Contents

1 Introduction to Quantum Computation 1
1.1 Quantum Bit and Elementary Operations 1
1.2 The Solovay–Kitaev Algorithm 4
1.3 Multi-Qubit Gates 6
1.4 Universal Quantum Computation 10
1.5 Quantum Algorithms 12
1.5.1 Indirect Measurement and the Hadamard Test 12
1.5.2 Phase Estimation, Quantum Fourier Transformation, and Factorization 14
1.5.3 A Quantum Algorithm to Approximate Jones Polynomial 16
1.6 Quantum Noise 21
1.7 Summary and Discussion 23

2 Stabilizer Formalism and Its Applications 24
2.1 Stabilizer Formalism 24
2.2 Clifford Operations 26
2.3 Pauli Basis Measurements 29
2.4 Gottesman–Knill Theorem 30
2.5 Graph States 32
2.6 Measurement-Based Quantum Computation 35
2.7 Quantum Error Correction Codes 43
2.8 Magic State Distillation 49
2.8.1 Knill-Laflamme-Zurek Protocol 49
2.8.2 Bravyi-Kitaev Protocol 51
2.9 Summary and Discussion 55

3 Topological Stabilizer Codes 56
3.1 Z_2 Chain Complex 57
3.2 A Bit-Flip Code: Exercise 60

3.3 Definition of Surface Codes 61
3.3.1 Surface Code on a Torus: Toric Code 61
3.3.2 Planar Surface Code 64
3.4 Topological Quantum Error Correction 65
3.5 Error Correction and Spin Glass Model 71
3.6 Other Topological Codes 76
3.7 Connection to Topological Order in Condensed Matter Physics 79
3.8 Summary and Discussion 84

4 Topological Quantum Computation with Surface Codes 86
4.1 Defect Pair Qubits 86
4.2 Defect Creation, Annihilation, and Movement 88
4.3 Logical Controlled-NOT Gate by Braiding 91
4.4 Magic State Injections and Distillation 93
4.5 Topological Calculus 96
4.6 Faulty Syndrome Measurements and Noise Thresholds 102
4.7 Summary and Discussion 106

5 Topologically Protected Measurement-Based Quantum Computation 107
5.1 Topological Cluster State in Three Dimensions 107
5.2 Vacuum, Defect, and Singular Qubit Regions 109
5.3 Elementary Operations in Topological Measurement-Based Quantum Computation 110
5.4 Topological Quantum Error Correction in Three Dimensions 115
5.5 Applications for Measurement-Based Quantum Computation on Thermal States 116
5.6 Summary and Discussion 121

Appendix A: Fault-Tolerant Quantum Computation 122

Appendix B: Decoding Stabilizer Codes 127

References 131

Index 137

http://www.springer.com/978-981-287-995-0

Quantum Computation with Topological Codes
From Qubit to Topological Fault-Tolerance
Fujii, K.
2015, X, 138 p. 131 illus., 113 illus. in color., Softcover
ISBN: 978-981-287-995-0