

# Preface

In 1982, Richard Feynman pointed out that a simulation of quantum systems on classical computers is generally inefficient because the dimension of the state space increases exponentially with the number of particles [1]. Instead, quantum systems could be simulated efficiently by other quantum systems. David Deutsch put this idea forward by formulating a quantum version of a Turing machine [2]. Quantum computation enables us to solve certain kinds of problems that are thought to be intractable with classical computers such as the prime factoring problem and an approximation of the Jones polynomial. It has a great possibility to disprove the extended (strong) Church–Turing thesis, i.e., that any computational process on realistic devices can be simulated efficiently on a probabilistic Turing machine.

However, for this statement to make sense, we need to determine whether or not quantum computation is a realistic model of computation. Rolf Landauer criticized it (encouragingly) by suggesting to put a footnote: “This proposal, like all proposals for quantum computation, relies on speculative technology, does not in its current form take into account all possible sources of noise, unreliability and manufacturing error, and probably will not work.” [3]. Actually, quantum coherence, which is essential for quantum computation is quite fragile against noise. If we cannot handle the effect of noise, quantum computation is of a limiting interest, like classical analog computers, as a realistic model of computation. To solve this, many researchers have investigated the fault-tolerance of quantum computation with developing quantum error correction techniques. One of the greatest achievements of this approach is topological fault-tolerant quantum computation using the surface code proposed by R. Raussendorf et al. [4–6]. It says that nearest-neighbor two-qubit gates and single-qubit operations on a two-dimensional array of qubits can perform universal quantum computation fault-tolerantly as long as the error rate per operation is less than  $\sim 1\%$ .

In this book, I present a self-consistent review of topological fault-tolerant quantum computation using the surface code. The book covers everything required to understand topological fault-tolerant quantum computation, ranging from the definition of the surface code to topological quantum error correction and

topological operations on the surface code. The basic concepts and powerful tools for understanding topological fault-tolerant quantum computation, such as universal quantum computation, quantum algorithms, stabilizer formalism, and measurement-based quantum computation, are also introduced in the first part (Chaps. 1 and 2) of the book. In particular, in Chap. 1, I also mentioned a quantum algorithm for approximating the Jones polynomials, which is also related to topological quantum computation with braiding non-Abelian anyons. In Chap. 3, the definition of the surface code and topological quantum error correction on it is explained. In Chap. 4, topological quantum computation on the surface code is described in the circuit-based model, where topological diagrams are introduced to understand the logical operations on the surface code diagrammatically. In Chap. 5, I explained the same thing in the measurement-based model, as done in the original proposal [4]. Hopefully, it would be easy to see how these two viewpoints are related.

Throughout the book, I have tried to explain the quantum operations using circuit and topological diagrams so that the readers can get a graphical understanding of the operations. The graphical understanding should be helpful to study the subjects more efficiently. Topological quantum error correction codes are a nice playground for studying the interdisciplinary connections between quantum information and other fields of physics, such as condensed matter physics and statistical physics. Actually, there is a nice correspondence between topological quantum error correction codes and topologically ordered systems in condensed matter physics. Furthermore, if we consider a decoding problem of a quantum error correction code, a partition function of a random statistical mechanical model is naturally appeared as a posterior probability for the decoding. These interdisciplinary topics are also included in Chap. 3.

Almost all topics, except for the basic concepts in the first part, are based on the results achieved after the appearance of the standard textbook of quantum information science entitled “Quantum Computation and Quantum Information” (Cambridge University Press 2000) by M.A. Nielsen and I.L. Chuang. In this sense, the present comprehensive review on these topics would be helpful to learn and update the recent progress efficiently. In this book, I concentrated on the quantum information aspect of topological quantum computation. Unfortunately, I cannot cover the more physical and condensed matter aspects of topological quantum computation, such as non-Abelian anyons and topological quantum field theory. In this sense, this book is complemented by the book “Introduction to Topological Quantum Computation” (Cambridge University Press 2012) written by J.K. Pachos. Readers who are interested in the more physical aspects of topological quantum computation are recommended to read it.

Hopefully, this review will encourage both theoretical and experimental researchers to find a more feasible way of quantum computation. It will also bring me great pleasure if this review provides an opportunity to reunify and refine various subdivided fields of modern physics in terms of quantum information.

Quantum Computation with Topological Codes

From Qubit to Topological Fault-Tolerance

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2015, X, 138 p. 131 illus., 113 illus. in color., Softcover

ISBN: 978-981-287-995-0