Chapter 3, Section 1

1. Identifying when the effect of a special cause may have entered the process and evaluate whether a process is stable with respect to aim or variation are the purposes of control charting.

2. Standards given implies $\mu \; and \; \sigma$ are known. Retrospective implies $\mu \; and \; \sigma$ are not known but must be estimated from process data.

3. Control charts are designed to detect special cause variation. Common cause variation is variation caused by factors built-into or assumed to be part of the process or system. Special cause variation is variation caused by factors not built-in or assumed part of the process or system.

4. Control limits get closer to the center line as subgroup size increases.

5. If the multiple changes from 3 to 2, more frequent false alarms will result. But if a change in the process occurs, the change will be detected sooner.

6. If the multiple increases above 3, less frequent false alarms will result. If a change in the process occurs, the change will not be detected as soon.

7. No, a retrospective chart could be stable and centered around, say, 10, but the target is 4. It is likely most items will be outside specs. Another scenario could be the retrospective or standards given chart determines a stable process with large variation. Thus, the specs may be tight enough that much of the stable process occurs outside specs.

8. If a process is judged unstable or out-of-control, estimating or predicting the % of items inside specs doesn't make sense, in other words, the prediction is not reliable. Moment to moment will produce different percentages of items inside specs. If σ is very small, it is possible a large % of items are inside specs, i.e., cycling or trending, thus unstable, yet all inside control limits, with a possible small σ if the R or s chart is stable but the chart for aim is not stable.

9. **a.** 10
b. 5
c. Analyst 2, eliminates known line to line variation.

Chapter 3 Section 2

1. **a.** A set of 9 containers sampled in a selected hour is the subgroup. Subgroup size is 9. 40 subgroups are selected.
b. $4 \pm 3 \frac{.1}{\sqrt{9}}$; $4 \pm .1$; $(3.9, \; 4.1)$; $LCL_{\bar{x}} = 3.9$; $UCL_{\bar{x}} = 4.1.$
c. Subgroup ranges; $D_1\sigma = .546(.1) = .0546$; $D_2\sigma = 5.394(.1) = .5394.$
d. In (b), it is Standards given, μ and σ known. In (c), it is

also standards given because μ and σ are known.

e. One decision rule implying stability is “all points within control limits”. However, there is a small probability a subgroup statistic falls outside the control limits even when the process is stable.

2. **a.** $\bar{\bar{x}} = 3.9, \bar{R} = .56, \bar{s} = .48,\quad LCL_R = .184(.56) = .10304,$
$UCL_R = 1.816(.56) = 1.0169.$

b. $3.9 \pm .337\,(.56)\,;\ 3.9 \pm .18872;\ LCL_{\bar{x}} = 3.7112,\ UCL_{\bar{x}} = 4.0887.$

c. LCL_s = .239(.48) = .1147; $\quad UCL_s = 1.7611(.48) = .8453.$

d. $LCL_{\bar{x}} = 3.9 - 1.032(.48) = 3.4046;$

$UCL_{\bar{x}} = 3.9 + 1.032(.48) = 4.3953.$

e. $\frac{\bar{R}}{d_2} = \frac{.56}{2.97} = .1885;\ \frac{\bar{s}}{c_4} = \frac{.48}{.9693} = .4952.\ \bar{\bar{x}} = 3.9$ is an estimate of the avg.

distance from bottom to handle.

3. **a.** $LCL_{\bar{x}} = 6 - 3\frac{1.5}{\sqrt{4}} = 3.75;$

$UCL_{\bar{x}} = 6 + 3\frac{1.5}{\sqrt{4}} = 8.25.$ No out of control points.

b. LCL_s = 0; $\quad UCL_s = 2.088(1.5) = 3.132;$ pt. 3 is outside limits.

c. $LCL_{\bar{x}} = 6.58 - 1.628(1.72) = 3.779;$

$UCL_{\bar{x}} = 6.58 + 1.628(1.72) = 9.38.$

LCL_s = 0; $\quad UCL_s = 2.266(1.72) = 3.897;$ No instability.

d. $\frac{\bar{s}}{c_4} = \frac{1.72}{.9213} = 1.867;\ E(R) = d_2 1.867 = 2.534(1.867) = 4.7307.$

4. **a.** The subgroup is a single Series XX transmission housing.

The subgroup size is 1.

b. 34

c. .0001, .0003

d. Individuals retrospective. Subgroup size is 1.

$\overline{MR} = \frac{.02472}{34} = .000727;\qquad \frac{\overline{MR}}{d_2} = \frac{.000727}{1.128} = .000645.$

$LCL_x = 3.7805 - 3(.000645) = 3.77856;$

$UCL_x = 3.7805 + 3(.000645) = 3.782435.$ All points inside limits.

Chapter 3 Section 3

1. **a.** Attributes because the variable values are counts.
b. Poisson ($\lambda = .05$)
c. 30(.05) = 1.5, Poisson ($\lambda = 1.5$)
d. $\sqrt{1.5} = 1.2247$
e. E(*X*/30) = 1.5/30 = .05; Var(*X*/30) = 1.5/900 = .0016667;
$\sigma\left(\frac{x}{30}\right) = .0408233.$
$LCL_{x/30} = 0;\ UCL_{x/30} = .05 + 3(.0408233) = .17247.$
No points outside limits.
f. u-chart
g. $\hat{\lambda}_{pooled} = \frac{8}{210} = .03809.$

$$LCL_{\frac{x}{30}} = .03809 - 3\sqrt{\frac{\hat{\lambda}_{pooled}}{30}} = 0;$$

$$UCL_{\frac{x}{30}} = .03809 + 3\sqrt{\frac{\hat{\lambda}_{pooled}}{30}} = .14498$$

2. **a.** Attribute because the variable values are counts.
b. Binomial, *n* = 30; *p* = .05.
c. $\mu_x = 30(.05) = 1.5.$
d. $\sigma_x = \sqrt{30(.05).95)}$ = 1.19373.
e. $LCL_{\hat{p}} = .05 - 3\sqrt{\frac{(.05).95)}{30}} = .05 - .11937 = 0.$

$UCL_{\hat{p}} = .05 + 3\sqrt{\frac{(.05).95)}{30}} = .05 + .11937 = .16937.$

f. $LCL_x = np - 3\sqrt{np(1-p)} = 1.5 - 3.5812 = 0.$

$UCL_x = np + 3\sqrt{np(1-p)} = 1.5 + 3.5812 = 5.0812.$

g. $LCL_x = n\hat{p} - 3\sqrt{n\hat{p}(1-\hat{p})} = 1.142857 - 3.14545 = 0.$

$UCL_x = n\hat{p} + 3\sqrt{n\hat{p}(1-\hat{p})} = 1.142857 + 3.14545 = 4.288.$

Chapter 3 Section 4

1. No patterns and all Q's are within control limits.

2. If only "outside control limits" rule is applied, it is possible for Q's to trend up or down or cycle and all be inside control limits. Using only the rule "outside limits" would incorrectly interpret these scenarios as stable.

3. The frequency of false alarms will increase if the extra alarm rules are used in addition to the "3 sigma" limits.

Chapter 3 Section 5

1. ARL means average run length until the first "believed"out-of-control point is identified. All OK ARL is the average run length until the first "believed" out-of-control point is identified, when, in fact, the process is stable or "All OK".

2. A long or large ARL is desired when a process is stable. Under non-stable situations, a short or small ARL is desired.

3. **a.** 370

b. $UCL_{\bar{x}} = 20 + 3\,\frac{4}{\sqrt{4}} = 26.$

$LCL_{\bar{x}} = 20 - 3\,\frac{4}{\sqrt{4}} = 14.$

$p = Prob\left(Z > \frac{26-21}{2}\right) + Prob\left(Z < \frac{14-21}{2}\right) = .0064$

$\frac{1}{p} = 156.25.$ When $\mu = 21,$ the ARL is about 156 or 157.

4. **a.** $UCL = 4 + 3\sqrt{4} = 10,\ LCL = 0.\ Letting\ \lambda = 4,\ \ p = 1 - P(X \le 10).$

$p = 1 - .99716 = .00284.\ \frac{1}{p} = 352.1\,.\ ARL = 352\ or\ 353.$

b. $p = 1 - P(X \le 10).\ \lambda_{new} = 8.$

$p = 1 - .815886 = .184114.\ \frac{1}{p} = 5.43\,.\ ARL = 5\ or\ 6.$

c. $Let\ X = \#\ items\ per\ 2\ units. So, X \backsim Poisson(8).\ UCL_x = 8 + 3\sqrt{8} = 16.485,\ LCL_x = 0.\ Letting\ \lambda\ per\ 2\ items = 8,\ All\ OK\ ARL,\ p = 1 - P(X \le 16).$

$p = 1 - .996282 = .003718.\ \frac{1}{p} = 268.96\,.\ ALL\ OK\ ARL = 268\ or\ 269.$

$Using\ UCL_x = 16.485\ and\ LCL_x = 0, and$

$Letting\ X = \#\ items\ per\ 2\ units\ and\ a\ new\ \lambda\ per\ item = 8\ \ X \backsim Poisson(16).\ p = 1 - P(X \le 16).\ p = 1 - .56596 = .4340.\ \frac{1}{p} = 2.30\,.\ ARL = 2\ or\ 3.$

5. (b)

6. **a**. Control limits are a function of *n*.

b. Because the control limits are the same number of $\frac{\sigma}{\sqrt{n}}$ from μ.

Chapter 3 Section 6

1. **a.** T(*t*) = 4. When the process is hitting T(*t*) = 4, the process is considered optimal.

b. E(*t* = 1) = 2; E(*t* = 2) = 3; E(*t* = 3) = 4. $\Delta E(2) = 3 - 2 = 1.$ $\Delta E(3) = 4 - 3 = 1. \Delta^2 E(3) = 1 - 1 = 0.$ E(*t*) is departure of the process from a targeted value. $\Delta E(t)$ is an indicator of how the departure from the process is changing. $\Delta^2 E(t)$ indicates how the changing departure from the process is changing.

c. $\Delta \mathrm{X}(3) = .8\Delta E(3) + 1.6E(3) \ + 1.9\, \Delta^2 E(3) = \ .8(1) + 1.6(4) + 1.9(0)$ = 7.2.

2. **a.** If no relationship exists between Y and X, most likely nothing will happen. The process will be similar to a random walk until deterioration of mechanics occur, then trending strongly one way or the other or wildly oscillating. If Y is inversely related to X and since the “first” $\Delta \mathrm{X}(3)$ is positive with all positive K’s, the Y’s will most likely be pushed down away from T(t), at least initially.

b. Do an experiment. Choose, say, 3 sets of K’s, each for *n* time periods. Calculate S_1, S_2, and S_3 where $S_i = \frac{1}{n}\sum_1^n E(t)^2$. The smallest S_i suggests where to start with a potentially effective set of K’s.

c. Control charts monitor a process and do not provide a real time adjustment to the process. Control charting can point to a point in time (or earlier) where efforts need to be made to find a cause for any unusual patterns on the chart.