\boldsymbol{r})\boldsymbol{J}_m(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{2.57}$$

and the overall Jacobian of the mapping from joint space to sensory space is defined as

$$\boldsymbol{J}(\boldsymbol{q}) = \boldsymbol{J}_s(\boldsymbol{r})\boldsymbol{J}_m(\boldsymbol{q}). \tag{2.58}$$

In the presence of uncertain kinematics or calibration errors, either the manipulator Jacobian matrix or the sensory Jacobian matrix is uncertain, and the overall Jacobian matrix is estimated as

$$\hat{\boldsymbol{J}}(\boldsymbol{q}) = \hat{\boldsymbol{J}}_s(\boldsymbol{r})\hat{\boldsymbol{J}}_m(\boldsymbol{q}), \tag{2.59}$$

where $\hat{\boldsymbol{J}}(\boldsymbol{q}) \in \Re^{m \times n}$ is an approximate matrix for $\boldsymbol{J}(\boldsymbol{q}) \in \Re^{m \times n}$.

When cameras are used to monitor the position of the end effector, the task coordinates are defined as image coordinates. The pinhole camera model can be used to represent the mapping from Cartesian space to image space. From Eq. (1.42) in Chap. 1, the relationship between image space and Cartesian space is represented by

$$\dot{\boldsymbol{x}} = \boldsymbol{Z}^{-1}(\boldsymbol{q})\boldsymbol{L}(\boldsymbol{x})\dot{\boldsymbol{r}}, \tag{2.60}$$

where

$$\boldsymbol{Z}^{-1}(\boldsymbol{q}) = \begin{bmatrix} \frac{1}{z_1(\boldsymbol{q})} & \boldsymbol{0} & \cdots & 0 \\ \boldsymbol{0} & \frac{1}{z_2(\boldsymbol{q})} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \frac{1}{z_m(\boldsymbol{q})} \end{bmatrix} \tag{2.61}$$

is a diagonal matrix that contains the depth information of the feature points with respect to the camera image frame, and $\boldsymbol{L}(\boldsymbol{x}) \in \Re^{m \times p}$ is a Jacobian matrix. The overall matrix $\boldsymbol{Z}^{-1}(\boldsymbol{q})\boldsymbol{L}(\boldsymbol{x})$ is the image Jacobian matrix or sensory Jacobian matrix.

The velocity of the image feature is then related to the joint-space velocity as

$$\dot{\boldsymbol{x}} = \underbrace{\boldsymbol{Z}^{-1}(\boldsymbol{q})\boldsymbol{L}(\boldsymbol{x})}_{\boldsymbol{J}_s}\boldsymbol{J}_m(\boldsymbol{q})\dot{\boldsymbol{q}}. \tag{2.62}$$

In the presence of uncertain kinematics and uncalibrated camera, the estimated image-space velocity of the end effector $\dot{\hat{\boldsymbol{x}}}$ is obtained as

$$\dot{\hat{\boldsymbol{x}}} = \hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q})\hat{\boldsymbol{L}}(\boldsymbol{x})\hat{\boldsymbol{J}}_m(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{2.63}$$

where $\hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q})$ is the approximate depth matrix, and $\hat{\boldsymbol{L}}(\boldsymbol{x})$ and $\hat{\boldsymbol{J}}_m(\boldsymbol{q})$ represent the approximate matrices of $\boldsymbol{L}(\boldsymbol{x})$ and $\boldsymbol{J}_m(\boldsymbol{q})$ respectively.

When a position sensor such as a PSD camera is used to measure the end effector's position directly in Cartesian space, the task coordinates are defined as the Cartesian coordinates. In this case,

$$\boldsymbol{J}(\boldsymbol{q}) = \boldsymbol{J}_m(\boldsymbol{q}). \tag{2.64}$$

If the kinematics is uncertain, the Jacobian matrix is also uncertain, which is specified as

$$\hat{\boldsymbol{J}}(\boldsymbol{q}) = \hat{\boldsymbol{J}}_m(\boldsymbol{q}), \tag{2.65}$$

where $\hat{\boldsymbol{J}}_m(\boldsymbol{q})$ is an estimated matrix of $\boldsymbol{J}_m(\boldsymbol{q})$.

Therefore, the task-space coordinates include the image coordinates and the Cartesian coordinates. In this section, we consider a setpoint control for a nonredundant robot with $m = n$ and a sensory feedback controller with approximate Jacobian matrix designed as

$$\boldsymbol{\tau} = -\hat{\boldsymbol{J}}^T(\boldsymbol{q})(\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}) + \boldsymbol{K}_v\dot{\boldsymbol{x}}) + \boldsymbol{g}(\boldsymbol{q}), \tag{2.66}$$

where $\dot{\boldsymbol{x}} \in \Re^n$ is the velocity of the end effector in sensory space, $\hat{\boldsymbol{J}}(\boldsymbol{q}) \in \Re^{n\times n}$ is the uncertain Jacobian matrix from joint-space to sensory space, $\boldsymbol{s}(\cdot) \in \Re^n$ is a saturated function to be defined, and $\boldsymbol{K}_p \in \Re^{n\times n}$ and $\boldsymbol{K}_v \in \Re^{n\times n}$ are diagonal and positive definite matrices. A block diagram for the controller in Eq. (2.66) is shown in Fig. 2.15.

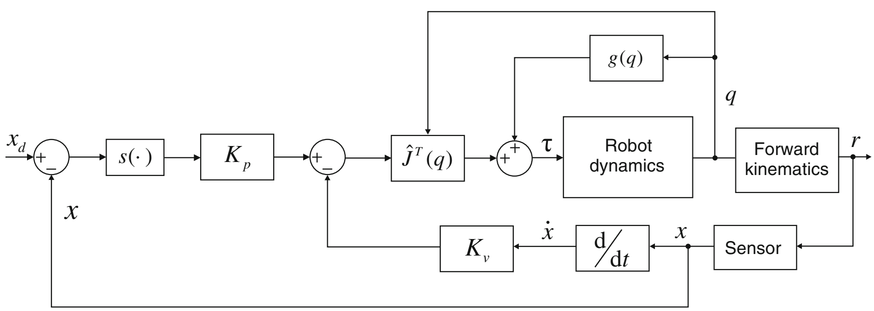


Fig. 2.15 Block diagram of the task-space sensory feedback control

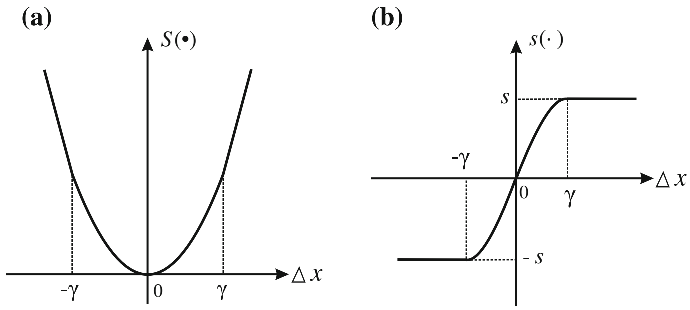


Fig. 2.16 **a** Quasinatural potential: $S(\Delta \boldsymbol{x})$. **b** Derivative of $S(\Delta \boldsymbol{x})$: $s(\Delta \boldsymbol{x})$

The approximate Jacobian matrix $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ is also chosen so that

$$\|\boldsymbol{I}_n - \boldsymbol{J}^{-T}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})\| \leq \bar{p}, \tag{2.67}$$

where $\boldsymbol{I}_n \in \Re^{n\times n}$ is the identity matrix, and $\bar{p}$ is a positive constant to be defined later.

First, let us define a scalar potential function $S_i(\Delta x_i)$ and its derivative $s_i(\Delta x_i)$, where Δx_i denotes the ith diagonal element of $\Delta \boldsymbol{x}$, as shown in Fig. 2.16, with the following properties:

1. $S_i(\Delta x_i) > 0$ for $\Delta x_i \neq 0$ and $S_i(0) = 0$.
2. $S_i(\Delta x_i)$ is twice continuously differentiable, and the derivative $s_i(\Delta x_i) = \frac{\mathrm{d}S_i(\Delta x_i)}{\mathrm{d}\Delta x_i}$ is strictly increasing in Δx_i for $|\Delta x_i| < \gamma_i$ with some γ_i and is saturated for $|\Delta x_i| \geq \gamma_i$, i.e., $s_i(\Delta x_i) = \pm s_i$ for $\Delta x_i \geq \gamma_i$ and $\Delta x_i \leq -\gamma_i$ respectively.
3. There are constants $\bar{c}_i > 0$, $d_i > 0$, $\bar{d}_i(> d_i) > 0$ such that

$$\bar{d}_i s_i^2(\Delta x_i) \geq \Delta x_i s_i(\Delta x_i) \geq d_i s_i^2(\Delta x_i) > 0, \quad S_i(\Delta x_i) \geq \bar{c}_i s_i^2(\Delta x_i), \tag{2.68}$$

for $\Delta x_i \neq 0$.

The saturation function is used to bound the position error $\Delta \boldsymbol{x}$ in the sensory feedback controller described by Eq. (2.66). Its role is quite similar to visually guided reaching tasks of human beings in the sense that when a person reaches for an object, only the directional information is required at the beginning, when the position error is large. That is, the magnitude is not important at the beginning stage and thus can be saturated, and only the directional information is focused on. The magnitude of the position error is monitored only when the target is near.

Next, the output is defined as

$$\boldsymbol{y} = \dot{\boldsymbol{q}} + \alpha\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\boldsymbol{s}(\Delta \boldsymbol{x}). \tag{2.69}$$

where joint-space velocity and sensory task-space position error are used, and an inverse of the approximate Jacobian matrix is used to transform the sensory feedback error to joint space.

Substituting Eq. (2.66) into Eq. (2.9), we have,

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})+\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}}+\hat{\boldsymbol{J}}^{T}(\boldsymbol{q})(\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x})+\boldsymbol{K}_v\dot{\boldsymbol{x}})=\boldsymbol{0}. \tag{2.70}$$

Similarly, the inner product of the output $\boldsymbol{y}$ described by Eqs. (2.69) and (2.70) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}})+W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}})=0, \tag{2.71}$$

where

$$\begin{aligned} V(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) &= \tfrac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+\alpha\boldsymbol{s}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad+\sum_{i=1}^{n}(k_{pi}+\alpha k_{vi})S_i(\Delta x_i) \end{aligned} \tag{2.72}$$

and

$$\begin{aligned} W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) &= \dot{\boldsymbol{q}}^T\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{K}_v\dot{\boldsymbol{x}}+\dot{\boldsymbol{q}}^T\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x})-\dot{\boldsymbol{x}}^T\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}) \\ &+\alpha\{\boldsymbol{s}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})(\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})-\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}))\dot{\boldsymbol{q}}-\dot{\boldsymbol{s}}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &-\boldsymbol{s}(\Delta\boldsymbol{x})^T\dot{\hat{\boldsymbol{J}}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\}+\alpha\boldsymbol{s}(\Delta\boldsymbol{x})^T\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}), \end{aligned} \tag{2.73}$$

where $\dot{\boldsymbol{q}}^T\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ equals zero by *Property* 1.2 in Chap. 1, and k_{pi}, k_{vi} denote the ith diagonal elements of $\boldsymbol{K}_p$ and $\boldsymbol{K}_v$ respectively. Since

$$\begin{aligned} &\tfrac{1}{4}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+\alpha\boldsymbol{s}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+\sum_{i=1}^{n}(k_{pi}+\alpha k_{vi})S_i(\Delta x_i) \\ &= \tfrac{1}{4}(\dot{\boldsymbol{x}}+2\alpha\boldsymbol{J}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x}))^T\boldsymbol{M}_x(\boldsymbol{q})(\dot{\boldsymbol{x}}+2\alpha\boldsymbol{J}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x}) \\ &-\alpha^2\boldsymbol{s}(\Delta\boldsymbol{x})^T(\boldsymbol{J}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}))^T\boldsymbol{M}_x(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x})+\sum_{i=1}^{n}(k_{pi}+\alpha k_{vi})S_i(\Delta x_i) \\ &\geq \sum_{i=1}^{n}(k_{pi}+\alpha k_{vi})\bar{c}_i s_i^2(\Delta x_i)-\alpha^2\boldsymbol{s}(\Delta\boldsymbol{x})^T(\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}))\boldsymbol{s}(\Delta\boldsymbol{x}) \\ &\geq \sum_{i=1}^{n}\{k_{pi}+\alpha(k_{vi}\bar{c}_i-\alpha\lambda_m)\}s_i^2(\Delta x_i), \end{aligned} \tag{2.74}$$

where $\boldsymbol{M}_x(\boldsymbol{q})=\boldsymbol{J}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q})$, $\lambda_m\triangleq\lambda_{\max}[\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})]$, and $\lambda_{\max}[\cdot]$ denotes the maximum eigenvalue of the matrix. Note from *Property* 1.1 in Chap. 1, $\boldsymbol{M}(\boldsymbol{q})$ is positive definite and hence $\boldsymbol{M}_x(\boldsymbol{q})$ is also positive definite. Substituting this into Eq. (2.72), we have

$$V(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{x}})\geq\tfrac{1}{4}\dot{\boldsymbol{x}}^T\boldsymbol{M}_x(\boldsymbol{q})\dot{\boldsymbol{x}}+\sum_{i=1}^{n}\{k_{pi}+\alpha(k_{vi}\bar{c}_i-\alpha\lambda_m)\}s_i^2(\Delta x_i)\geq 0, \tag{2.75}$$

when

$$k_{vi}\bar{c}_i - \alpha\lambda_m > 0. \tag{2.76}$$

By letting $\Delta_J = \boldsymbol{I}_n - \boldsymbol{J}^{-T}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})$, Eq. (2.73) becomes

$$\begin{aligned} W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{x}}) = \dot{\boldsymbol{x}}^T((\boldsymbol{I}_n - \Delta_J)\boldsymbol{K}_v)\dot{\boldsymbol{x}} + \alpha\boldsymbol{s}(\Delta\boldsymbol{x})^T\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}) \\ - \dot{\boldsymbol{x}}^T(\Delta_J\boldsymbol{K}_p)\boldsymbol{s}(\Delta\boldsymbol{x}) - \alpha h(\boldsymbol{x}, \dot{\boldsymbol{q}}), \end{aligned} \tag{2.77}$$

where

$$\begin{aligned} h(\boldsymbol{x}, \dot{\boldsymbol{q}}) = \boldsymbol{s}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) - \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \dot{s}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ + \boldsymbol{s}(\Delta\boldsymbol{x})^T\dot{\hat{\boldsymbol{J}}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}. \end{aligned} \tag{2.78}$$

Since the position error is bounded by the saturation function, i.e., $\boldsymbol{s}(\Delta\boldsymbol{x})$ is bounded, and the Jacobian matrix is nonsingular, there exists a constant c_0 such that

$$\alpha h(\boldsymbol{x}, \dot{q}) \geq -\alpha c_0\|\dot{\boldsymbol{x}}\|^2. \tag{2.79}$$

Substitution of this into Eq. (2.73) yields

$$\begin{aligned} W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{x}}) \geq \dot{\boldsymbol{x}}^T((\boldsymbol{I}_n - \Delta_J)\boldsymbol{K}_v - \alpha c_0\boldsymbol{I}_n)\dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}^T(\Delta_J\boldsymbol{K}_p)\boldsymbol{s}(\Delta\boldsymbol{x}) \\ + \alpha\boldsymbol{s}(\Delta\boldsymbol{x})^T\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}). \end{aligned} \tag{2.80}$$

The existence of Δ_J such that $W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{x}}) \geq 0$ can be clearly seen from Eq. (2.80). In the following development, we derived a sufficient condition to guarantee $W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{x}}) \geq 0$. From Eq. (2.80), we have

$$\begin{aligned} W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{x}}) \geq (\lambda_{\min}[\boldsymbol{K}_v] - \bar{p}\lambda_{\max}[\boldsymbol{K}_v] - \alpha c_0)\|\dot{\boldsymbol{x}}\|^2 - \bar{p}\lambda_{\max}[\boldsymbol{K}_p]\|\boldsymbol{s}(\Delta\boldsymbol{x})\|\|\dot{\boldsymbol{x}}\| \\ + \alpha\lambda_{\min}[\boldsymbol{K}_p]\|\boldsymbol{s}(\Delta\boldsymbol{x})\|^2. \end{aligned} \tag{2.81}$$

Note that,

$$-\bar{p}\lambda_{\max}[\boldsymbol{K}_p]\|\boldsymbol{s}(\Delta\boldsymbol{x})\| \cdot \|\dot{\boldsymbol{x}}\| \geq -\tfrac{1}{2}\bar{p}\lambda_{\max}[\boldsymbol{K}_p](\|\boldsymbol{s}(\Delta\boldsymbol{x})\|^2 + \|\dot{\boldsymbol{x}}\|^2). \tag{2.82}$$

Hence,

$$W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{x}}) \geq \lambda_{\max}[\boldsymbol{K}_v]l_1\|\dot{\boldsymbol{x}}\|^2 + a\lambda_{\max}[\boldsymbol{K}_v]l_2\|\boldsymbol{s}(\Delta\boldsymbol{x})\|^2, \tag{2.83}$$

where

$$l_1 = \frac{\lambda_{\min}[\boldsymbol{K}_v]}{\lambda_{\max}[\boldsymbol{K}_v]} - \frac{\alpha c_0}{\lambda_{\max}[\boldsymbol{K}_v]} - \bar{p}(1 + \frac{a}{2}),$$

$$l_2 = \alpha \frac{\lambda_{\min}[\boldsymbol{K}_p]}{\lambda_{\max}[\boldsymbol{K}_p]} - \frac{\bar{p}}{2} \tag{2.84}$$

and $a = \frac{\lambda_{\max}[\boldsymbol{K}_p]}{\lambda_{\max}[\boldsymbol{K}_v]}$. Hence if

$$\min\left\{\frac{2}{2+a}\left(\frac{\lambda_{\min}[\boldsymbol{K}_v]}{\lambda_{\max}[\boldsymbol{K}_v]} - \frac{\alpha c_0}{\lambda_{\max}[\boldsymbol{K}_v]}\right), 2\alpha\frac{\lambda_{\min}[\boldsymbol{K}_p]}{\lambda_{\max}[\boldsymbol{K}_p]}\right\} > \bar{p}, \tag{2.85}$$

and $\frac{\lambda_{\min}[\boldsymbol{K}_v]}{\lambda_{\max}[\boldsymbol{K}_v]} - \frac{\alpha c_0}{\lambda_{\max}[\boldsymbol{K}_v]} > 0$, then $l_1 > 0$ and $l_2 > 0$, and hence $W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{x}})$ is positive definite in $\dot{\boldsymbol{x}}$ and $\boldsymbol{s}(\Delta\boldsymbol{x})$.

Similarly, we let $\boldsymbol{K}_v = k_v\boldsymbol{I}_n$, $\boldsymbol{K}_p = ak_v\boldsymbol{I}_n$. Then condition (2.85) becomes

$$\bar{p} < \min\left\{\frac{2}{a+2}\left(1 - \frac{\alpha c_0}{k_v}\right), 2\alpha\right\}. \tag{2.86}$$

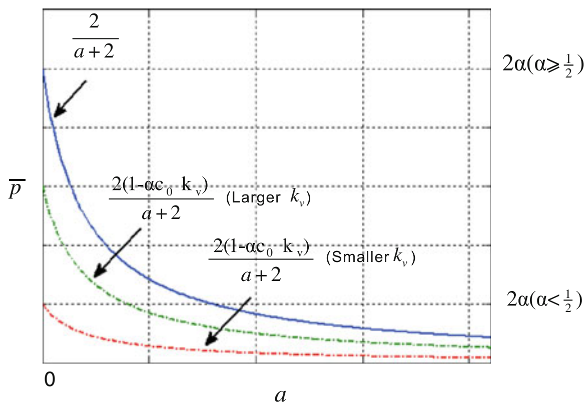
The relationship between $\bar{p}$ and a described by inequality (2.86) is shown in Fig. 2.17.

From Fig. 2.17, it can be seen that *both the feedback-gains ratio $a = k_p/k_v$ and the absolute value of k_v play an important role in stabilizing the system in the presence of uncertain kinematics*. As seen from the figure, increasing k_v will shift the curve $2(1 - \frac{\alpha c_0}{k_v})/(a+2)$ upward, resulting in higher $\bar{p}$ for the same feedback ratio a. That is, larger k_v would allow a higher margin of uncertainty of the approximate Jacobian matrix for the same a. When k_v is so large that $\alpha c_0/k_v$ is negligible, condition (2.86) reduces to

$$\min\left\{\frac{2}{a+2}, 2\alpha\right\} > \bar{p}. \tag{2.87}$$

Conversely, if $\bar{p}$ is small, a smaller controller gain k_v is required. The relationship between $\bar{p}$ and a described by inequality (2.87) is shown in Fig. 2.18.

Fig. 2.17 Variation of $\bar{p}$ with a



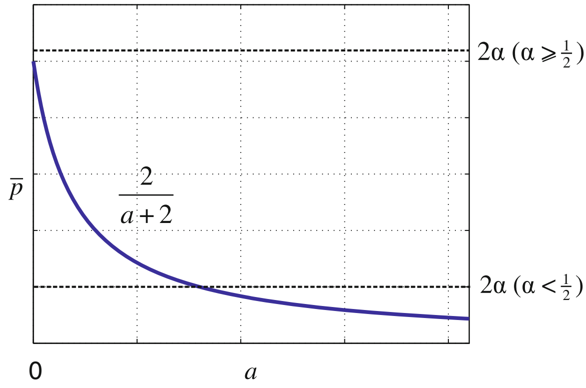


Fig. 2.18 Variation of $\bar{p}$ with a (k_v is very large)

Next, it is interesting to note the effect of a on the stability of the system. As seen from Fig. 2.17, a larger range of a can be chosen for a smaller $\bar{p}$. In fact, in the extreme case $\bar{p} = 0$, a could be chosen as any value. However, when $\bar{p}$ is larger, a narrower range of a is allowed. Hence, while for the case of perfect kinematics, increasing $\boldsymbol{K}_p$ improves steady-state performance by reducing error, this is no longer true for the case of uncertain kinematics. As seen from the figure, for any chosen $\boldsymbol{K}_v$, increasing $\boldsymbol{K}_p$ increases a and hence results in a smaller bound on the Jacobian uncertainty $\bar{p}$. This means that the robot system with uncertain Jacobian matrix could become unstable if a is too large. Therefore, a should be kept smaller so that the allowable bound of the Jacobian uncertainty is larger. This can be achieved by either increasing $\boldsymbol{K}_v$ or reducing $\boldsymbol{K}_p$. Though the condition is a sufficient condition, it is natural and reasonable because it simply means that

1. if you are not sure, then do it at a slower speed (i.e., if $\bar{p}$ is large, then more damping is required);
2. if you are sure, then you can do it at any speed (i.e., if $\bar{p}$ is small, then a wide range a is allowed).

Another important and practical conclusion of the result is that when the system is unstable, redesign of $\hat{\boldsymbol{J}}(\boldsymbol{q})$ or calibrations may not be necessary, since the instability may be due to the feedback gain k_v or the feedback gain ratio a not being tuned properly. In practice, we should therefore try to stabilize the system or increase the margin of stability first by increasing feedback gains or reducing a.

The following theorem states the results:

Theorem 2.2 *The sensory feedback controller with uncertain Jacobian matrix* (2.66) *for the robot system* (2.9) *gives rise to the convergence of the sensory-space position error and velocity such that* $\Delta\boldsymbol{x} \to \boldsymbol{0}$ *and* $\dot{\boldsymbol{x}} \to \boldsymbol{0}$ *as* $t \to \infty$ *if the feedback gains* $\boldsymbol{K}_p$ *and* $\boldsymbol{K}_v$ *are chosen to satisfy conditions* (2.76), (2.85), *and* $\hat{\boldsymbol{J}}(\boldsymbol{q})$ *is chosen so that*

$$\|\boldsymbol{I}_n - \boldsymbol{J}^{-T}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})\| \le \bar{p}. \tag{2.88}$$

Proof Since $V(\boldsymbol{s}(\Delta \boldsymbol{x}), \dot{\boldsymbol{x}})$ and $W(\boldsymbol{s}(\Delta \boldsymbol{x}), \dot{\boldsymbol{x}})$ are positive definite in $\boldsymbol{s}(\Delta \boldsymbol{x})$ and $\dot{\boldsymbol{x}}$, both $\boldsymbol{s}(\Delta \boldsymbol{x})$ and $\dot{\boldsymbol{x}}$ are bounded. The boundedness of $\dot{\boldsymbol{x}}$ ensures the boundedness of $\dot{\boldsymbol{q}}$, since it is assumed that $\boldsymbol{J}^{-1}(\boldsymbol{q})$ exists. From Eq. (2.70), it is seen that $\ddot{\boldsymbol{q}}$ is bounded. The boundedness of $\ddot{\boldsymbol{q}}$ also ensures the boundedness of $\ddot{\boldsymbol{x}}$, since $\ddot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}$. Since both $\ddot{\boldsymbol{x}}$ and $\dot{\boldsymbol{x}}$ are bounded, $\dot{\boldsymbol{x}}$ and $\boldsymbol{s}(\Delta \boldsymbol{x})$ are uniformly continuous. From Eqs. (2.71) and (2.83), we have $\dot{\boldsymbol{x}}, \boldsymbol{s}(\Delta \boldsymbol{x}) \in L_2(0, +\infty)$. Therefore, from Barbalat's emma, $\boldsymbol{s}(\Delta \boldsymbol{x}) \to \boldsymbol{0}$ and $\dot{\boldsymbol{x}} \to \boldsymbol{0}$ as $t \to \infty$, and hence $\Delta \boldsymbol{x} \to \boldsymbol{0}$. △△△

Given the desired end effector's position $\boldsymbol{x}_d$, the manipulator has its own configuration $\boldsymbol{q}_d$ at joint space. The positive definiteness of V in the state variables $\Delta \boldsymbol{q} = \boldsymbol{q} - \boldsymbol{q}_d$ and $\dot{\boldsymbol{q}}$ cannot be concluded in general. However, for a finite task space such that the kinematics mapping between task space and joint space is one-to-one and the Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ is nonsingular, then the Lyapunov method can be used directly to conclude the asymptotic stability of the equilibrium state.

When cameras are used to specify the position of the robot end effector, the sensory space is defined as the image coordinates. In the presence of an uncalibrated camera, the image Jacobian matrix is uncertain, as indicated by $\hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q})\hat{\boldsymbol{L}}(\boldsymbol{x})\hat{\boldsymbol{J}}_m(\boldsymbol{q})$ in Eq. (2.63). A vision-based controller using the approximate Jacobian matrix can be designed as follows:

$$\boldsymbol{\tau} = -\hat{\boldsymbol{J}}_m^T(\boldsymbol{q})\hat{\boldsymbol{L}}^T(\boldsymbol{x})\hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q})(\boldsymbol{K}_p \boldsymbol{s}(\Delta \boldsymbol{x}) + \boldsymbol{K}_v \dot{\boldsymbol{x}}) + \boldsymbol{g}(\boldsymbol{q}). \tag{2.89}$$

If the approximate image Jacobian matrix is chosen such that

$$\| \boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{L}^T(\boldsymbol{x})\boldsymbol{Z}^{-1}(\boldsymbol{q}) - \hat{\boldsymbol{J}}_m^T(\boldsymbol{q})\hat{\boldsymbol{L}}^T(\boldsymbol{x})\hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q}) \| \leq \bar{p}, \tag{2.90}$$

then it can be proved similarly that the vision-based controller in Eq. (2.89) ensures the convergence of the image-space error.

2.3.3 Sensory Feedback Control with Uncertain Gravitational Force

The control input (2.66) requires exact knowledge of the gravitational force $\boldsymbol{g}(\boldsymbol{q})$ or else could be implemented on a gravity-free robot. In principle, the parameters of the gravitational force can be identified offline by commanding the robot to move with arbitrary joint motions without solving the inverse kinematics problem. However, when the robot is required to handle objects of various lengths and masses, the gravitational force could vary according to different tasks. Therefore, it is important to compensate the gravitational force online without exact knowledge of the kinematics and Jacobian matrix.

In this section, we consider the sensory feedback control problem with uncertain gravitational force compensation and with uncertain Jacobian using the concept of gravity regressor. Note that the gravity term can be completely characterized by a set of parameters $\boldsymbol{\theta}_g = (\theta_{g1}, \cdots, \theta_{gn_g})^T \in \Re^{n_g}$ (see *Property* 1.4) as

$$\boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{Y}_g(\boldsymbol{q})\boldsymbol{\theta}_g, \tag{2.91}$$

where $\boldsymbol{Y}_g(\boldsymbol{q}) \in \Re^{n \times n_g}$ is the gravity regressor. Consider the following control input:

$$\boldsymbol{\tau} = -\hat{\boldsymbol{J}}^T(\boldsymbol{q})(\boldsymbol{K}_p \boldsymbol{s}(\Delta \boldsymbol{x}) + \boldsymbol{K}_v \dot{\boldsymbol{x}}) + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g, \tag{2.92}$$

$$\dot{\hat{\boldsymbol{\theta}}}_g = \boldsymbol{L}_g \boldsymbol{Y}_g^T(\boldsymbol{q})\boldsymbol{y}, \tag{2.93}$$

where $\boldsymbol{y} = \dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\boldsymbol{s}(\Delta \boldsymbol{x})$ is the output, and $\boldsymbol{L}_g \in \Re^{n_g \times n_g}$ is a positive definite matrix. Note that the output vector defined in Eq. (2.69) is now used in the design of the update law in Eq. (2.93). A block diagram of the controller in Eq. (2.92) is shown in Fig. 2.19.

Substituting Eqs. (2.92) and (2.91) into Eq. (2.9) yields

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left(\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \hat{\boldsymbol{J}}^T(\boldsymbol{q})(\boldsymbol{K}_p \boldsymbol{s}(\Delta \boldsymbol{x}) + \boldsymbol{K}_v \dot{\boldsymbol{x}}) + \boldsymbol{Y}_g(\boldsymbol{q})\Delta\boldsymbol{\theta}_g = \boldsymbol{0}, \tag{2.94}$$

where $\Delta \boldsymbol{\theta}_g = \boldsymbol{\theta}_g - \hat{\boldsymbol{\theta}}_g \in \Re^{n_g \times n_g}$.

The convergence of the task-space position error and velocity is specified by the following theorem:

Theorem 2.3 *The sensory feedback controller with uncertain gravitational force* (2.93) *for the robot system* (2.9) *gives rise to the convergence of the sensory-space position error and velocity such that* $\Delta \boldsymbol{x} \to \boldsymbol{0}$ *and* $\dot{\boldsymbol{x}} \to \boldsymbol{0}$ *as* $t \to \infty$ *if the feedback gains* $\boldsymbol{K}_p$ *and* $\boldsymbol{K}_v$ *are chosen to satisfy conditions* (2.76), (2.85), *and* $\hat{\boldsymbol{J}}(\boldsymbol{q})$ *is chosen to satisfy condition* (2.67).

Proof Multiplying both sides of Eq. (2.94) by $\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})$, taking the inner product with the output $\boldsymbol{y}$, and using Eq. (2.93) leads to

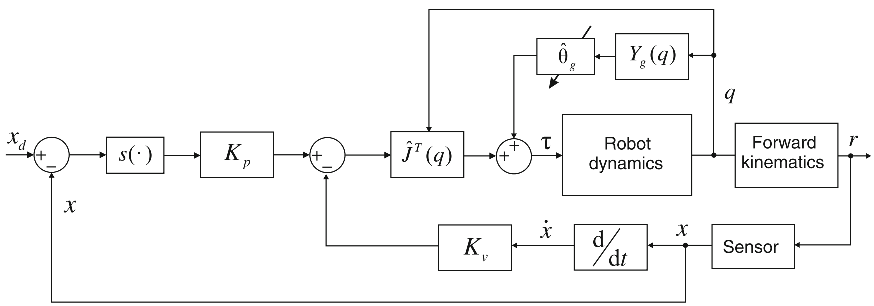


Fig. 2.19 A block diagram of a sensory feedback control with uncertain gravitational force

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \alpha\boldsymbol{s}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \sum_{i=1}^{n}(k_{pi}+\alpha k_{vi})S_i(\Delta x_i)\right.$$
$$\left. + \frac{1}{2}\Delta\boldsymbol{\theta}_g^T\boldsymbol{L}_g^{-1}\Delta\boldsymbol{\theta}_g\right\} + \dot{\boldsymbol{x}}^T\boldsymbol{J}^{-T}(\boldsymbol{x})\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{K}_v\dot{\boldsymbol{x}} + \alpha\boldsymbol{s}(\Delta\boldsymbol{x})^T\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x})$$
$$-\dot{\boldsymbol{x}}^T(\boldsymbol{I}_n - \boldsymbol{J}^{-T}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q}))\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}) + \alpha\left\{\boldsymbol{s}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\left(\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\right.\right.$$
$$\left.\left.-\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} - \dot{\boldsymbol{s}}(\Delta\boldsymbol{x})^T\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \boldsymbol{s}(\Delta\boldsymbol{x})^T\dot{\hat{\boldsymbol{J}}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\right\} = 0, \quad (2.95)$$

which is equal to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(V(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{x}}) + \frac{1}{2}\Delta\boldsymbol{\theta}_g^T\boldsymbol{L}_g^{-1}\Delta\boldsymbol{\theta}_g\right) = -W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{x}}). \quad (2.96)$$

Both $V(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{x}})$ and $W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{x}})$ are positive definite in $\boldsymbol{s}(\Delta\boldsymbol{x})$ and $\dot{\boldsymbol{x}}$. From Eq. (2.96), $\boldsymbol{s}(\Delta\boldsymbol{x})$, $\dot{\boldsymbol{x}}$, and $\Delta\boldsymbol{\theta}_g$ are bounded. The boundedness of $\dot{\boldsymbol{x}}$ ensures the boundedness of $\dot{\boldsymbol{q}}$, since it is assumed that $\boldsymbol{J}^{-1}(\boldsymbol{q})$ exists. The boundedness of $\dot{\boldsymbol{q}}$ and $\boldsymbol{s}(\Delta\boldsymbol{x})$ ensures the boundedness of the output $\boldsymbol{y}$. From Eq. (2.94), it is seen that $\ddot{\boldsymbol{q}}$ is bounded. The boundedness of $\ddot{\boldsymbol{q}}$ also ensures the boundedness of $\ddot{\boldsymbol{x}}$, since $\ddot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}$. Since both $\ddot{\boldsymbol{x}}$ and $\dot{\boldsymbol{x}}$ are bounded, $\dot{\boldsymbol{x}}$ and $\boldsymbol{s}(\Delta\boldsymbol{x})$ are uniformly continuous. From Eqs. (2.95) and (2.96), we have $\dot{\boldsymbol{x}}$, $\boldsymbol{s}(\Delta\boldsymbol{x}) \in L_2(0, +\infty)$. Therefore, from Barbalat's lemma, $\boldsymbol{s}(\Delta\boldsymbol{x}) \to \boldsymbol{0}$ and $\dot{\boldsymbol{x}} \to \boldsymbol{0}$ as $t \to \infty$, and hence $\Delta\boldsymbol{x} \to \boldsymbol{0}$. △△△

2.4 Sensory-Space Setpoint Control with Joint-Space Damping

Note that the controllers described by Eqs. (2.66) and (2.92) require the task-space velocity $\dot{\boldsymbol{x}}$. The task-space velocity is obtained from numerical differentiation of the task-space position which is usually noisy. In addition, in the case of redundant robot, the convergence of the task-space velocity to zero does not necessary ensure the convergence of the joint-space velocity to zero, even if the Jacobian matrix is nonsingular.

2.4.1 Sensory Feedback Control with Exact Gravity Compensation

Due to the possible existence of self motions in the case of redundant robot, the joint-space damping is used directly to damp out the joint motion. First, we consider a simple approximate Jacobian feedback controller by using joint-space damping as follows:

$$\boldsymbol{\tau} = -\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}) - \boldsymbol{K}_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}), \quad (2.97)$$

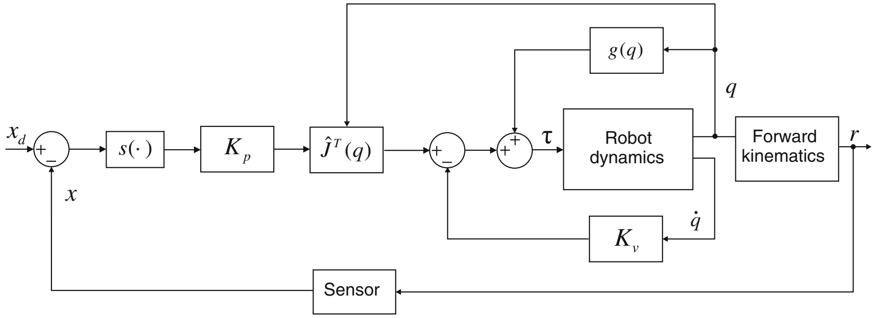


Fig. 2.20 A block diagram of the approximate Jacobian feedback control with joint-space damping and gravitational force compensation

where $\boldsymbol{K}_p = k_p \boldsymbol{I}_n$, $\boldsymbol{K}_v = k_v \boldsymbol{I}_n$, and $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ is chosen so that

$$\|\boldsymbol{J}^T(\boldsymbol{q}) - \hat{\boldsymbol{J}}^T(\boldsymbol{q})\| \leq p. \tag{2.98}$$

A block diagram of the controller in Eq. (2.97) is shown in Fig. 2.20.

The closed-loop equation of the system is obtained by substituting Eq. (2.97) into Eq. (2.9),

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{K}_p \boldsymbol{s}(\Delta\boldsymbol{x}) + \boldsymbol{K}_v\dot{\boldsymbol{q}} = \boldsymbol{0}. \tag{2.99}$$

Let us define an output vector $\boldsymbol{y}$ of the form

$$\boldsymbol{y} = \dot{\boldsymbol{q}} + \alpha\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x}). \tag{2.100}$$

where joint-space velocity and sensory task-space position error are used and a transpose of the approximate Jacobian is now used to transform the task-space position error to joint space. The above output does not require the use of the inverse or pseudo-inverse of the approximate Jacobian matrix and hence is simpler and more useful.

To carry out the stability analysis for the closed-loop system with the approximate Jacobian controller, we take the inner product between the output vector $\boldsymbol{y}$ described by Eq. (2.100) and the closed-loop Eq. (2.99) and make use of *Properties* 1.1 and 1.2 of the robot dynamics to give,

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) + W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) = 0, \tag{2.101}$$

where

$$\begin{aligned} V(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) = {} & \tfrac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \alpha\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x}) \\ & + \sum_{i=1}^{m}(k_p + \alpha k_v)S_i(\Delta x_i), \end{aligned} \tag{2.102}$$

and

$$\begin{aligned} W(s(\Delta x), \dot{q}) &= k_v \dot{q}^T \dot{q} + \alpha k_p s^T(\Delta x) \hat{J}(q) \hat{J}^T(q) s(\Delta x) \\ &\quad -(k_p + \alpha k_v) \dot{q}^T (J^T(q) - \hat{J}^T(q)) s(\Delta x) \\ &\quad -\alpha \Big\{ s^T(\Delta x) \hat{J}(q) \left(\tfrac{1}{2}\dot{M}(q) - S(q, \dot{q})\right) \dot{q} + \dot{s}^T(\Delta x) \hat{J}(q) M(q) \dot{q} \\ &\quad + s^T(\Delta x) \dot{\hat{J}}(q) M(q) \dot{q}. \Big\} \end{aligned} \tag{2.103}$$

To show that $V(s(\Delta x), \dot{q})$ in Eq. (2.102) is positive definite in $\dot{q}$ and $s(\Delta x)$, we note that

$$\begin{aligned} &\tfrac{1}{4}\dot{q}^T M(q) \dot{q} + \alpha \dot{q}^T M(q) \hat{J}^T(q) s(\Delta x) + \sum_{i=1}^{m} (k_p + \alpha k_v) S_i(e_i) \\ &= \tfrac{1}{4}(\dot{q} + 2\alpha \hat{J}^T(q) s(\Delta x))^T M(q) (\dot{q} + 2\alpha \hat{J}^T(q) s(\Delta x)) \\ &\quad -\alpha^2 s^T(\Delta x) \hat{J}(q) M(q) \hat{J}^T(q) s(\Delta x) + \sum_{i=1}^{m} (k_p + \alpha k_v) S_i(e_i) \\ &\geq \sum_{i=1}^{m} \{k_p \bar{c}_i + \alpha(k_v \bar{c}_i - \alpha \lambda_m)\} s_i^2(\Delta x_i) \end{aligned} \tag{2.104}$$

where $\lambda_m \overset{\triangle}{=} \lambda_{\max}[\hat{J}(q) M(q) \hat{J}^T(q)]$. Substituting the above equation into Eq. (2.102) yields:

$$V(s(\Delta x), \dot{q}) \geq \tfrac{1}{4}\dot{q}^T M(q) \dot{q} + \sum_{i=1}^{m} \{k_p \bar{c}_i + \alpha(k_v \bar{c}_i - \alpha \lambda_m)\} s_i^2(\Delta x_i) \geq 0, \tag{2.105}$$

where k_v and α can be chosen so that

$$k_v \bar{c}_i - \alpha \lambda_m > 0. \tag{2.106}$$

Therefore, the function $V(s(\Delta x), \dot{q})$ is positive definite in $s(\Delta x)$ and $\dot{q}$ when k_v is chosen sufficiently large and α is set sufficiently small.

Next, we proceed to show that the time derivative of the function $V(s(\Delta x), \dot{q})$ is negative definite in $s(\Delta x)$ and $\dot{q}$. As seen from Eq. (2.101), this is to show that $W(s(\Delta x), \dot{q})$ is positive definite in $s(\Delta x)$ and $\dot{q}$. From the last term on the right-hand side of Eq. (2.103), since $s(\Delta x)$ is bounded, there exist a constant $c_0 > 0$ such that:

$$\begin{aligned} &\alpha | s^T(\Delta x) \hat{J}(q) \left(\tfrac{1}{2}\dot{M}(q) - S(q, \dot{q})\right) \dot{q} + \dot{s}^T(\Delta x) \hat{J}(q) M(q) \dot{q} + s^T(\Delta x) \dot{\hat{J}}(q) M(q) \dot{q} | \\ &\leq \alpha c_0 \|\dot{q}\|^2. \end{aligned} \tag{2.107}$$

Substituting inequality (2.107) into Eq. (2.103) yields

$$W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) \geq \dot{\boldsymbol{q}}^T(k_v\boldsymbol{I}_n - \alpha c_0\boldsymbol{I}_n)\dot{\boldsymbol{q}} + \alpha k_p\boldsymbol{s}^T(\Delta\boldsymbol{x})\hat{\boldsymbol{J}}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x}) \\ -(k_p+\alpha k_v)\dot{\boldsymbol{q}}^T(\boldsymbol{J}^T(\boldsymbol{q}) - \hat{\boldsymbol{J}}^T(\boldsymbol{q}))\boldsymbol{s}(\Delta\boldsymbol{x}). \tag{2.108}$$

Now, letting $\bar{\Delta}_J \overset{\triangle}{=} \boldsymbol{J}^T(\boldsymbol{q}) - \hat{\boldsymbol{J}}^T(\boldsymbol{q})$, we have

$$W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) \geq \dot{\boldsymbol{q}}^T(k_v\boldsymbol{I}_n - \alpha c_0\boldsymbol{I}_n)\dot{\boldsymbol{q}} + \alpha k_p\boldsymbol{s}^T(\Delta\boldsymbol{x})\hat{\boldsymbol{J}}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x}) \\ -(k_p+\alpha k_v)\dot{\boldsymbol{q}}^T\bar{\Delta}_J\boldsymbol{s}(\Delta\boldsymbol{x}). \tag{2.109}$$

The existence of a $\bar{\Delta}_J$ ensures that $W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}})$ is positive definite in $\boldsymbol{s}(\Delta\boldsymbol{x})$ and $\dot{\boldsymbol{q}}$, as seen from Eq. (2.109). In the following development, a sufficient condition is derived to guarantee the positive definiteness of $W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}})$ in $\boldsymbol{s}(\Delta\boldsymbol{x})$ and $\dot{\boldsymbol{q}}$. Note that

$$W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) \geq (k_v - \alpha c_0)\|\dot{\boldsymbol{q}}\|^2 - p(k_p+\alpha k_v)\|\boldsymbol{s}(\Delta\boldsymbol{x})\|\cdot\|\dot{\boldsymbol{q}}\| \\ +\alpha k_p\lambda_{\hat{J}}\|\boldsymbol{s}(\Delta\boldsymbol{x})\|^2, \tag{2.110}$$

where $\lambda_{\hat{J}} = \lambda_{\min}[\hat{\boldsymbol{J}}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})]$. Next, note that

$$-\|\boldsymbol{s}(\Delta\boldsymbol{x})\|\cdot\|\dot{\boldsymbol{q}}\| \geq -\tfrac{1}{2}(\|\boldsymbol{s}(\Delta\boldsymbol{x})\|^2 + \|\dot{\boldsymbol{q}}\|^2). \tag{2.111}$$

Substituting inequality (2.111) into Eq. (2.110) gives,

$$W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}}) \geq k_v l_1\|\dot{\boldsymbol{q}}\|^2 + k_v l_2\|\boldsymbol{s}(\Delta\boldsymbol{x})\|^2, \tag{2.112}$$

where

$$l_1 = 1 - \tfrac{\alpha c_0}{k_v} - \tfrac{p}{2}(\bar{a}+\alpha), \quad l_2 = \alpha\bar{a}\lambda_{\hat{J}} - \tfrac{p}{2}(\bar{a}+\alpha), \tag{2.113}$$

and $\bar{a} = \frac{k_p}{k_v}$. Hence, if

$$\min\left\{\frac{2(1-\frac{\alpha c_0}{k_v})}{\bar{a}+\alpha}, \frac{2\bar{a}\alpha\lambda_{\hat{J}}}{\bar{a}+\alpha}\right\} > p, \tag{2.114}$$

and $k_v > \alpha c_o$, then $l_1 > 0$ and $l_2 > 0$, and hence $W(\boldsymbol{s}(\Delta\boldsymbol{x}),\dot{\boldsymbol{q}})$ is positive definite in $\dot{\boldsymbol{q}}$ and $\boldsymbol{s}(\Delta\boldsymbol{x})$. The relationship between p and $\bar{a}$ can be similarly illustrated in the following (Fig. 2.21).

We are now in a position to prove the following theorem:

Theorem 2.4 *The sensory feedback controller in Eq.* (2.97) *for the robot system described by Eq.* (2.9) *ensures the convergence of sensory-space position error and*

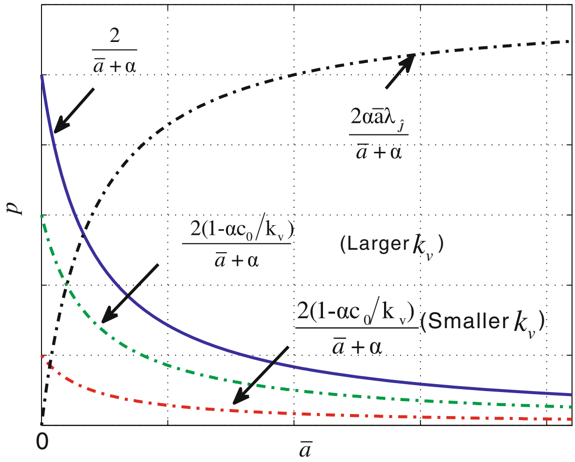


Fig. 2.21 Variation of p with $\bar{a}$

the joint-space velocity is guaranteed, if the feedback gains $\boldsymbol{K}_p$ and $\boldsymbol{K}_v$ are chosen to satisfy conditions (2.106)*,* (2.114)*, and $\hat{\boldsymbol{J}}(\boldsymbol{q})$ is chosen to satisfy condition* (2.98).

Proof Since both $V(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{q}})$ and $W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{q}})$ are positive definite in $\dot{\boldsymbol{q}}$ and $\boldsymbol{s}(\Delta\boldsymbol{x})$, from Eq. (2.101), we have

$$\tfrac{\mathrm{d}}{\mathrm{d}t} V(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{q}}) = -W(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{q}}) < 0. \tag{2.115}$$

Hence, $V(\boldsymbol{s}(\Delta\boldsymbol{x}), \dot{\boldsymbol{q}})$ is a Lyapunov-like function whose time derivative is negative in $\boldsymbol{s}(\Delta\boldsymbol{x})$ and $\dot{\boldsymbol{q}}$. Therefore, both $\boldsymbol{s}(\Delta\boldsymbol{x})$ and $\dot{\boldsymbol{q}}$ are bounded. From Eq. (2.99), it is seen that $\ddot{\boldsymbol{q}}$ is bounded. The boundedness of $\dot{\boldsymbol{q}}$ also ensures the boundedness of $\dot{\boldsymbol{x}}$ since $\boldsymbol{J}(\boldsymbol{q})$ are trigonometric functions of joint angles. Since both $\ddot{\boldsymbol{q}}$ and $\dot{\boldsymbol{x}}$ are bounded, $\dot{\boldsymbol{q}}$ and $\boldsymbol{s}(\Delta\boldsymbol{x})$ are uniformly continuous. From Eqs. (2.101) and (2.112), we have $\dot{\boldsymbol{q}}, \boldsymbol{s}(\Delta\boldsymbol{x}) \in L_2(0, +\infty)$. Therefore, from Barbalat's lemma, $\boldsymbol{s}(\Delta\boldsymbol{x}) \to \boldsymbol{0}$ and $\dot{\boldsymbol{q}} \to \boldsymbol{0}$ as $t \to \infty$, and hence $\Delta\boldsymbol{x} \to \boldsymbol{0}$. △△△

If the robot is non-redundant so that the dimension of task space is equal to the dimension of joint space and the kinematics mapping between task space and joint space is one-to-one, then Lyapunov method can be used directly to conclude the asymptotical stability if the Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ is nonsingular.

When cameras are used to specified the position of the robot end effector, the sensory space is defined as image coordinates. In the presence of uncalibrated camera, the image Jaocbian matrix is uncertain as denoted as $\hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q})\hat{\boldsymbol{L}}(\boldsymbol{x})\hat{\boldsymbol{J}}_m(\boldsymbol{q})$ in Eq. (2.63). A vision-based controller by using the joint-space damping can be designed as follows:

$$\boldsymbol{\tau} = -\hat{\boldsymbol{J}}_m^T(\boldsymbol{q})\hat{\boldsymbol{L}}^T(\boldsymbol{x})\hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q})\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}) - \boldsymbol{K}_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}). \tag{2.116}$$

If the approximate Jacobian matrix is chosen such that:

$$\|\boldsymbol{J}_m^T(\boldsymbol{q})\boldsymbol{L}^T(\boldsymbol{x})\boldsymbol{Z}^{-1}(\boldsymbol{q}) - \hat{\boldsymbol{J}}_m^T(\boldsymbol{q})\hat{\boldsymbol{L}}^T(\boldsymbol{x})\hat{\boldsymbol{Z}}^{-1}(\boldsymbol{q})\| \le p, \tag{2.117}$$

then the vision-based controller in Eq. (2.116) ensures the convergence of image-space error and joint velocity to zero.

2.4.2 Sensory Feedback Control in Presence of Uncertain Gravitational Force

Next, consider the problem of sensory task-space setpoint control with uncertain gravitational force compensation and Jacobian matrix. The control input is designed as

$$\boldsymbol{\tau} = -\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x}) - \boldsymbol{K}_v\dot{\boldsymbol{q}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g, \tag{2.118}$$

$$\dot{\hat{\boldsymbol{\theta}}}_g = -\boldsymbol{L}\boldsymbol{Y}_g^T(\boldsymbol{q})\boldsymbol{y}, \tag{2.119}$$

where $\boldsymbol{y} = \dot{\boldsymbol{q}} + \alpha\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{x})$ is the output defined in Eq. (2.100). Note that the above controller does not require the use of generalized inverse of the approximate Jacobian matrix, which could become very large when the robot approaches any singularity configuration. A block diagram of the controller in Eq. (2.118) is shown in Fig. 2.22.

Substituting Eq. (2.118) into Eq. (2.9), we have the closed-loop equation,

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})+\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}}+\hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{K}_p\boldsymbol{s}(\Delta\boldsymbol{x})+\boldsymbol{K}_v\dot{\boldsymbol{q}}+\boldsymbol{Y}_g(\boldsymbol{q})\Delta\boldsymbol{\theta}_g=\boldsymbol{0}, \tag{2.120}$$

The following theorem states the result of sensory-space setpoint control with uncertain gravitational force compensation:

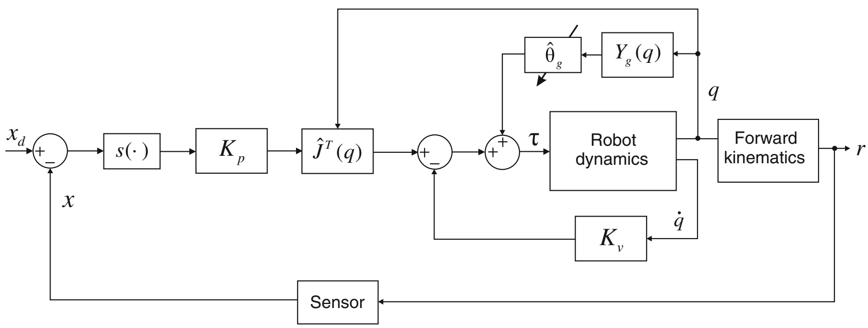


Fig. 2.22 A block diagram of the approximate Jacobian feedback control with joint-space damping and update of uncertain parameters in the estimated gravitational force

Theorem 2.5 *The sensory feedback controller with uncertain gravitational force* (2.118) *and the update law* (2.119) *for the robot system* (2.9) *ensures the convergence of the sensory-space position error and the joint-space velocity, if the feedback gains* $\boldsymbol{K}_p$, $\boldsymbol{K}_v$ *and* $\hat{\boldsymbol{J}}(\boldsymbol{q})$ *are chosen as in Theorem* 2.4.

Proof Taking the inner product of Eq. (2.120) with $\boldsymbol{y} = \dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^T(\boldsymbol{q})\boldsymbol{s}(\Delta \boldsymbol{x})$ and using Eq. (2.119), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(V(\boldsymbol{s}(\Delta \boldsymbol{x}), \dot{\boldsymbol{q}}) + \tfrac{1}{2}\Delta\boldsymbol{\theta}_g^T \boldsymbol{L}_g^{-1}\Delta\boldsymbol{\theta}_g) = -W(\boldsymbol{s}(\Delta \boldsymbol{x}), \dot{\boldsymbol{q}}) \leq 0. \tag{2.121}$$

Hence, $V(\boldsymbol{s}(\Delta \boldsymbol{x}), \dot{\boldsymbol{q}})$ is a Lyapunov-like function whose time derivative is negative in $\boldsymbol{s}(\Delta \boldsymbol{x})$ and $\dot{\boldsymbol{q}}$. Therefore, $\boldsymbol{s}(\Delta \boldsymbol{x})$, $\dot{\boldsymbol{q}}$, and $\Delta\boldsymbol{\theta}_g$ are bounded. The boundedness of $\dot{\boldsymbol{q}}$ and $\boldsymbol{s}(\Delta \boldsymbol{x})$ ensures the boundedness of the output $\boldsymbol{y}$. From Eq. (2.120), it is seen that $\ddot{\boldsymbol{q}}$ is bounded. Since both $\ddot{\boldsymbol{q}}$ and $\dot{\boldsymbol{x}}$ are bounded, $\dot{\boldsymbol{q}}$ and $\boldsymbol{s}(\Delta \boldsymbol{x})$ are uniformly continuous. Similarly, it can be shown that $\dot{\boldsymbol{q}}, \boldsymbol{s}(\Delta \boldsymbol{x}) \in L_2(0, +\infty)$. Therefore, $\boldsymbol{s}(\Delta \boldsymbol{x}) \to \boldsymbol{0}$ and $\dot{\boldsymbol{q}} \to \boldsymbol{0}$ as $t \to \infty$, and hence $\Delta \boldsymbol{x} \to \boldsymbol{0}$. △△△

2.5 Unified Analysis of Sensory-Space Setpoint Control

Task-space regulation of robots is classified into two basic approaches, namely transpose Jacobian regulation and inverse Jacobian regulation. In the previous sections, various transpose Jacobian setpoint controllers using task-space sensory feedback are considered. The basic idea is to form a control force of the robot end-effector using the sensory task-space error, and then transform it into joint torques by using the transpose of the approximate Jacobian matrix. In the inverse Jacobian regulation method, the task-space error is transformed into the joint error directly by means of the inverse Jacobian matrix. The basic idea is to relate an infinitesimal displacement in joint space to an infinitesimal displacement in task space directly, by using the inverse Jacobian matrix. An illustration of the concept is shown in Fig. 2.23. In this section, a unified approach for the analysis and design of both the transpose Jacobian and inverse Jacobian setpoint controllers using task-space sensory feedback is introduced. Based on the unified analysis, a fundamental property in the task-space regulation problem, namely the duality property, is presented.

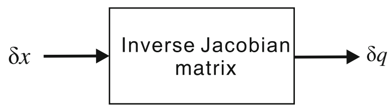


Fig. 2.23 The infinitesimal displacement in joint space is related to an infinitesimal displacement in task space by using the inverse Jacobian matrix

2.5.1 Generalized Approximate Jacobian Setpoint Control

In this section, we consider a generalised approximate Jacobian controller with gravitational force compensation as:

$$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{D}}_1(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \hat{\boldsymbol{D}}_2(\boldsymbol{q})\dot{\hat{\boldsymbol{x}}} + \boldsymbol{g}(\boldsymbol{q}) \tag{2.122}$$

where $\hat{\boldsymbol{D}}_1(\boldsymbol{q})$ and $\hat{\boldsymbol{D}}_2(\boldsymbol{q})$ are approximate transformations in the presence of uncertain kinematics, $\dot{\hat{\boldsymbol{x}}} = \hat{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}$ is an estimated task-space velocity, $\hat{\boldsymbol{J}}(\boldsymbol{q}) \in \Re^{n\times n}$ is an approximate Jacobian matrix for non-redundant robot where $m = n$.

The matrices $\hat{\boldsymbol{D}}_1(\boldsymbol{q})$ and $\hat{\boldsymbol{D}}_2(\boldsymbol{q})$ can be either the approximate transpose Jacobian matrix $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ or the approximate inverse Jacobian matrix $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$. That is,

$$\begin{aligned} \hat{\boldsymbol{D}}_1(\boldsymbol{q}) &: \hat{\boldsymbol{J}}^T(\boldsymbol{q}) \Longleftrightarrow \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}) \\ \hat{\boldsymbol{D}}_2(\boldsymbol{q}) &: \hat{\boldsymbol{J}}^T(\boldsymbol{q}) \Longleftrightarrow \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}) \end{aligned} \tag{2.123}$$

There exists four combinations of approximate Jacobian controllers as shown in Table 2.1. Therefore, there is a duality property in task-space regulation in the sense that the transpose Jacobian matrix $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ can be replaced by the inverse Jacobian matrix $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$ and vice versa. The two transformations, the transpose Jacobian and the inverse Jacobian, are said to be dual.

The closed-loop equation of the system is obtained by substituting Eq. (2.122) into Eq. (2.9),

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + k_p \hat{\boldsymbol{D}}_1(\boldsymbol{q})\Delta\boldsymbol{x} + k_v \hat{\boldsymbol{D}}_2(\boldsymbol{q})\dot{\hat{\boldsymbol{x}}} = \boldsymbol{0}. \tag{2.124}$$

To prove the stability, a Lyapunov-like candidate is defined as:

$$\begin{aligned} V(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) &= \tfrac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad + \tfrac{1}{2}k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\Delta\boldsymbol{x} + \tfrac{1}{2}\alpha k_v\Delta\boldsymbol{x}^T\Delta\boldsymbol{x}, \end{aligned} \tag{2.125}$$

where $\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})$ is the inverse matrix of $\hat{\boldsymbol{D}}_2(\boldsymbol{q})$, and $\boldsymbol{q}_d$ is the desired joint configuration that corresponds to $\boldsymbol{x}_d$. Note that $\boldsymbol{q}_d$ is not required in the controller (2.122) and is defined for analysis only. Similarly, Eq. (2.125) can be expressed as:

Table 2.1 Various combinations of approximate Jacobian controllers with exact gravitational force

	$\hat{\boldsymbol{D}}_1(\boldsymbol{q})$	$\hat{\boldsymbol{D}}_2(\boldsymbol{q})$	Controller
$(B1)$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p\hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x} - k_v\hat{\boldsymbol{J}}^T(\boldsymbol{q})\dot{\hat{\boldsymbol{x}}} + \boldsymbol{g}(\boldsymbol{q})$
$(B2)$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p\hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x} - k_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q})$
$(B3)$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x} - k_v\hat{\boldsymbol{J}}^T(\boldsymbol{q})\dot{\hat{\boldsymbol{x}}} + \boldsymbol{g}(\boldsymbol{q})$
$(B4)$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x} - k_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q})$

$$
\begin{aligned}
V(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) &= \tfrac{1}{4}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+\tfrac{1}{4}(\dot{\boldsymbol{q}}+2\alpha(\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q}))^T\Delta\boldsymbol{x})^T\boldsymbol{M}(\boldsymbol{q})(\dot{\boldsymbol{q}}+2\alpha(\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q}))^T\Delta\boldsymbol{x}) \\
&-\alpha^2\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})(\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q}))^T\Delta\boldsymbol{x}+\tfrac{1}{2}k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\Delta\boldsymbol{x}+\tfrac{1}{2}\alpha k_v\Delta\boldsymbol{x}^T\Delta\boldsymbol{x} \\
&\geq \tfrac{1}{4}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+\tfrac{1}{2}k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\Delta\boldsymbol{x}+\alpha\left(\frac{k_v}{2}-\alpha\lambda_m\right)\|\Delta\boldsymbol{x}\|^2
\end{aligned}
\tag{2.126}
$$

where $\lambda_m \overset{\triangle}{=} \lambda_{\max}[\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})(\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q}))^T]$. Since $\hat{\boldsymbol{D}}_1(\boldsymbol{q})$ is either $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ or $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$, the matrix $\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)$ in (2.125) is either an identity matrix $\boldsymbol{I}_n$ or $(\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d))^T\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)$. Hence it is always symmetric and positive definite if the approximate Jacobian matrix is non singular at the desired position. Therefore k_v must be chosen so that

$$
k_v - 2\alpha\lambda_m > 0. \tag{2.127}
$$

Differentiating Eq. (2.125) with respect to time and substituting $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}$ from Eq. (2.124) into it yields,

$$
\frac{\mathrm{d}}{\mathrm{d}t}V(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) = -W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}), \tag{2.128}
$$

where

$$
\begin{aligned}
W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) &= k_v\dot{\boldsymbol{q}}^T\hat{\boldsymbol{D}}_2(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}+\alpha k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\hat{\boldsymbol{D}}_1(\boldsymbol{q})\Delta\boldsymbol{x} \\
&+k_p\Delta\boldsymbol{x}^T(\hat{\boldsymbol{D}}_1^T(\boldsymbol{q})-\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q}))\dot{\boldsymbol{q}}+\alpha k_v\Delta\boldsymbol{x}^T(\hat{\boldsymbol{J}}(\boldsymbol{q})-\boldsymbol{J}(\boldsymbol{q}))\dot{\boldsymbol{q}} \\
&-\alpha\Big\{\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})-\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}}+\dot{\boldsymbol{q}}^T\boldsymbol{J}^T(\boldsymbol{q})\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\
&+\Delta\boldsymbol{x}^T\dot{\hat{\boldsymbol{D}}}_2^{-1}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\Big\}.
\end{aligned}
\tag{2.129}
$$

Here, it is assumed that $\boldsymbol{x}$ is bounded due to the finite workspace of robot manipulator. From the last term on the right-hand side of Eq. (2.129), there exist a constant $c_1 > 0$ such that:

$$
\begin{aligned}
&\alpha\left\{\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})-\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}}+\dot{\boldsymbol{q}}^T\boldsymbol{J}^T(\boldsymbol{q})\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+\Delta\boldsymbol{x}^T\dot{\hat{\boldsymbol{D}}}_2^{-1}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\right\} \\
&\qquad \leq c_1\|\dot{\boldsymbol{q}}\|^2.
\end{aligned}
\tag{2.130}
$$

Substituting inequality (2.130) into Eq. (2.129) yields:

$$
\begin{aligned}
W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) \geq \dot{\boldsymbol{q}}^T(k_v\hat{\boldsymbol{D}}_2(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})-\alpha c_1\boldsymbol{I}_n)\dot{\boldsymbol{q}}&+\alpha k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\hat{\boldsymbol{D}}_1(\boldsymbol{q})\Delta\boldsymbol{x} \\
&+k_p\Delta\boldsymbol{x}^T(\hat{\boldsymbol{D}}_1^T(\boldsymbol{q})-\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q}))\dot{\boldsymbol{q}} \\
&+\alpha k_v\Delta\boldsymbol{x}^T(\hat{\boldsymbol{J}}(\boldsymbol{q})-\boldsymbol{J}(\boldsymbol{q}))\dot{\boldsymbol{q}}.
\end{aligned}
\tag{2.131}
$$

As the approximate transformation $\hat{\boldsymbol{D}}_2(\boldsymbol{q})$ is either $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ or $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$, the matrix $\hat{\boldsymbol{D}}_2(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})$ in (2.131) is either $\hat{\boldsymbol{J}}^T(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})$ or $\boldsymbol{I}_n$ and is always symmetric and positive definite if the approximate Jacobian matrix is non singular. Similarly, $\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\hat{\boldsymbol{D}}_1(\boldsymbol{q})$ is either $\boldsymbol{I}_n$, $\hat{\boldsymbol{J}}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ or $(\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}))^T\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$ which is also symmetric and positive definite. From Eq. (2.131), we have

$$W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) \geq (k_v\lambda_3 - \alpha c_1)\|\dot{\boldsymbol{q}}\|^2 - (k_p\rho_1 + \alpha k_v\rho_2)\|\Delta\boldsymbol{x}\|\cdot\|\dot{\boldsymbol{q}}\| + \alpha k_p\lambda_4\|\Delta\boldsymbol{x}\|^2, \quad (2.132)$$

where

$$\|\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}) - \hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q})\| \leq \rho_1 \quad (2.133)$$

$$\|\hat{\boldsymbol{J}}(\boldsymbol{q}) - \boldsymbol{J}(\boldsymbol{q})\| \leq \rho_2 \quad (2.134)$$

and $\lambda_3 \triangleq \lambda_{\min}[\hat{\boldsymbol{D}}_2(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})]$ and $\lambda_4 \triangleq \lambda_{\min}[\hat{\boldsymbol{D}}_2^{-1}(\boldsymbol{q})\hat{\boldsymbol{D}}_1(\boldsymbol{q})]$. Note that $-\|\Delta\boldsymbol{x}\|\cdot\|\dot{\boldsymbol{q}}\| \geq \frac{1}{2}(\|\Delta\boldsymbol{x}\|^2 + \|\dot{\boldsymbol{q}}\|^2)$. Therefore, we have:

$$\begin{aligned} W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) \geq & \left(k_v\lambda_3 - \frac{k_p}{2}\rho_1 - \alpha\frac{k_v}{2}\rho_2 - \alpha c_1\right)\|\dot{\boldsymbol{q}}\|^2 \\ & + \left(\alpha k_p\lambda_4 - \frac{k_p}{2}\rho_1 - \alpha\frac{k_v}{2}\rho_2\right)\|\Delta\boldsymbol{x}\|^2, \end{aligned} \quad (2.135)$$

Let $\rho_a = \max(\rho_1, \rho_2)$, if

$$\begin{aligned} \rho_a &< \frac{2\left(\lambda_3 - \frac{\alpha c_1}{k_v}\right)}{a+\alpha} \\ \rho_a &< \frac{2\alpha\lambda_4 a}{a+\alpha} \end{aligned} \quad (2.136)$$

then $W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}})$ is positive definite in $\Delta\boldsymbol{x}$ and $\dot{\boldsymbol{q}}$.

The stability of the closed-loop control system is specified by the following theorem:

Theorem 2.6 *Consider the generalised task-space PD controller* (2.122)*, the closed-loop system gives rise to the convergence of the sensory-space position error and the joint-space velocity, if the feedback gains k_p and k_v are chosen to satisfy conditions* (2.127) *and* (2.136) *and the approximate Jacobian matrix is chosen to satisfy conditions* (2.133) *and* (2.134).

Proof If the feedback gains k_p and k_v are chosen to satisfy conditions (2.127), (2.136), then both $V(\Delta\boldsymbol{x},\dot{\boldsymbol{q}})$ and $W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}})$ are positive definite in $\dot{\boldsymbol{q}}$ and $\Delta\boldsymbol{x}$. From Eq. (2.128), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) = -W(\Delta\boldsymbol{x},\dot{\boldsymbol{q}}) < 0. \quad (2.137)$$

Hence, $V(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}})$ is a Lyapunov-like candidate whose time derivative is negative definite in $\Delta \boldsymbol{x}$ and $\dot{\boldsymbol{q}}$. Therefore, both $\Delta \boldsymbol{x}$ and $\dot{\boldsymbol{q}}$ are bounded. The boundedness of $\dot{\boldsymbol{q}}$ ensures the boundedness of $\dot{\hat{\boldsymbol{x}}}$. From Eq. (2.124), it is seen that $\ddot{\boldsymbol{q}}$ is bounded. The boundedness of $\dot{\boldsymbol{q}}$ also ensures the boundedness of $\dot{\boldsymbol{x}}$ since $\boldsymbol{J}(\boldsymbol{q})$ are trigonometric functions of joint angles. Since both $\ddot{\boldsymbol{q}}$ and $\dot{\boldsymbol{x}}$ are bounded, $\dot{\boldsymbol{q}}$ and $\Delta \boldsymbol{x}$ are uniformly continuous. From Eqs. (2.128) and (2.135), we have $\dot{\boldsymbol{q}}$, $\Delta \boldsymbol{x} \in L_2(0, +\infty)$. Therefore, from Barbalat's lemma, $\Delta \boldsymbol{x} \to \boldsymbol{0}$ and $\dot{\boldsymbol{q}} \to \boldsymbol{0}$ as $t \to \infty$. $\triangle\triangle\triangle$

Similarly, the approximate transformations, transpose Jacobian, and inverse Jacobian matrix are said to be dual, as specified by the following corollary:

Corollary 2.1 *For the approximate Jacobian controller* (2.122)*, a duality property exists in the sense that the approximate Jacobian transpose* $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ *can be replaced by the approximate inverse Jacobian* $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$ *and conversely.*

Proof Let $\bar{\lambda}_m = \max\{\lambda_{\max}[\hat{\boldsymbol{J}}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})], \lambda_{\max}[\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})]\}$. Then from Eq. (2.126), it is possible to choose k_v and α such that

$$k_v - 2\alpha\bar{\lambda}_m > 0 \tag{2.138}$$

for both approximate inverse and transpose Jacobian matrices. Next, we let $\underline{\lambda}_3 = \min\{\lambda_{\min}[\hat{\boldsymbol{J}}^T(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})], 1\}$ and $\underline{\lambda}_4 = \min\{\lambda_{\min}[\hat{\boldsymbol{J}}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})], \lambda_{\min}[(\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}))^T\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})], 1\}$. Then Eq. (2.135) becomes

$$\begin{aligned} W(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}}) \geq & \left(k_v\underline{\lambda}_3 - \tfrac{k_p}{2}\bar{\rho}_a - \alpha\tfrac{k_v}{2}\bar{\rho}_a - \alpha c_1\right)\|\dot{\boldsymbol{q}}\|^2 \\ & + \left(\alpha k_p\underline{\lambda}_4 - \tfrac{k_p}{2}\bar{\rho}_a - \alpha\tfrac{k_v}{2}\bar{\rho}_a\right)\|\Delta \boldsymbol{x}\|^2, \end{aligned} \tag{2.139}$$

where $\bar{\rho}_a = \max(\rho_1, \rho_2)$ and

$$\|(\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}))^T - (\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d))^T\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q})\| \leq \rho_1 \tag{2.140}$$

$$\|\hat{\boldsymbol{J}}(\boldsymbol{q}) - \boldsymbol{J}(\boldsymbol{q})\| \leq \rho_2. \tag{2.141}$$

Hence if

$$\begin{aligned} \bar{\rho}_a &< \frac{2\left(\underline{\lambda}_3 - \frac{\alpha c_1}{k_v}\right)}{a + \alpha} \\ \bar{\rho}_a &< \frac{2\alpha\underline{\lambda}_4 a}{a + \alpha}, \end{aligned} \tag{2.142}$$

then $W(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}})$ is positive definite in $\dot{\boldsymbol{q}}$ and $\Delta \boldsymbol{x}$ for both approximate inverse Jacobian and approximate transpose Jacobian. The controller gains can therefore be chosen such that $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ can be replaced by $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$ and conversely. $\triangle\triangle\triangle$

2.5.2 Generalized Approximate Jacobian Adaptive Setpoint Control

Next, a generalized approximate Jacobian adaptive setpoint controller with uncertain gravitational force is considered as follows:

$$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{D}}_1(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \hat{\boldsymbol{D}}_2(\boldsymbol{q})\dot{\boldsymbol{x}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g, \tag{2.143}$$

$$\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{D}}_3(\boldsymbol{q})\Delta\boldsymbol{x}), \tag{2.144}$$

where $\bar{\boldsymbol{L}}_g \in \Re^{n_g \times n_g}$ is a positive definite matrix, $\boldsymbol{Y}_g(\boldsymbol{q}) \in \Re^{n \times n_g}$ is the gravity regressor (see *Property* 1.4), $\hat{\boldsymbol{\theta}}_g$ is an estimated parameter updated by (2.144), and $\hat{\boldsymbol{D}}_3(\boldsymbol{q})$ is an approximate transformation that is defined similarly to $\hat{\boldsymbol{D}}_1(\boldsymbol{q})$ and $\hat{\boldsymbol{D}}_2(\boldsymbol{q})$. As shown in Table 2.2, there are eight combinations of the approximate Jacobian adaptive controllers.

The closed-loop equation of the system is obtained by substituting Eq. (2.143) into Eq. (2.9),

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \left(\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + k_p \hat{\boldsymbol{D}}_1(\boldsymbol{q})\Delta\boldsymbol{x} + k_v \hat{\boldsymbol{D}}_2(\boldsymbol{q})\dot{\boldsymbol{x}} + \boldsymbol{Y}_g(\boldsymbol{q})\Delta\boldsymbol{\theta}_g = \boldsymbol{0}, \tag{2.145}$$

Table 2.2 Various combinations of approximate Jacobian adaptive control

	$\hat{\boldsymbol{D}}_1(\boldsymbol{q})$	$\hat{\boldsymbol{D}}_2(\boldsymbol{q})$	$\hat{\boldsymbol{D}}_3(\boldsymbol{q})$	Controller
$(C1)$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \hat{\boldsymbol{J}}^T(\boldsymbol{q})\dot{\boldsymbol{x}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x})$
$(C2)$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \hat{\boldsymbol{J}}^T(\boldsymbol{q})\dot{\boldsymbol{x}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x})$
$(C3)$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \dot{\boldsymbol{q}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x})$
$(C4)$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \dot{\boldsymbol{q}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x})$
$(C5)$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \hat{\boldsymbol{J}}^T(\boldsymbol{q})\dot{\boldsymbol{x}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x})$
$(C6)$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \hat{\boldsymbol{J}}^T(\boldsymbol{q})\dot{\boldsymbol{x}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x})$
$(C7)$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^T(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \dot{\boldsymbol{q}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^T(\boldsymbol{q})\Delta\boldsymbol{x})$
$(C8)$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$	$\boldsymbol{\tau} = -k_p \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x} - k_v \dot{\boldsymbol{q}} + \boldsymbol{Y}_g(\boldsymbol{q})\hat{\boldsymbol{\theta}}_g$ $\dot{\hat{\boldsymbol{\theta}}}_g = -\bar{\boldsymbol{L}}_g \boldsymbol{Y}_g^T(\boldsymbol{q})(\dot{\boldsymbol{q}} + \alpha \hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})\Delta\boldsymbol{x})$

where $\Delta\boldsymbol{\theta}_g = \boldsymbol{\theta}_g - \hat{\boldsymbol{\theta}}_g$. We define a generalized Lyapunov function candidate as

$$V(\Delta\boldsymbol{x}, \dot{\boldsymbol{q}}, \Delta\boldsymbol{\theta}_g) = \tfrac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \alpha\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}$$
$$+\tfrac{1}{2}k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\Delta\boldsymbol{x}+\tfrac{1}{2}\alpha k_v\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_3^T(\boldsymbol{q}_d)\hat{\boldsymbol{D}}_2(\boldsymbol{q}_d)\Delta\boldsymbol{x}+\tfrac{1}{2}\Delta\boldsymbol{\theta}_g^T\bar{\boldsymbol{L}}_g^{-1}\Delta\boldsymbol{\theta}_g. \quad (2.146)$$

Again, since $\hat{\boldsymbol{D}}_2(\boldsymbol{q})$ and $\hat{\boldsymbol{D}}_3(\boldsymbol{q})$ are either $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ or $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$, it can be easily verified that $\hat{\boldsymbol{D}}_3^T(\boldsymbol{q}_d)\hat{\boldsymbol{D}}_2(\boldsymbol{q}_d)$ is always symmetric and positive definite if the approximate Jacobian matrix is nonsingular at the desired position. Following a similar argument to one in previous subsections, we have

$$V(\Delta\boldsymbol{x}, \dot{\boldsymbol{q}}, \Delta\boldsymbol{\theta}_g) \geq \tfrac{1}{4}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \tfrac{1}{2}k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\Delta\boldsymbol{x}$$
$$+\alpha\left(\tfrac{1}{2}k_v\lambda_5 - \alpha\lambda_{m2}\right)\|\Delta\boldsymbol{x}\|^2 + \tfrac{1}{2}\Delta\boldsymbol{\theta}_g^T\bar{\boldsymbol{L}}_g^{-1}\Delta\boldsymbol{\theta}_g, \quad (2.147)$$

where $\lambda_5 \triangleq \lambda_{\min}[\hat{\boldsymbol{D}}_3^T(\boldsymbol{q}_d)\hat{\boldsymbol{D}}_2(\boldsymbol{q}_d)]$ and $\lambda_m \triangleq \lambda_{\max}[\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{D}}_3(\boldsymbol{q})]$. Hence k_v and α must be chosen such that

$$k_v\lambda_5 - 2\alpha\lambda_m > 0. \quad (2.148)$$

Differentiating Eq. (2.146) with respect to time and substituting Eqs. (2.145) and (2.144) into it yields

$$\tfrac{\mathrm{d}}{\mathrm{d}t}V(\Delta\boldsymbol{x}, \dot{\boldsymbol{q}}, \Delta\boldsymbol{\theta}_g) = -W(\Delta\boldsymbol{x}, \dot{\boldsymbol{q}}), \quad (2.149)$$

where

$$W(\Delta\boldsymbol{x}, \dot{\boldsymbol{q}}) = k_v\dot{\boldsymbol{q}}^T\hat{\boldsymbol{D}}_2(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}+\alpha k_p\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\hat{\boldsymbol{D}}_1(\boldsymbol{q})\Delta\boldsymbol{x}+k_p\Delta\boldsymbol{x}^T(\hat{\boldsymbol{D}}_1^T(\boldsymbol{q})$$
$$-\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q}))\dot{\boldsymbol{q}}+\alpha k_v\Delta\boldsymbol{x}^T(\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\hat{\boldsymbol{D}}_2(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})-\hat{\boldsymbol{D}}_3^T(\boldsymbol{q}_d)\hat{\boldsymbol{D}}_2(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q}))\dot{\boldsymbol{q}}$$
$$-\alpha\left\{\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) - \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}}\right.$$
$$\left.+\dot{\boldsymbol{q}}^T\boldsymbol{J}^T(\boldsymbol{q})\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \Delta\boldsymbol{x}^T\dot{\hat{\boldsymbol{D}}}_3(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\right\}. \quad (2.150)$$

From the last term on the right-hand side of Eq. (2.150), there exists a constant $c_2 > 0$ such that

$$\alpha\left\{\Delta\boldsymbol{x}^T\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\left(\tfrac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) - \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{J}^T(\boldsymbol{q})\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\right.$$
$$\left.+\Delta\boldsymbol{x}^T\dot{\hat{\boldsymbol{D}}}_3(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\right\} \leq \alpha c_2\|\dot{\boldsymbol{q}}\|^2. \quad (2.151)$$

Hence

$$W(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}}) \geq (k_v\lambda_3 - \alpha c_2)\|\dot{\boldsymbol{q}}\|^2 - (k_p\rho_1 + \alpha k_v\rho_3)\|\Delta \boldsymbol{x}\| \cdot \|\dot{\boldsymbol{q}}\| + \alpha k_p\lambda_6\|\Delta \boldsymbol{x}\|^2, \quad (2.152)$$

where

$$\|\hat{\boldsymbol{D}}_1^T(\boldsymbol{q}) - \hat{\boldsymbol{D}}_1^T(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q})\| \leq \rho_1 \quad (2.153)$$

$$\|\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\hat{\boldsymbol{D}}_2(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q}) - \hat{\boldsymbol{D}}_3^T(\boldsymbol{q}_d)\hat{\boldsymbol{D}}_2(\boldsymbol{q}_d)\boldsymbol{J}(\boldsymbol{q})\| \leq \rho_3 \quad (2.154)$$

and $\lambda_6 \overset{\triangle}{=} \lambda_{\min}[\hat{\boldsymbol{D}}_3^T(\boldsymbol{q})\hat{\boldsymbol{D}}_1(\boldsymbol{q})]$. Next, note that $-\|\Delta \boldsymbol{x}\| \cdot \|\dot{\boldsymbol{q}}\| \geq \frac{1}{2}(\|\Delta \boldsymbol{x}\|^2 + \|\dot{\boldsymbol{q}}\|^2)$, and therefore,

$$\begin{aligned} W(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}}) \geq & \left(k_v\lambda_3 - \frac{k_p}{2}\rho_1 - \alpha\frac{k_v}{2}\rho_3 - \alpha c_2\right)\|\dot{\boldsymbol{q}}\|^2 \\ & + \left(\alpha k_p\lambda_6 - \frac{k_p}{2}\rho_1 - \alpha\frac{k_v}{2}\rho_3\right)\|\Delta \boldsymbol{x}\|^2. \end{aligned} \quad (2.155)$$

Let $\bar{\rho}_2 = \max(\rho_1, \rho_3)$. If

$$\begin{aligned} \bar{\rho}_2 &< \frac{2\left(\lambda_3 - \frac{\alpha c_2}{k_v}\right)}{a+\alpha} \\ \bar{\rho}_2 &< \frac{2\alpha\lambda_6 a}{a+\alpha}, \end{aligned} \quad (2.156)$$

then $W(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}})$ is positive definite in $\dot{\boldsymbol{q}}$ and $\Delta \boldsymbol{x}$.

We are now ready to state the following theorem:

Theorem 2.7 *For the generalized approximate Jacobian adaptive controller describedby Eqs.* (2.143) *and* (2.144)*, the closed-loop system gives rise to the convergence of* $\boldsymbol{x}_d - \boldsymbol{x} \to \boldsymbol{0}$, $\dot{\boldsymbol{q}} \to \boldsymbol{0}$ *as* $t \to \infty$ *if the feedback gains* k_p *and* k_v *are chosen to satisfy conditions* (2.148) *and* (2.156) *and the approximate Jacobian matrix is chosen to satisfy conditions* (2.153) *and* (2.154).

Proof Since $V(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}}, \Delta\boldsymbol{\theta}_g)$ is positive definite in $\Delta \boldsymbol{x}$, $\dot{\boldsymbol{q}}$ and $\Delta\boldsymbol{\theta}_g$, and $W(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}})$ is positive definite in $\Delta \boldsymbol{x}$, $\dot{\boldsymbol{q}}$, from Eq. (2.149), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}}) = -W(\Delta \boldsymbol{x}, \dot{\boldsymbol{q}}) \leq 0. \quad (2.157)$$

Therefore, $\Delta \boldsymbol{x}$, $\dot{\boldsymbol{q}}$, and $\Delta\boldsymbol{\theta}_g$ are bounded. The boundedness of $\dot{\boldsymbol{q}}$ also ensures the boundedness of $\dot{\hat{\boldsymbol{x}}}$. From Eq. (2.145), it is seen that $\ddot{\boldsymbol{q}}$ is bounded. Since both $\ddot{\boldsymbol{q}}$ and $\dot{\boldsymbol{x}}$ are bounded, $\dot{\boldsymbol{q}}$ and $\Delta \boldsymbol{x}$ are uniformly continuous. From Eqs. (2.149) and (2.155), it is seen that $\dot{\boldsymbol{q}}$, $\Delta \boldsymbol{x} \in L_2(0, +\infty)$. Therefore, $\Delta \boldsymbol{x} \to \boldsymbol{0}$ and $\dot{\boldsymbol{q}} \to \boldsymbol{0}$ as $t \to \infty$. $\triangle\triangle\triangle$

Similarly, the approximate transformations, transpose Jacobian $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$, and inverse Jacobian matrix in the adaptive controllers are said to be dual, as specified by the following corollary:

Corollary 2.2 *For the generalized approximate Jacobian adaptive controller described by Eqs.* (2.143) *and* (2.144)*, a duality property exists in the sense that the approximate Jacobian transpose* $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ *can be replaced by the approximate inverse Jacobian* $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$ *and conversely.*

Proof Let $\underline{\lambda}_5 = \min\{1, \lambda_{\min}[\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}_d)], \lambda_{\min}[\hat{\boldsymbol{J}}(\boldsymbol{q}_d)\hat{\boldsymbol{J}}^T(\boldsymbol{q}_d)]\}$, and $\bar{\lambda}_{m2} = \max\{\lambda_{\max}[\hat{\boldsymbol{J}}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})], \lambda_{\max}[\hat{\boldsymbol{J}}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})]\}$. Then from Eq. (2.147), it is possible to chose k_v and α such that

$$k_v\underline{\lambda}_5 - 2\alpha\bar{\lambda}_{m2} > 0. \tag{2.158}$$

Next, let $\underline{\lambda}_3 = \min\{\lambda_{\min}[\hat{\boldsymbol{J}}^T(\boldsymbol{q})\hat{\boldsymbol{J}}(\boldsymbol{q})], 1\}$ and $\underline{\lambda}_6 = \min\{\lambda_{\min}[\hat{\boldsymbol{J}}(\boldsymbol{q})\hat{\boldsymbol{J}}^T(\boldsymbol{q})], \lambda_{\min}[(\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q}))^T\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})], 1\}$. Then Eq. (2.155) becomes

$$\begin{aligned} W(\Delta\boldsymbol{x}, \dot{\boldsymbol{q}}) \geq & \left(k_v\underline{\lambda}_3 - \frac{k_p}{2}\bar{\rho}_a - \alpha\frac{k_v}{2}\bar{\rho}_a - \alpha c_2\right)\|\dot{\boldsymbol{q}}\|^2 \\ & + \left(\alpha k_p\underline{\lambda}_6 - \frac{k_p}{2}\bar{\rho}_a - \alpha\frac{k_v}{2}\bar{\rho}_a\right)\|\Delta\boldsymbol{x}\|^2, \end{aligned} \tag{2.159}$$

where $\bar{\rho}_a = \max(\rho_1, \rho_3)$, and if

$$\begin{aligned} \bar{\rho}_a &< \frac{2\left(\underline{\lambda}_3 - \frac{\alpha c_2}{k_v}\right)}{a + \alpha} \\ \bar{\rho}_a &< \frac{2\alpha\underline{\lambda}_6 a}{a + \alpha}, \end{aligned} \tag{2.160}$$

then $W(\Delta\boldsymbol{x}, \dot{\boldsymbol{q}})$ is positive definite in $\dot{\boldsymbol{q}}$ and $\Delta\boldsymbol{x}$ for both approximate inverse Jacobian and approximate transpose Jacobian. The controller gains can therefore be chosen such that $\hat{\boldsymbol{J}}^T(\boldsymbol{q})$ can be replaced by $\hat{\boldsymbol{J}}^{-1}(\boldsymbol{q})$ and conversely. △△△

2.6 Summary and Notes

The dynamics of a robot manipulator is highly nonlinear with couplings between joints. It was first shown in Takegaki and Arimoto (1981); Arimoto and Miyazaki (1985); Arimoto (1996) using the Lyapunov method that a simple PD controller with gravity compensation is effective for setpoint control of a robot manipulator. The gravity compensation term can also be computed offline using the desired position (Tomei 1991; Kelly 1993; Arimoto 1996). A main drawback of the PD plus gravity controller is that exact knowledge of the gravitational force of the robot manipulator is required. The existence of modeling uncertainty in the gravitational term results in steady-state position error. One way to alleviate the problem is to increase the P control gain, which may lead to saturation of the control torques. A common practice to remove the steady-state error is to use a PID controller (Arimoto and Miyazaki 1983; Arimoto 1996).

A linear PID robot control system is asymptotically stable in a local sense. To achieve global asymptotic stability, a saturated or bounded nonlinear function of the position error can be used (Arimoto et al. 1994; Arimoto 1996). Other regulators for robot manipulators that do not use the exact knowledge of the gravitational force can be found in Kelly (1998), Ortega et al. (1995). If the structure of the gravitational force is known, an adaptive PD controller with gravity regressor can also be used to eliminate the steady-state position error in the presence of uncertain parameters (Tomei 1991; Kelly 1993; Arimoto 1996). The aforementioned controllers were formulated in joint space. This book focuses on sensory task-space feedback control of robot manipulators, and more detail on joint-space control can be found in Lewis et al. (1993), Arimoto (1996), Kelly et al. (2005). The use of the inner product for the derivation of Lyapunov functions can be found in Arimoto (1996).

Task-space feedback control laws with an imperfect Jacobian matrix were first considered in Miyazaki and Masutani (1990), where a sensor coordinate is defined as the task space and the exact knowledge of the Jacobian matrix of the mapping from Cartesian space to sensor space is not required. However, the exact model of the manipulator Jacobian matrix from joint space to Cartesian space is assumed to be known exactly. A fundamental benefit of task-space feedback control is its ability to deal with kinematic uncertainty. Setpoint controllers for robot manipulators with uncertain kinematics and dynamics were first proposed in Cheah et al. (1998). The approximate Jacobian setpoint controller (2.66) with task-space damping was proposed in Cheah et al. (1999). The controller requires the measurement of a task-space position using a task-space sensor such as a vision system. The task-space velocity is obtained by numerical differentiation of the position. In Cheah et al. (2003), an approximate Jacobian feedback controller using only joint-space damping was developed. The joint-space velocity is obtained by numerical differentiation of the joint position, which is usually less noisy than that in task space. In the presence of uncertainty in the gravitational force, the update law (2.93) or the integration control term Cheah et al. (1999) also requires the use of the inverse Jacobian matrix. An approximate Jacobian controller with adaptive gravity compensation was developed in Cheah et al. (2003). The main advantages of the controller are that the task-space velocity and the inverse of the approximate Jacobian matrix are not required. An update law can also be used to update the kinematic parameters of the estimated Jacobian matrix online (Cheah 2003; Dixon 2004; Dixon 2007) for setpoint control, and simpler stability conditions than those obtained using a fixed approximate Jacobian matrix can be obtained.

The duality property in task-space regulation of nonredundant robots was analyzed in Cheah (2008a), and it was shown that the transpose Jacobian matrix can be replaced by the inverse Jacobian matrix and conversely.

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