Chapter 5 Section 1

1. $y_{ij} \sim N(\mu_i\,,\ \sigma^2)$

2. s_p estimates σ .

3. $s^2_p = 16.444$. $s_p = 4.055$.

4. $3 = 2\frac{s_p}{\sqrt{n_1}};\ 4 = 2\ \frac{s_p}{\sqrt{n_2}};\quad \frac{9}{4} + 4 = \ s_p^2\left(\frac{1}{n_1} + \ \frac{1}{n_2}\right);\ \frac{5}{2} = \quad \sqrt{s_p^2\left(\frac{1}{n_1} + \ \frac{1}{n_2}\right)}\ ;$

$\bar{y}_1 - \ \bar{y}_2\ \pm 2(\frac{5}{2})$ or $\bar{y}_1 - \ \bar{y}_2\ \pm 5$

5. **a.** 12 = 4(3) = df., $s_p = 3$, $\bar{y}_i \pm t_{12;.975}\ \frac{3}{\sqrt{4}}$; $t_{12;.975} = 2.179$;

$\Delta = (2.179)\frac{3}{2} = 3.2685$

b. $1 - 4(1-.95) \geq\ \gamma,\ where\ \ \gamma\ is\ the\ confidence\ level.\ So,\ \ .80\ \geq\ \gamma$ because of Bonferroni. So, no, confidence is not 95% all include the parameters of interest. Individually they are 95% confident but simultaneous inclusion reduces the “family- wise” confidence because the confidence must take into account the joint multiple confidences all occurring at once.

6. **a.** $s^2_p = 45.585$; $s_p = 6.7516$.

b. $35\ \pm\ t_{8;.975}(6.7516/\sqrt{3})$; $t_{8;.975} = 2.306$;

$35\ \pm\ 8.988$; $(26.011\ ,\ 43.988)$; 95% confidence

c. $\bar{y}_4 - \ \bar{y}_1\ \pm\ \ t_{8;.975}\sqrt{s_p^2\left(\frac{1}{n_1} + \ \frac{1}{n_2}\right)}$ becomes $-15\ \ \pm\ \ 12.712$ or (-27.71, -2.288)

d. $\frac{1}{2}\ (\mu_2 +\ \mu_4) - \ \frac{1}{2}(\ \mu_3 +\ \mu_5)$ corresponds to Design X minus Design Y.

e. Individual measurements for each design come from different prototypes. This permits legitimate inference to performance of new or old prototypes for the given designs.

7. **a**. $-22.5\ \pm\ 9.063$ or (-31.563, -13.437); Design X minus Design Y.

b. $\binom{5}{2} = 10$.

c. $1 - 10(1-\alpha) = 1 - 10(1-.01) \geq .90;\ \gamma \geq .90$. So, each must be 99%.

Chapter 5 Section 2

1. Two-way factorial implies there are two factors, each having perhaps a different number of levels. Perhaps one factor is Pressure and the other is Moisture. Pressure could be at, say, hi, med or low and Moisture could be at 2%, 4%, 6%, 8%, 10%. So a total of 15 = 3 X 5 treatment combinations.

2. **a.** $\bar{y} + a_3 + b_3 + ab_{33} = 10 + -5 + -1 + -1 = 3$

b. $\Delta = t_{9;.975}(2)\sqrt{\frac{4}{(3)(3)(2)}} = 2.1326$; $t_{9;.975} = 2.262$.

c. Yes, some departure from parallelism because some $\widehat{\alpha\beta}_{ij}$ *exceed* 2.1326 *in* absolute value, i.e., significant interaction exists.

3. **a.** No, only A effect at a selected level of B. The simple effects of A are different from level to level of B. It is possible both "simple" effects of A are of the same "sign", meaning one could average both simple effects and make a general inference about an A effect independent of level of B.

b. Yes, we have an interaction effect so if there is an A effect it changes for different levels of B.

c. No, we have interaction implying simple effects of A change for different levels of B.

4. **a.** No, the .0008 inch std. dev. is understandably smaller than the $s_p = .0017$ because the .0008 value came from repeat measurements on the same item, whereas the s_p value came from measurements on different copies of the same CAD drawing pooled across different machine/enlargements.

b. -.0018, .0042, .0012, -.0008, -.0028

c. Yes, both fit and assumed common variance can be evaluated.

d. ab_{13} = .00438, ab_{23} = -.00302, ab_{33} = -.00136, ab_{32} = .00104, ab_{31} = .00032

e. $\Delta = t_{36;.975}(.0017)\sqrt{\frac{4}{(3)(3)(5)}}$ = .001029; $t_{36;\,.975}$ = 2.03. Yes, the absolute value of most estimated interaction effects exceed .001029**.**

f. Estimated $\alpha_1 - \alpha_2 = .00799$. Estimated std. dev. Is $(.0017)(6/45)^{.5}$ = **.**0006208. So, $\Delta = 2.03(.0006208) = .00126$. .00799 $\pm$ $.00126$ or (.00673, .00925); Not credible to use for every enlargement level because important interaction exists.

g. Plot not given here.

<u>Chapter 5 Section 3</u>

1. 2^5 = 32 treatments

2. **a.** $\hat{\mu} = 118.125$; $\hat{\alpha}_2 = 11.375$; $\hat{\beta}_2 = -14.625$; $\widehat{\alpha\beta}_{22} = -6.875$; $\hat{\gamma}_2 = 55.125$; $\widehat{\alpha\gamma}_{22} = -3.125$; $\widehat{\beta\gamma}_{22} = 1.375$; $\widehat{\alpha\beta}\gamma_{222} = 4.625$.

b. $\Delta = t_{56;.975}\,(7.2)\frac{1}{2^3}\sqrt{\frac{8}{8}} = \frac{(2.005)(7.2)}{8} = 1.8045$. All fitted effects are significant except $\beta\gamma_{22}$.

c. Grand Avg**. +** Hi A (11.375) + Lo B (14.625) + Hi C (55.125) + HiA/LoB(6. 875) = 118.125 + 88 = 206.125.

3. $\hat{\alpha}_2 = 3$; $\hat{\beta}_2 = 1$;

4. 5 cycles, divided by 32.

5. $\Delta = t_{24;.975}\,(s_p)\frac{1}{2^3}\sqrt{\frac{8}{4}} = 2.064\left(\frac{1}{8}\right)s_p\sqrt{2} = .3648\,s_p$

6. (a)(b)

7. (a)

8. (a) (b) (c)