

Chapter 2

Some Remarks on the Transmission Line Matrix (TLM) Method and Its Application to Transient EM Fields and to EMC Problems

Peter Russer and Johannes A. Russer

Abstract Wolfgang J.R. Hoefer has pioneered the Transmission Line Matrix (TLM) method and made it a powerful tool for time-domain modeling of electromagnetic fields. In his scientific work, Wolfgang Hoefer always is placing a strong focus on imagery thinking and geometric and physical understanding of the electromagnetic phenomena. In this contribution, we invite the apt reader to stroll with us through the garden of TLM and would like to share with him some thoughts on the origin of the TLM method and also present some specific applications. We discuss the relation of the TLM method to Christian Huygens' model of light propagation and show how the TLM method can be deduced on the basis Huygens' model by application of network theory. We show how the TLM scheme can be embedded in a general discrete time circuit concept. The application of the TLM method to electromagnetic compatibility (EMC) problems is discussed. As a time-domain method, the TLM method is optimally suited to model broadband and transient electromagnetic phenomena and therefore, combining the TLM method with the Integral Equation method yields a powerful tool for the modeling of complex electromagnetic structures separated by large distances in free space. Introducing network models allows the application of correlation matrix methods for the modeling of stochastic fields.

Keywords Transmission line matrix • Electromagnetic compatibility (EMC) • Huygens' principle • Stochastic electromagnetic fields • Hybrid methods

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2.1 Introduction

Combining electromagnetic field theory, numerical methods, experimental techniques, and practical engineering principles, Wolfgang J.R. Hoefer has given numerous groundbreaking contributions to microwave circuit modeling and computer-aided design. Presumably, Wolfgang J.R. Hoefer's major contribution to knowledge is his work in computational electromagnetics, specifically in time-domain modeling of electromagnetic fields and its applications to high frequency analog and high-speed digital circuits. He pioneered the *Transmission Line Matrix* (TLM) method as a space- and time discrete model of electromagnetic phenomena. The TLM method first published by Johns and Beurle in 1971 [1, 2], and then further developed by Wolfgang Hoefer [3–7] is a space- and time discretizing method for time-domain modeling of electromagnetic structures. The TLM method not only has evolved into a powerful method for numerical electromagnetic field computation but also provides an imagery representation of the electromagnetic phenomena supporting their understanding and creative design of electromagnetic structures.

Wolfgang J.R. Hoefer gave seminal contributions to the TLM method and its application to Microwave and Millimeter-Wave circuit modeling [8], and he was the first to model nonlinear electromagnetic structures [9, 10], arbitrary frequency-dispersive boundaries, and materials [11, 12] in the time-domain using TLM diakoptics [13, 14] and numerical convolution techniques.

Time-domain methods for electromagnetic field computation are excellently suited for the treatment of broadband impulsive and transient phenomena. The electromagnetic full-wave transient modeling of electromagnetic structures by computing the field impulse response for a single impulse excitation allows the broadband characterization of the electromagnetic structures with a single computational run [3, 4, 15–17]. The data obtained by this way also allow the generation of compact lumped element equivalent circuits [18, 19]. Especially in the case of periodically space-discrete materials as for example negative refractive index metamaterials the TLM method has proven to be an excellently problem-matched tool [20].

Due to its appropriateness for the modeling of transient and impulsive broadband electromagnetic phenomena the TLM method is excellently suited for the treatment of *electromagnetic interference* (EMI) problems [21]. Combining the TLM method with the Integral Equation method yields versatile hybrid methods for the modeling of complex electromagnetic structures separated by large distances of free space [22–29]. In EMI we usually have to deal with stochastic electromagnetic fields. Applying the TLM method converts the EM field problem into a network problem. This facilitates an efficient application of network and correlation matrix methods for the numerical simulation of noisy electromagnetic fields, accounting for arbitrary correlations between stochastic radiation sources [30, 31].

In this contribution, we first focus on the basic principle of the TLM method which is strongly related to the concept of light propagation formulated by Christian Huygens more than 300 years ago. We show that we can find the TLM scheme combining basic concepts of Huygens with circuit theory without access to Maxwell's equations. So the TLM scheme can be considered as a fundamental concept of discrete electrodynamics. We further stress the relationship between the TLM concept and circuit theory and show that the *wave digital filter* concept provides a unified framework for a combined discrete field and circuit theory.

2.2 Maxwell's Theory and Huygens' Theory of Light

The poetry of physics is revealed in its great equations. Like fine poetry which can give language the most concise, concentrated, and beautiful form and expression, also equations representing physical laws may reveal an extraordinary power and beauty. A great equation casts a natural law ascertained by observation and recognition of the patterns of natural phenomena into a concise mathematical form. Great equations spread an inexhaustible plethora of new perceptions and insights enabling scientists to find out things which have been beyond all means at the time when these equations have been established. Eugene P. Wigner has reasoned that *it is not at all natural that "laws of nature" exist, much less that man is able to discover them* [32].

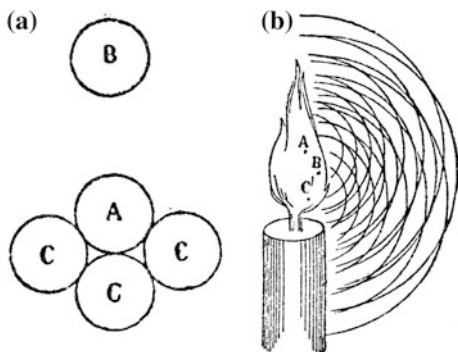
Providing the fundamental analytic and conceptual framework for the understanding and evaluating of electromagnetic phenomena Maxwell's equations are an outstanding example of such great equations [33]. They not only allow a complete description of the phenomena of electrodynamics but also are a basis of mental imagery supporting creative solution of problems. Heinrich Hertz wrote in his treatise on Maxwell's equations:

One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser than their discoverers, that we get more out of them than was originally put into them [34, 35].

Frank Wilczek added *The power of equations can seem magical. Like the brooms created by the Sorcerer's Apprentice, they can take on a power and life of her own, giving birth to consequences that their creator did not expect* [35].

The analog world of Maxwell's theory is based on continuous representation of space, time and field amplitudes. In the pre-computer age, scientists and engineers were reliant on analytic methods and highly sophisticated analytic tools for solving Maxwell's equations have been developed, see for example [36–39]. Increasing bandwidth and data rates of modern electronic circuits and systems demand more and more a design of circuit structures on the basis of electromagnetic full-wave modeling. Today's computer-oriented numerical methods allow the analysis of complex geometries with the ability to globally model and optimize large electromagnetic structures [40, 41]. Numerical methods for electromagnetic field

Fig. 2.1 Huygens' model of light propagation: **a** Ether particles, **b** Superposition of elementary waves [45]



computation are essentially based on the solution of algebraic equations, either obtained from analytic schemes as for example the integral equation method [39], by method of moments [42], or by direct discretization of Maxwell's equation's as for example in finite difference [43] or finite integration [44] schemes.

As imagery thinking is highly important for creative problem solution this raises the question whether we can also develop an imagery understanding of electromagnetic phenomena on the basis of discrete concepts. In this context it is worth noting that Christian Huygens has given a lucid imagery concept of light propagation in his *Traité de la lumière*, which he presented in 1690, explaining light propagation by a model looking like a billiard game of small ether spheres [45]. As done in the Maxwell theory later on, Huygens' model follows the principle of locality which states that an object is influenced directly only by its immediate surroundings. Huygens imagined light propagation mediated by particles of ether with perfect hardness, however possessing an elasticity, and arranged randomly so that one of them touches several others. This does not hinder them from transmitting their movement and from spreading it always forward. Figure 2.1a shows Huygens' drawing of the interacting ether particles. Huygens states: *When the sphere A here, touches several other similar spheres CCC, if it is struck by another sphere B in such a way as to exert an impulse against all the spheres CCC which touch it, it transmits to them the whole of its movement, and remains after that motionless like the sphere B.* Figure 2.1b shows Huygens drawing illustrating his famous principle formulated as follows: *...each little region of a luminous body, such as the sun, a candle or a burning coal, generates its own waves of which that region is the center. Thus in the flame of a candle, having distinguished the points A,B,C, concentric circles described about each of these points represent the waves which come from them.* Although Huygens model cannot describe polarization and interference it is an important step toward the understanding of the propagation of light and was able to give an explanation of most phenomena of light propagation known at Huygens' time. Moreover, we notice that Huygens' model implies the fundamental physical principle of locality which states that an object is influenced directly only by its immediate surroundings and it suggests a finite velocity of light propagation.

2.3 TLM: A Discrete Scheme of Electromagnetism

In this section, we will introduce a space- and time-discrete scheme, the so-called *Transmission Line Matrix* (TLM) scheme describing the dynamics of the electromagnetic field. With regard to electromagnetics Wolfgang Hoefer has given with his work on the TLM method groundbreaking contributions on discrete modeling of electromagnetic fields by creating powerful and versatile computational tools and by contributing to the understanding of the discrete models [3, 4]. The concept of the TLM method based on discretization, local interaction, and scattering relates closely to Huygens' reasoning.

Historically, the TLM scheme, first published by Peter B. Johns in 1971 [1, 2] is more than a century younger than Maxwell's equations. Furthermore, James Clerk Maxwell has established his equations in 1865 [46, 47] prior to the experimental demonstration of the existence of electromagnetic waves by Heinrich Hertz in 1888 [48], whereas the TLM scheme already is founded on the concept of wave propagation. Nevertheless, it may be interesting to put the question whether it would have been possible to find the TLM scheme without any knowledge of Maxwell's theory. These considerations will show that the TLM concept besides being an efficient tool for numerical computations also can be considered a fundamental theoretical concept fostering physical understanding of electromagnetic phenomena.

In his paper "The unreasonable effectiveness of mathematics in natural sciences" [32] Eugene P. Wigner raised the question: *How do we know that, if we made a theory which focusses its attention on phenomena we disregard and disregards some of the phenomena now commanding our attention, that we could not build another theory which has little in common with the present one but which, nevertheless, explains just as many phenomena as the present theory?* In some sense the TLM scheme can be considered as such an alternative scheme to Maxwell's theory. Like Maxwell's theory it gives a physically correct description of electromagnetic phenomena. Different from Maxwell's theory which is a continuum theory, the TLM scheme is discrete in space and time. We could consider this to be a drawback. However, since theoretical investigations usually end up in numerical computations and all numerical values are discrete also discrete schemes are at least potentially exact. We discard the argument of space- and time-discretization due to a physical elementary length since it is meaningless in the context of a classical theory of electromagnetism.

With the TLM scheme, we establish a space- and time-discrete model of electrodynamics based on the interaction of finite size space cells. In the TLM scheme the electromagnetic field is modeled by wave pulses propagating in a mesh of transmission lines and subsequently scattered in the mesh nodes. A simple 2D-simulator designed as an educational tool shows that TLM is strongly related to the Huygens billiard game [49].

The TLM scheme is a discrete scheme describing electrodynamics in discretized space and time. It may be derived from Maxwell's equations as it has been

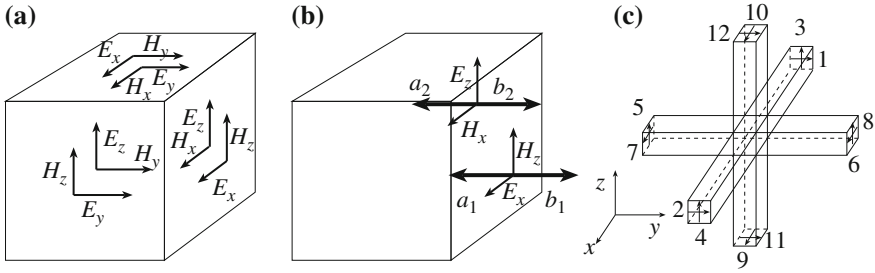


Fig. 2.2 Schematic of the TLM cell: **a** Space cell with samples of the tangential electric and magnetic field values, **b** Wave pulse amplitudes, **c** TLM node

done for example using a propagator integral approach [50], the method of moments [51] or by finite integration [52, 53]. However, we will show in the following that for discretized space the TLM scheme can be established using circuit theory on the basis of first principles, i.e., symmetry and energy conservation without recourse to Maxwell's equations. Network-based circuit theory, based on Ohm's law (1927) and Kirchhoff's laws (1845), is older than Maxwell's theory [54, 55]. These laws grew from an empirical base, and can now be derived from Maxwell's equations and the constitutive equations (see, e.g., [53, pp. 28, 38–40]). However, Kirchhoff's laws and Ohm's law can also be formulated on an axiomatic basis independently from Maxwell's equations [56]. The idea is to discretize the space into cubical cells and then to consider these cubical cells as multiports. Each cubical cell has six surfaces. Considering polarizations at each surface we have to model these cells by twelve-ports.

We will guide our search for a space- and time-discrete scheme by a few basic facts like symmetry and energy conservation. In analogy to Huygens' scheme, we subdivide the 3D-space into cubical cells as shown in Fig. 2.2a. In TLM, we use electromagnetic wave amplitudes as the basic variables. Figure 2.2b exemplifies the assignment of incident and scattered wave amplitudes to the electromagnetic field amplitudes. For a detailed description of the formalism, see for example [57]. We note that the replacement of field amplitudes by wave amplitudes requires the reference of a given transverse plane which is in our case the respective cell surface plane. Each cell may interact with each of its six neighbor cells. Since the waves are incident from six sides and in two polarizations the TLM node is represented by a twelve-port.

In the TLM scheme the electromagnetic field is modeled by wave pulses represented in a Cartesian mesh of transmission lines and being scattered in the mesh nodes. In a TLM mesh with a total number of N nodes we have to consider $12 N$ incident and $12 N$ scattered wave pulses. Figure 2.2c shows the schematic representation of a symmetric condensed TLM node. To account for the two transverse polarizations a pair of transmission lines is assigned to every branch of the mesh. In a compact formulation of the TLM scheme, we summarize all

12 N incident wave pulses in the vector $|_ka\rangle$ and all 12 N scattered wave pulses in the vector $|_kb\rangle$. The index k enumerates the discrete time step. We can formulate the TLM scheme in the compact notation [53, 57, 58]:

$$|_{k+1}b\rangle = \mathbf{S}|_ka\rangle, \quad (2.1a)$$

$$|_ka\rangle = \mathbf{\Gamma}|_kb\rangle, \quad (2.1b)$$

where the scattering matrix \mathbf{S} describes the instantaneous scattering of the wave pulses in the TLM node. The TLM cell consists of the TLM node and the transmission line arms. Due to the propagation through transmission line arms the scattered pulses are delayed by one discrete time step and k is incremented by 1. The connection matrix $\mathbf{\Gamma}$ describes the connection of the TLM cells. The matrices \mathbf{S} and $\mathbf{\Gamma}$ describe connection circuits and must be symmetric, real, Hermitian, unitary, and orthogonal. For a detailed description of this formalism see [53, 57, 58].

A fully symmetric twelve-port is described by the \mathbf{S} matrix

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_d & \mathbf{S}_0 & \mathbf{S}_0^T \\ \mathbf{S}_0^T & \mathbf{S}_d & \mathbf{S}_0 \\ \mathbf{S}_0 & \mathbf{S}_0^T & \mathbf{S}_d \end{bmatrix}, \quad (2.2a)$$

with the submatrices

$$\mathbf{S}_d = \begin{bmatrix} \rho & \tau_1 & 0 & 0 \\ \tau_1 & \rho & 0 & 0 \\ 0 & 0 & \rho & \tau_1 \\ 0 & 0 & \tau_1 & \rho \end{bmatrix}, \quad \mathbf{S}_0 = \begin{bmatrix} 0 & 0 & -\tau_3 & \tau_3 \\ 0 & 0 & \tau_3 & -\tau_3 \\ \tau_2 & \tau_2 & 0 & 0 \\ \tau_2 & \tau_2 & 0 & 0 \end{bmatrix}. \quad (2.2b)$$

For a lossless structure we have to impose the unitarity condition.

$$\mathbf{S}^\dagger \mathbf{S} = \mathbf{1}. \quad (2.3)$$

With this we obtain the elements of the \mathbf{S} matrix fulfilling the unitarity condition summarized in Table 2.1. From these 16 solutions only the first one gives the physically correct description of electromagnetic field propagation in discretized free space. We have considered that we cannot *derive* a fundamental natural law, rather we have to *find* it. Assuming that the node affecting the scattering of the wave pulses must be a twelve-port and by our symmetry and energy conservation considerations we have been guided up to the 16-fold crossroad summarized in Table 2.1. Now we have to choose the solution which is in coincidence with the observed wave phenomenon. We will clearly end up with the solution given in the first column.

Table 1 Elements of the S matrix fulfilling the unitarity condition

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
ρ	0	0	0	0	0	0	1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
τ_1	0	0	0	0	1	-1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
τ_2	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
τ_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0

The scattering matrix S of the TLM node therefore is given by

$$S = \begin{bmatrix} 0 & S_0 & S_0^T \\ S_0^T & 0 & S_0 \\ S_0 & S_0^T & 0 \end{bmatrix}, \quad (2.4a)$$

with the submatrices S_0 given by

$$S_0 = \begin{bmatrix} 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}. \quad (2.4b)$$

The TLM scheme as formulated in (2.1a), (2.1b) is a compact representation of discrete electrodynamics. The scattering matrix S_0 describes the instantaneous scattering of the TLM signal pulses in the twelve-port TLM node. We note that S_0 is real, symmetric, Hermitian, orthogonal, and unitary. The twelve-port TLM node can be described by a connection network exhibiting only connections and ideal transformers [59, 60]. Rotating the polarizations by 45° yields a decomposition of the twelve-port node into two unconnected six-ports which can be represented without ideal transformers by simply connecting these ports [61, 62].

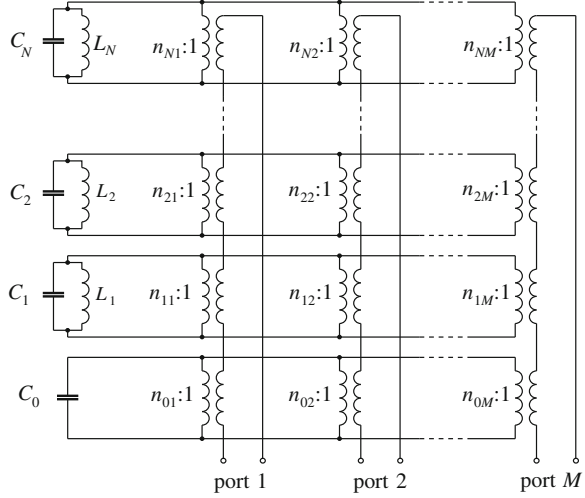
The TLM scheme summarized in the compact form of Eq. (2.1a), (2.1b) is the time-discrete counterpart to Maxwell's equation. It provides a comprehensive description of the dynamics of the discretized electromagnetic field on the basis of Huygens' principle.

2.4 System Identification and Global Network Modeling

In general, distributed circuits can be modeled by a lumped element network with arbitrary accuracy. However, since an accurate reproduction of the transfer function is only needed for a given frequency band, the number of lumped circuit elements can be limited. The global models are very compact and allow an accurate network modeling of the electromagnetic structures with a minimum number of circuit elements.

The first step in global network modeling for a distributed circuit described either by measured data or data obtained from numerical full-wave modeling usually is to build a system model by system identification (SI) methods [63]. SI can be used to extract admittance or impedance matrices of electromagnetic systems [15, 16, 18]. In the following the z -domain transfer function $\tilde{H}(z)$

Fig. 2.3 Canonical foster impedance multiport [53, p. 445]



represents a matrix element of the z -domain impedance matrix $\tilde{Z}(z)$ or admittance matrix $\tilde{Y}(z)$, respectively.

Applying the singularity expansion method [64], one can express the electromagnetic system response function $\tilde{H}(z)$ in the form

$$\tilde{H}(z) = \tilde{H}'(z) + \tilde{H}''(z) = \sum_{n=1}^N \frac{A_n(z)}{z - z_n} + \tilde{H}''(z), \quad (2.5)$$

where $\tilde{H}'(z)$ and $\tilde{H}''(z)$ denotes the transient and driven parts, respectively, of the system response. $\tilde{H}''(z)$ is formed when the excitation wavefront is interacting with the system.

Based on the network description by positive rational functions obtained from system identification methods applied to the signal transfer characteristics obtained from numerical full-wave simulation, equivalent lumped element networks can be synthesized. Reciprocal passive lossless structures, for example, can be modeled by Foster lumped element equivalent circuits and radiating structures by Cauer-type lumped element equivalent circuits [53, 55, 65–67].

Figure 2.3 shows the canonical Foster impedance multiport realization of a reciprocal passive multiport [53, p. 445]. Each parallel resonant circuit represents a pair of frequency poles describing an eigenfrequency of the circuit to be modeled. Figure 2.4 shows the canonical Foster admittance multiport realization of a reciprocal passive multiport which is dual to the Foster impedance multiport realization [53, p. 443]. These Foster equivalent circuits are global model of the electromagnetic structure since each eigenfrequency depends on the complete structure.

The Cauer-type ladder network depicted in Fig. 2.5 models a spherical radiating mode [68], [69, p. 279], [53, p. 447]. Also this equivalent circuit provides a global model since every mode extends over the complete problem space.

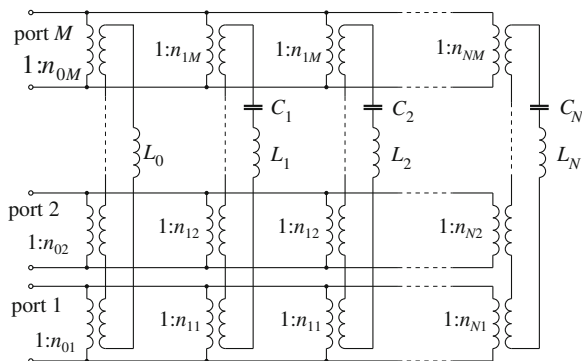


Fig. 2.4 Canonical foster impedance multiport [53, p. 443]

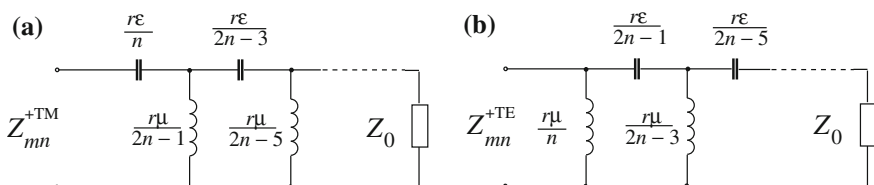


Fig. 2.5 Lumped Element equivalent circuits of **a** Transverse magnetic and **b** Transverse electric spherical waves [68], [69, p. 279], [53, p. 447]

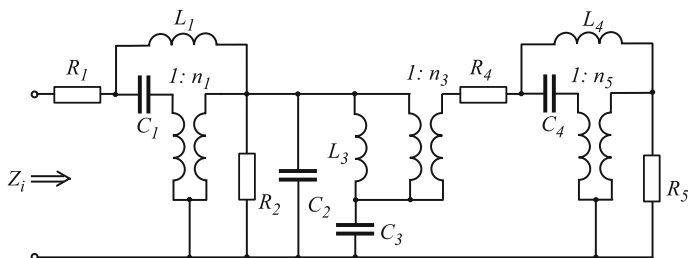


Fig. 2.6 The equivalent circuit of the driving-point admittance [73]

While purely reactive electromagnetic systems can be modeled using aforementioned Foster multiport realization, a Brune's synthesis approach can be followed to deal with lossy structures [70]. The Brune's procedure yields a lumped element network with minimum number of elements [71, 72]. It realizes an admittance or impedance matrix of order N as a lossless two-port terminated by an admittance or impedance of the order $N - 2$. The example of a lumped element equivalent circuit synthesis of a four-port microwave structure can be found in [73]. Figure 2.6 gives an example of Brune's realization of the driving-point impedance of a one-port. It can be seen that a passive equivalent circuit contains not only R, L, C elements but also ideal transformers. Brune synthesis of two-ports

was described in [74, 75] and in [72, 76, 77] Brune's multiport synthesis was treated systematically. The method allows to synthesize linear passive reciprocal multiport circuits using lumped element RLC networks containing only positive circuit elements.

2.5 The TLSC Scheme

In the following, we will give a generalization of the TLM scheme that yields a discrete time description of general linear reciprocal networks with the TLM network as a special case. We use the z-transform to convert the discrete time-domain signal $x[k]$, which is a sequence of real numbers and where k is the integer discrete time parameter, into a complex frequency-domain representation [78, 79]. To discretize a time-continuous variable $x(t)$ in time we take samples at integer multiples $n\tau$ of the chosen sampling time interval τ . The z-transform of $x[k]$ is given by

$$\tilde{X}(z) \equiv Z\{x_k\} = \sum_{k=0}^{\infty} z^{-k} x_k \quad \text{with } x_k \equiv x(k\tau) . \quad (2.6)$$

In the z-domain we can express the TLM scheme (2.1a), (2.1b) by

$$|\tilde{b}(z)\rangle = \frac{1}{z} \mathbf{S} |\tilde{a}(z)\rangle, \quad (2.7a)$$

$$|\tilde{a}(z)\rangle = \Gamma |\tilde{b}(z)\rangle, \quad (2.7b)$$

where $|\tilde{a}(z)\rangle$ and $|\tilde{b}(z)\rangle$ are the z-domain TLM state vectors. We already have shown that discrete time-domain lumped element networks, consisting of inductors, capacitors, resistors, and exciting sources embedded in a connection circuit with ideal transformers, can be represented in the z-domain and combined with the TLM scheme [80–82]. Applying the Richards transformation [83], inductors and capacitors are replaced by shorted and open transmission line stubs, yielding a time-discrete *transmission line segment circuit* (TLSC) approach for efficient time-domain modeling of electromagnetic structures including also lumped elements.

To do this, we have to bear in mind that due to Bartlett's theorem a transmission line segment can be represented by a simple network of open and shorted stubs. Figure 2.7b shows the stub equivalent circuit for the transmission line segment in Fig. 2.7a. The stub equivalent circuit contains two open stubs and two shorted stubs, both exhibiting half the length of the transmission line segment in Fig. 2.7a.

Replacing the transmission line segments in a TLM mesh by stub equivalent circuits yields a TLM network containing only open and shorted stubs of uniform length and an interconnection circuit. This interconnection circuit is represented by

Fig. 2.7 **a** Transmission line segment two-port, **b** Stub equivalent circuits

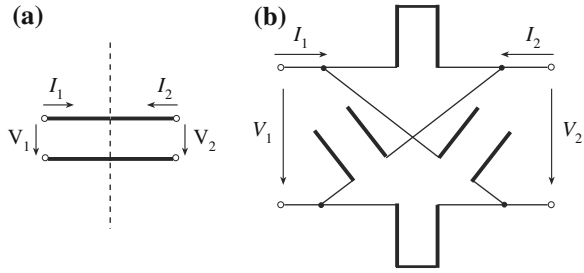
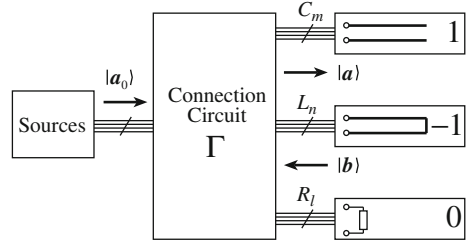


Fig. 2.8 The TLSC scheme



a scattering matrix which again is symmetric, real, Hermitian, unitary, and orthogonal and exhibits only the eigenvalues ± 1 . Replacing the transmission line segments by the stub equivalent circuit yields the very interesting possibility to modify the characteristic impedance of the open and shorted stubs independently. Increasing the characteristic impedance of the shorted stubs simulates an increase of the material permeability and reducing the characteristic impedance of the open stubs models increased material permittivity.

We also can represent the TLM scheme by a transmission line segment circuit (TLSC) as depicted in Fig. 2.8. We have added also resistive terminations to account for material losses. In z -domain, the scattering matrix representing the stubs and the matched terminations exhibits the diagonal form

$$\tilde{\mathbf{S}}(z) = z^{-1} \mathbf{S} \quad \text{with} \quad \mathbf{S} = \text{diag}[1, -1, 0], \quad (2.8)$$

where the diagonal submatrices 1 , -1 and 0 of dimension $M \times M$, $N \times N$, and $L \times L$, respectively represent M open stubs, N shorted stubs, and L matched terminations. Summarizing all incident wave pulses in \mathbf{S} , and all scattered wave pulses in $|\tilde{\mathbf{b}}\rangle$, denoting with $|\tilde{\mathbf{a}}\rangle_s$ the wave pulses exciting the structure, and with $|\tilde{\mathbf{b}}\rangle_r$ the port responses, we obtain the *transmission line segment circuit* (TLSC) scheme represented by the state equations

$$|\tilde{\mathbf{b}}\rangle = z^{-1} \mathbf{S} |\tilde{\mathbf{a}}\rangle, \quad (2.9a)$$

$$|\tilde{\mathbf{a}}\rangle = [\mathbf{\Gamma}_0, \mathbf{\Gamma}_s] \begin{bmatrix} |\tilde{\mathbf{b}}\rangle \\ |\tilde{\mathbf{a}}\rangle_s \end{bmatrix}, \quad (2.9b)$$

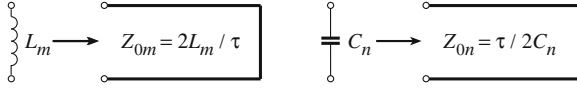


Fig. 2.9 The Richards transformation

$$|\tilde{\mathbf{b}}\rangle_r = \mathbf{\Gamma}_Q |\tilde{\mathbf{a}}\rangle_s, \quad (2.9c)$$

where the connection matrix $\mathbf{\Gamma}$ has been split into the parts $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_s$, and $\mathbf{\Gamma}_Q$ is the output matrix. This is formally identical with the transmission line matrix (TLM) scheme already discussed in Sect. 2.3. The TLM and TLSC scheme are topological schemes. That means, the model is completely defined by the interconnect structure or network structure of the scheme.

From the state Eq. (2.9a)–(2.9c) we obtain the response function

$$\tilde{\mathbf{H}} = \mathbf{\Gamma}_Q (z\mathbf{I} - \mathbf{\Gamma}_0 \mathbf{S})^{-1} \mathbf{\Gamma}_R = \sum_{k=1}^{\infty} z^{-k} \mathbf{\Gamma}_Q (\mathbf{\Gamma}_0 \mathbf{S})^{k-1} \mathbf{\Gamma}_R \quad (2.10)$$

relating the port response vector $|\tilde{\mathbf{a}}\rangle_r$ to the excitation vector $|\tilde{\mathbf{a}}\rangle_s$ by

$$|\tilde{\mathbf{a}}\rangle_r = \tilde{\mathbf{H}} |\tilde{\mathbf{a}}\rangle_s. \quad (2.11)$$

An interesting option is to establish transmission line segment circuit models representing global lumped element models. This can be done by applying the Richards transform [83] to LC models and replacing the capacitors and inductors by open and shorted stubs. Figure 2.9 shows the correspondence between LC and stub elements. The mathematical equivalent of the Richards transform is the z-transform:

$$s \rightarrow \begin{array}{cc} \text{Richards transform} & \text{z-transform} \\ \frac{2e^{s\tau} - 1}{\tau e^{s\tau} + 1} & \frac{2z - 1}{\tau z + 1} \end{array}. \quad (2.12)$$

Figure 2.10 shows a Cauer type TLSC equivalent circuits corresponding to the lumped element equivalent circuits of spherical waves depicted in Fig. 2.5.

An example for an efficient hybridization of TLM and TLSC modeling is combining TLM modeling of an electromagnetic structure embedded in a spherical region with TLSC modeling of the radiated field modes outside the sphere [84]. Figure 2.11a shows a bow-tie antenna embedded in a virtual sphere. This spherical region is discretized by a TLM mesh.

Figure 2.11b shows the input impedance of the bow-tie antenna computed by hybrid TLM-TLSC modeling in comparison with the results obtained by Method of

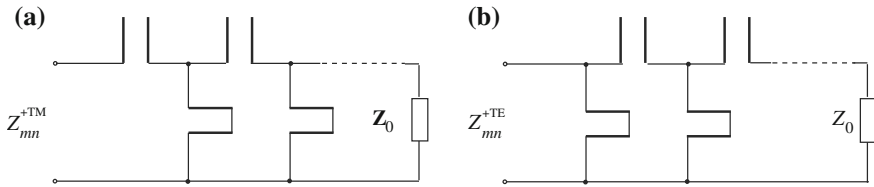


Fig. 2.10 Lumped element equivalent circuits of **a** Transverse magnetic and **b** Transverse electric spherical waves

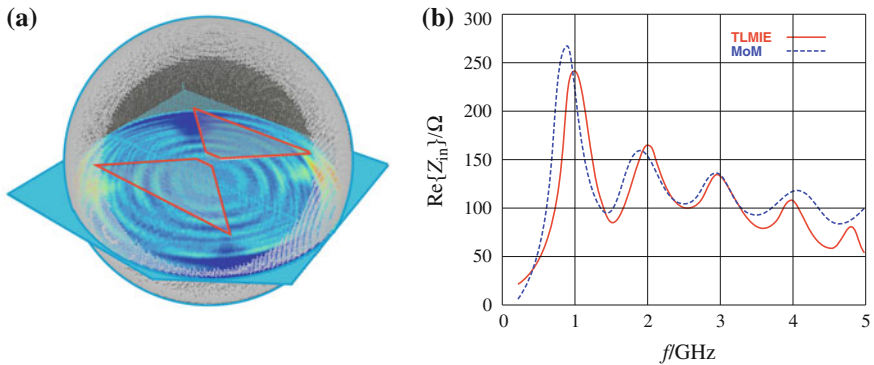


Fig. 2.11 **a** Bow-tie antenna, **b** Input Impedance of the bow-tie antenna [84]

Moments (MoM) modeling. Different from MoM modeling the hybrid TLM-TLSC approach can be applied to complex three-dimensional radiating electromagnetic structures.

2.6 TLM-Integral Equation Schemes

Simulation tools suitable for solving EMC problems provide for modeling of transient electromagnetic fields over a broad band and must be applicable to dispersive, nonuniform, nonlinear, and electrically large electromagnetic structures. In many EMC and EMI problems one has to deal with interacting objects of arbitrary shape separated by large distances in free space.

The hybrid TLM-IE methods are very well suited for the numerical modeling of the electromagnetic interaction between complex objects separated by large free space regions. While the field inside the objects is modeled by the TLM method, the electromagnetic interaction between the objects is described via Green's functions [22–29]. The empty space outside the TLM subdomain is not discretized and therefore does not contribute to the total computational effort.

A hybrid method combining the TLM method and the time-domain method of moments allowing the accurate modeling of the transient interference between a perfectly conducting thin surface of arbitrary shape and a complex object separated by large free space is described in [25]. In order to obtain the field radiated from the TLM subdomain, we replace the sources within the TLM subdomain by equivalent current sources on the interface of the TLM subdomain. Using dyadic free space Green's function, the radiated electric and magnetic field components \mathbf{E}^r and \mathbf{H}^r are obtained from

$$\begin{aligned} \mathbf{E}^r(\mathbf{r}, t) = & \frac{T}{4\pi} \iint_S dS' \left\{ \frac{-\mu}{R} \frac{\partial}{\partial t} (\mathbf{n}(\mathbf{r}') \times \mathbf{H}_{B \text{ tot}}(\mathbf{r}', \tau)) + \frac{1}{R^3} \left[1 + (t - \tau) \frac{\partial}{\partial t} \right] \right. \\ & \times [(\mathbf{n}(\mathbf{r}') \times \mathbf{E}_{B \text{ tot}}(\mathbf{r}', \tau)) \times \mathbf{R} + (\mathbf{n}(\mathbf{r}') \times \mathbf{E}_{B \text{ tot}}(\mathbf{r}', \tau)) \cdot \mathbf{R}] \Big\}_{\tau=t-\frac{R}{c}}, \end{aligned} \quad (2.13a)$$

$$\begin{aligned} \mathbf{H}^r(\mathbf{r}, t) = & \frac{T}{4\pi} \iint_S dS' \left\{ \frac{\varepsilon}{R} \frac{\partial}{\partial t} (\mathbf{n}(\mathbf{r}') \times \mathbf{E}_{B \text{ tot}}(\mathbf{r}', \tau)) + \frac{1}{R^3} \left[1 + (t - \tau) \frac{\partial}{\partial t} \right] \right. \\ & \times [(\mathbf{n}(\mathbf{r}') \times \mathbf{H}_{B \text{ tot}}(\mathbf{r}', \tau)) \times \mathbf{R} + (\mathbf{n}(\mathbf{r}') \times \mathbf{H}_{B \text{ tot}}(\mathbf{r}', \tau)) \cdot \mathbf{R}] \Big\}_{\tau=t-\frac{R}{c}}. \end{aligned} \quad (2.13b)$$

where $\mathbf{E}_{B \text{ tot}}$ and $\mathbf{H}_{B \text{ tot}}$ are the total electric and magnetic field and the subdomain's boundary. The total tangential field $\mathbf{E}_{B \text{ tot}}$ is expanded using space and time retarded pulse functions, P and Q , and the unknown expansion coefficients $\mathbf{E}_\varphi(u, j)$,

$$\mathbf{E}_{B \text{ tot}}(\mathbf{r}, \tau) = \sum_{v=1}^V \sum_{i=1}^N \mathbf{E}_\varphi(\mathbf{r}_v, t_i) P(\mathbf{r} - \mathbf{r}_v) Q(t - t_i), \quad (2.14)$$

where V is the number of patches on the interface and N are the time steps under consideration. Using a time marching scheme, we obtain the unknown expansion coefficients of each subsequent time step by solving

$$\begin{aligned} \mathbf{E}_\varphi(u, j) = & {}^{TLM} \mathbf{E}_B^{\text{inc}}(u, j) + \mathbf{E}_{BC}^r(u, j) \\ & + \sum_{v=1}^V \sum_{i=1}^N \{ \mathbf{G}_E(u, v, j - i) \mathbf{E}_\varphi(v, i) + \mathbf{G}_H(u, v, j - i) \mathbf{H}_\varphi(v, i) \}, \end{aligned} \quad (2.15)$$

where ${}^{TLM} \mathbf{E}_B^{\text{inc}}$ is the tangential field at the interface due to sources within the TLM subdomain, \mathbf{E}_{BC}^r are contributions to the radiated field originating from sources or scatterers outside the TLM subdomain, and \mathbf{G}_E and \mathbf{G}_H are operators representing the Green's function formulation to account for interaction of the electromagnetic field on surface patches of the TLM subdomain with each other. In order to handle late time instabilities, an averaging scheme

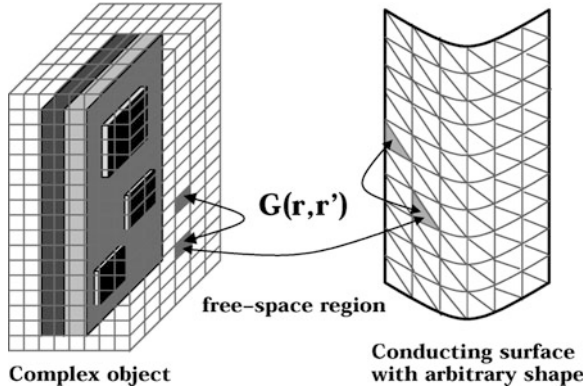


Fig. 2.12 Electromagnetic coupling between a complex object and arbitrarily shaped conducting surface [26]

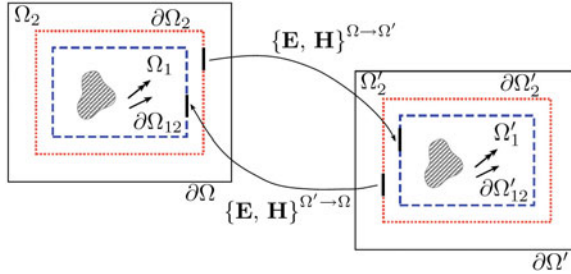


Fig. 2.13 Electromagnetically coupled TLM subdomains Ω and Ω' [28]

$$\tilde{E}_{\phi u, j} = \frac{1}{4} (\tilde{E}_{\phi u, j-1} + 2E_{\phi u, j} + E_{\phi u, j}) \quad (2.16)$$

is implemented. Figure 2.12 shows a complex object embedded into a discretized TLM region and an arbitrarily shaped surface on the right hand side. Inside the TLM region the TLM method is applied, whereas the interaction between the surface of the TLM subregion is modeled using the time-domain electric field integral equation, which is solved using the marching-on-in-time method given in (2.15).

A hybrid TD-IE/TLM method allowing the efficient electromagnetic coupling of distributed remote TLM subdomains is presented in [28]. Figure 2.13 shows two coupled TLM subdomains Ω and Ω' . On the surfaces $\partial\Omega_2$ and $\partial\Omega'_2$ the electric and magnetic fields are probed and are used as source terms in the time-domain Stratton–Chu integral equation [85] to compute the TLM input quantities on $\partial\Omega'_1$ and $\partial\Omega_1$, respectively.

Fig. 2.14 UWB antenna link consisting of a TEM horn and a dipole antenna. The separation is $d = 2.5$ m [28]

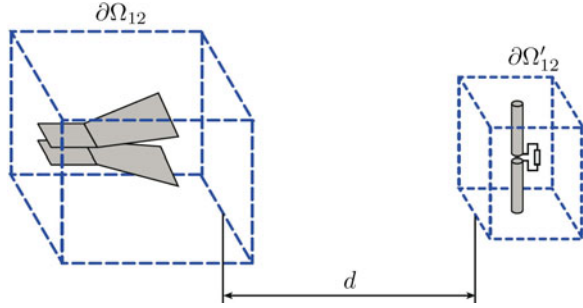
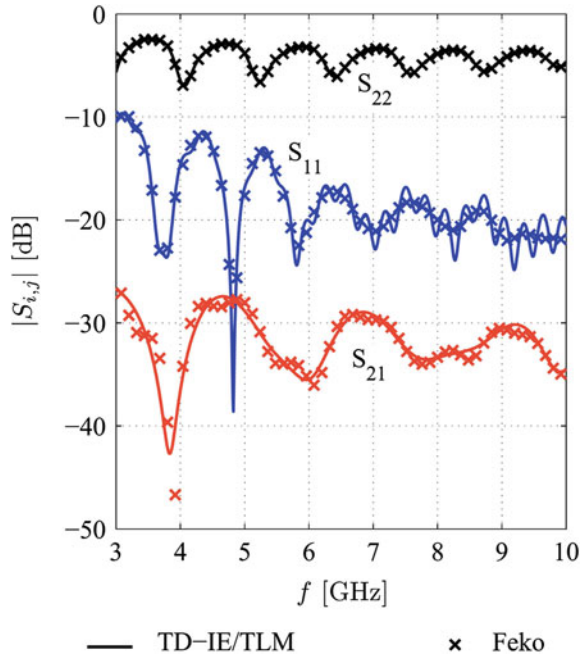


Fig. 2.15 Scattering parameters of UWB link [28]



The method has been applied to the modeling of an ultra wide band (UWB) antenna link consisting of a TEM horn and a dipole antenna as drawn in Fig. 2.14. The details of the structure are given in [28]. The scattering parameters computed in the frequency range from 3 GHz to 10 GHz are shown in Fig. 2.15.

2.7 Stochastic Electromagnetic Fields

Stochastic electromagnetic fields cannot be described numerically by specifying field amplitudes or amplitude spectra since these quantities are not determined. However, it is possible to assign numerical values to energy and power spectra. A complete description of stochastic electromagnetic fields with Gaussian amplitude distribution can be given by the autocorrelation and cross-correlation spectra of the field variables evaluated for distinct observation points. By considering arbitrary correlations between the noise radiation sources, the spatial distribution of the spectral energy density can be computed. The auto and cross-correlation spectra of the far field are computed from the auto- and cross-correlation spectra of the source distribution [86].

Consider a current density vector $\mathbf{J}(\mathbf{x}, \omega)$ describing the source of the electromagnetic field. The electric field excited from $\mathbf{J}(\mathbf{x}, \omega)$ is given by

$$\mathbf{E}(\mathbf{x}, \omega) = \int_V \mathbf{G}(\mathbf{x}, \mathbf{x}', \omega) \mathbf{J}(\mathbf{x}', \omega) d^3 x', \quad (2.17)$$

where $\mathbf{G}(\mathbf{x}, \mathbf{x}', \omega)$ is the total Green's dyadic and the integration is extended over the whole volume V where $\mathbf{J}(\mathbf{x}, \omega)$ is nonvanishing [87, p. 306].

Stochastic source currents can be described by the dyadic

$$\Gamma_J(\mathbf{x}_1, \mathbf{x}_2, \omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle \mathbf{J}_T(\mathbf{x}_1, \omega) \mathbf{J}_T^\dagger(\mathbf{x}_2, \omega) \rangle, \quad (2.18)$$

where $\mathbf{J}_T(\mathbf{x}, \omega)$ is the time-windowed current density and $\mathbf{J}_T^\dagger(\mathbf{x}, \omega)$ is its Hermitian conjugate. The stochastic electric field can be described by the dyadic

$$\Gamma_E(\mathbf{x}_1, \mathbf{x}_2, \omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle \mathbf{E}_T(\mathbf{x}_1, \omega) \mathbf{E}_T^\dagger(\mathbf{x}_2, \omega) \rangle, \quad (2.19)$$

where $\mathbf{E}_T(\mathbf{x}, \omega)$ is the time-windowed electric field. From (2.17), (2.18) and (2.19) we obtain

$$\Gamma_E(\mathbf{x}_1, \mathbf{x}_2, \omega) = \int_V \int_V \mathbf{G}(\mathbf{x}_1, \mathbf{x}'_1) \Gamma_J(\mathbf{x}_1, \mathbf{x}_2, \omega) \mathbf{G}^\dagger(\mathbf{x}_2, \mathbf{x}'_2) d^3 x'_1 d^3 x'_2. \quad (2.20)$$

With this we obtain from the correlation dyadic $\Gamma_J(\mathbf{x}_1, \mathbf{x}_2, \omega)$ of the source currents the correlation dyadic of the electric field $\Gamma_E(\mathbf{x}_1, \mathbf{x}_2, \omega)$. The spectral electric energy density $W_E(\mathbf{x}, \omega)$ is given by

$$W_E(\mathbf{x}, \omega) = \frac{\varepsilon}{2} \Gamma_E(\mathbf{x}, \mathbf{x}, \omega), \quad (2.21)$$

where ε is the permittivity of the medium.

In order to obtain the correlation matrix of the field samples, a sampling of the electric or magnetic field values in all pairs of a set of sampling points is required. The numerical computation of these stochastic fields is discussed in the following.

2.8 Numerical Computation of Stochastic Fields

As a time-domain computational method the TLM method is excellently suited for the modeling of impulsive and transient electromagnetic fields as occurring in EMC problems. EMC studies usually include in addition to normal signal conditions a variety of superimposed signals either from interfering systems or from natural phenomena [21]. The numerical computation of stochastic electromagnetic fields can be performed in an efficient way by transforming the field problem to a network problem [88]. Like in the case of deterministic electromagnetic fields, also in the case of stochastic electromagnetic fields network methods can reduce the computational effort considerably and beyond this can contribute to compact physical wireless link model generation. Network methods for deterministic fields already have been described in [53, 67].

In the following, we describe the computation of stochastic electromagnetic fields by the Method of Moments (MoM). The MoM allows to transform a field problem into a network-like problem described by algebraic equations [42].

Let us first apply the MoM to compute the integral expression (2.17) for deterministic fields. We expand the field functions $\mathbf{J}(\mathbf{x}, \omega)$ and $\mathbf{E}(\mathbf{x}, \omega)$ into basis functions

$$\mathbf{J}(\mathbf{x}, \omega) = \sum_n I_n(\omega) \mathbf{u}_n(\mathbf{x}), \quad (2.22a)$$

$$\mathbf{E}(\mathbf{x}, \omega) = \sum_n V_n(\omega) \mathbf{u}_n(\mathbf{x}), \quad (2.22b)$$

where the $\mathbf{u}_n(\mathbf{x})$ are vectorial basis functions and $I_n(\omega)$ and $V_n(\omega)$ are the expansion coefficients. We can consider $I_n(\omega)$ and $V_n(\omega)$ as generalized voltages and currents, respectively. If we use a complete set of basis functions, the series expansions will converge to the exact value. However, to facilitate a numerical treatment of the problem we have to truncate the series expansion after a finite number of elements. Inserting these series expansions into (2.17) yields

$$\sum_n V_n(\omega) \mathbf{u}_n(\mathbf{x}) = \sum_n I_n(\omega) \int_V \mathbf{G}(\mathbf{x}, \mathbf{x}', \omega) \mathbf{u}_n(\mathbf{x}') d^3 x'. \quad (2.23)$$

Using expansion functions $\mathbf{u}_n(\mathbf{x})$ with the property

$$\int_V \mathbf{u}_m^\dagger(\mathbf{x}) \mathbf{u}_n(\mathbf{x}) d^3 x = \delta_{mn}, \quad (2.24)$$

where δ_{mn} is the Kronecker delta. Multiplying (2.23) from the left with $\mathbf{u}_m^\dagger(\mathbf{x})$ and integrating over V yields

$$V_m(\omega) = \sum_n Z_{mn}(\omega) I_n(\omega). \quad (2.25)$$

The matrix elements $Z_{mn}(\omega)$ are given by

$$Z_{mn}(\omega) = \iint_V \mathbf{u}_m^\dagger(\mathbf{x}) \mathbf{G}(\mathbf{x}, \mathbf{x}', \omega) \mathbf{u}_n(\mathbf{x}') d^3 x d^3 x'. \quad (2.26)$$

For a chosen dimension N of the series expansions (2.22a) and (2.22b) we introduce the generalized current and voltage vectors

$$\mathbf{I}(\omega) = [I_1(\omega) \dots I_N(\omega)]^T, \quad (2.27a)$$

$$\mathbf{V}(\omega) = [V_1(\omega) \dots V_N(\omega)]^T \quad (2.27b)$$

and the impedance matrix

$$\mathbf{Z}(\omega) = \begin{bmatrix} Z_{11}(\omega) & \dots & Z_{1N}(\omega) \\ \vdots & \ddots & \vdots \\ Z_{N1}(\omega) & \dots & Z_{NN}(\omega) \end{bmatrix} \quad (2.28)$$

and can write (2.25) as

$$\mathbf{V}(\omega) = \mathbf{Z}(\omega) \mathbf{I}(\omega). \quad (2.29)$$

Let us now consider stochastic signals. We can expand the correlation dyadics $\Gamma_J(\mathbf{x}_1, \mathbf{x}_2, \omega)$ and $\Gamma_E(\mathbf{x}_1, \mathbf{x}_2, \omega)$ introduced in (2.18) and (2.19) into basis functions and obtain the correlation matrices $\mathbf{C}_I(\omega)$ and $\mathbf{C}_V(\omega)$ with the matrix elements

$$C_{I,mn}(\omega) = \iint_V \mathbf{u}_m^\dagger(\mathbf{x}) \Gamma_J(\mathbf{x}, \mathbf{x}', \omega) \mathbf{u}_n(\mathbf{x}') d^3x d^3x', \quad (2.30a)$$

$$C_{V,mn}(\omega) = \iint_V \mathbf{u}_m^\dagger(\mathbf{x}) \Gamma_E(\mathbf{x}, \mathbf{x}', \omega) \mathbf{u}_n(\mathbf{x}') d^3x d^3x'. \quad (2.30b)$$

These matrix elements can be summarized in the matrices

$$\mathbf{C}_I(\omega) = \begin{bmatrix} C_{I,11}(\omega) & \dots & C_{I,1N}(\omega) \\ \vdots & \ddots & \vdots \\ C_{I,N1}(\omega) & \dots & C_{I,NN}(\omega) \end{bmatrix}, \quad (2.31a)$$

$$\mathbf{C}_V(\omega) = \begin{bmatrix} C_{V,11}(\omega) & \dots & C_{V,1N}(\omega) \\ \vdots & \ddots & \vdots \\ C_{V,N1}(\omega) & \dots & C_{V,NN}(\omega) \end{bmatrix}. \quad (2.31b)$$

We can also obtain these correlation matrices from the time windowed current and voltage amplitudes $\mathbf{I}_T(\omega)$ and $\mathbf{V}_T(\omega)$ via

$$\mathbf{C}_I(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle \mathbf{I}_T(\omega) \mathbf{I}_T^\dagger(\omega) \rangle, \quad (2.32a)$$

$$\mathbf{C}_V(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle \mathbf{V}_T(\omega) \mathbf{V}_T^\dagger(\omega) \rangle. \quad (2.32b)$$

From (2.29), (2.32a) and (2.32b) we obtain

$$\mathbf{C}_V(\omega) = \mathbf{Z}(\omega) \mathbf{C}_I(\omega) \mathbf{Z}^\dagger(\omega). \quad (2.33)$$

This equation allows to compute the correlation matrix coefficients for the series expansion of the field correlation dyadic from the expansion coefficients from the source correlation dyadic. The transformation matrix \mathbf{Z} describing the field response is the same as for the computation of the deterministic field response. The matrix \mathbf{Z} can be computed assuming deterministic fields, e.g., by computing the field response for each basis function excitation independently.

2.9 Examples of Stochastic Field Modeling

Examples for the numerical simulation of noisy electromagnetic fields, accounting for arbitrary correlations between the noise radiation sources, have been presented in [30, 31, 89–91]. The method presented in Sect. 2.8 allows to compute the spatial distribution of the spectral energy density of noisy electromagnetic sources.

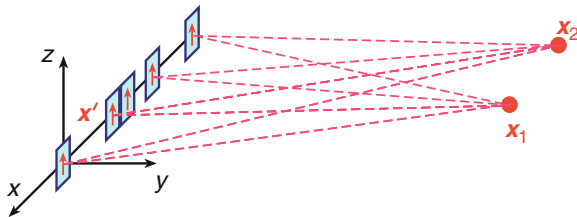


Fig. 2.16 Nonuniform linear array of Hertzian dipoles positioned at $x' = 0, -2.5\lambda, -3.25\lambda, -4.5\lambda, -6.5\lambda$

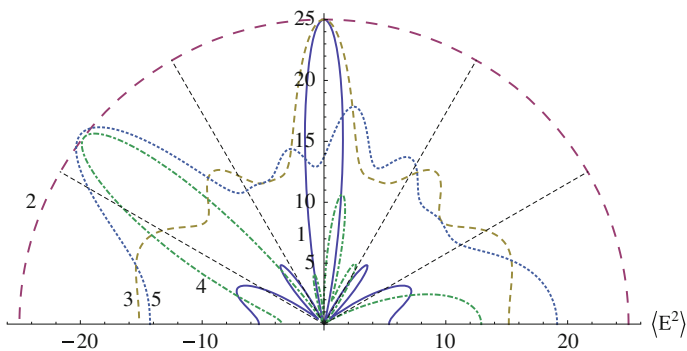


Fig. 2.17 Polar plot of the angular distribution of the magnitude $\langle |E_z(\varphi)|^2 \rangle$ electric field spectral energy density in the far field

Consider for example the nonuniform linear array of parallel Hertzian dipoles positioned at $x'/\lambda = 0, -2.5, -3.25, -4.5, -6.5$ as depicted in Fig. 2.16. The electric field $\mathbf{E}(\mathbf{x})$ excited by the array is given by

$$\mathbf{E}(\mathbf{x}) = \mathbf{M}(\vartheta, \varphi) \mathbf{I}, \quad (2.34)$$

where \mathbf{I} is the vector summarizing the excitation currents of the array. The transfer matrix \mathbf{M} can be computed by successive unit excitation of the dipole elements. For excitation of the dipoles with stochastic currents, described by the correlation matrix \mathbf{C}_I , we obtain the spectral electric energy density of the electric field [30, 31]

$$W_E(\mathbf{x}) = \frac{\varepsilon}{2} \left| \mathbf{M}(\mathbf{x}) \mathbf{C}_I \mathbf{M}^\dagger(\mathbf{x}) \right|. \quad (2.35)$$

Figure 2.17 shows the polar plot of the angular distribution of the magnitude $\langle |E_z(\varphi)|^2 \rangle$ of the electric energy density in the xy -plane and at a given distance r_0 from the origin for the following five cases of excitation of the Hertzian dipoles with

1. Correlated currents of equal amplitude and equal phase.
2. Mutually uncorrelated currents with equal rms values.
3. Superposition of correlated and uncorrelated currents.
4. Mutually correlated currents of equal amplitude and variable phase (0.2π , 0 , 0.6π , 1.7π , 1.3π).
5. A superposition of correlated currents with nonuniform phase according to (4) and uncorrelated currents.

This example shows that the spatial distribution of the electromagnetic field excited by stochastic sources strongly depends on the correlation of the sources.

2.10 Conclusion

We have shown that the TLM scheme is a fundamental approach for discrete time modeling of electromagnetic structures. Since the time-discretization can be made arbitrarily fine the method is potentially exact. Based on Huygens' principle, the TLM method provides an imagery access to the physical principles of the considered electromagnetic phenomena. We have shown that the TLM scheme can be considered as a spacial case of a time-discrete circuit theory.

The TLM method known as a powerful versatile for solving of electromagnetic field problems also is extremely well suited to deal with stochastic electromagnetic fields and EMC problems. The TLM method is excellently suited for the modeling of broadband and transient fields. The TLM method follows a circuitual approach and allows to represent electromagnetic field problems as circuit problems. Combining the TLM method with the Integral Equation method yields versatile hybrid methods for the modeling of complex electromagnetic structures separated by large distances of free space. Introducing network models allows the application of correlation matrix methods for the modeling of stochastic electromagnetic fields.

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