<u>Chapter 6 Section 1</u>

1. **a.** $2^3 = 8$

b. $\Delta = t_{3;.975}\, s_p \frac{1}{2^3}\sqrt{\frac{3}{2}+5}$ = (3.182)(.31868)s_p = 1.014s_p

c. 1 generator

d. $I \leftrightarrow ABCD$ is the defining relation and the generator is

$D \leftrightarrow ABC$. So, from the problem, the following are judged detectable:

$\alpha_2 + \beta\gamma\delta_{222}$
$\delta_2 + \alpha\beta\gamma_{222}$
$\beta\gamma_{22} + \alpha\delta_{22}$

Further, assuming all two-factor and higher interactions are negligible, the A effect and D effect are what is driving differences in the responses.

2. Hi A, Hi B, Hi C and Low D are recommended. This combination is not represented in the fractional factorial, 2^{4-1} where the generator is $D \leftrightarrow ABC$. All hi for A, B, C and D occurs in the experimental setup.

3. **a.** 9 factors

b. $2^9 = 512$ combinations

c. $2^{9-1} = 256$ combinations

d. 16

e. 1/32

f. 5 generators

g. 31 = $2^q - 1 = 2^5 - 1$

4. **a.**

E	F	G	H	J
-	-	+	-	+
+	-	-	+	-

b. CDG, DH, BJ

c. Assuming all two-factor and higher interactions are not important implies the following, Hi D and Hi H important influences on y.

-6.38 estimates δ_2 + all higher order aliases, and

-10.13 estimates $\alpha\delta_{22} + h_2 + a$ll other two factor and higher aliases.

Since the two-factor and higher order interactions are all assumed unimportant, -6.38 estimates δ_2 and -10.13 estimates h_2.

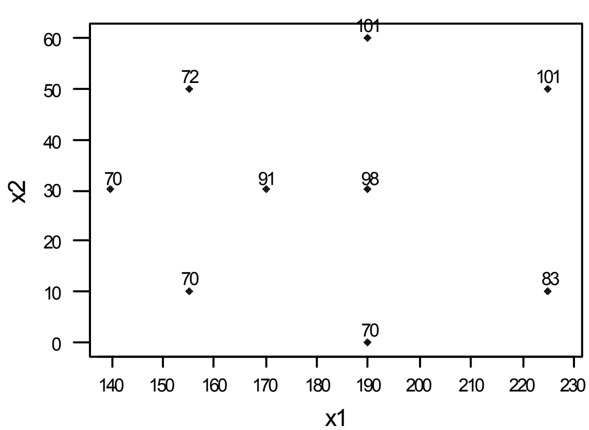
d. From **c**. we select Hi D, Hi H. As for the others, since all two-factor and higher interactions are judged not important, -1.25 estimates j_2 so select Hi level of J, -.75 estimates e_2 so select Hi level of E, .13 estimates f_2 so select Lo F, .13 estimates g_2 so select Lo G, 3.75 estimates α_2, select Lo A and 1.25 estimates β_2, select Lo B. Finally select either Hi or Lo C.

$$\hat{y} = 92 - 3.75 + -1.25 + 0 + -6.38 - .75 - .13 - .13 - 10.38 - 1.25$$

$$\hat{y} = 92 - 24.02 = 67.98$$

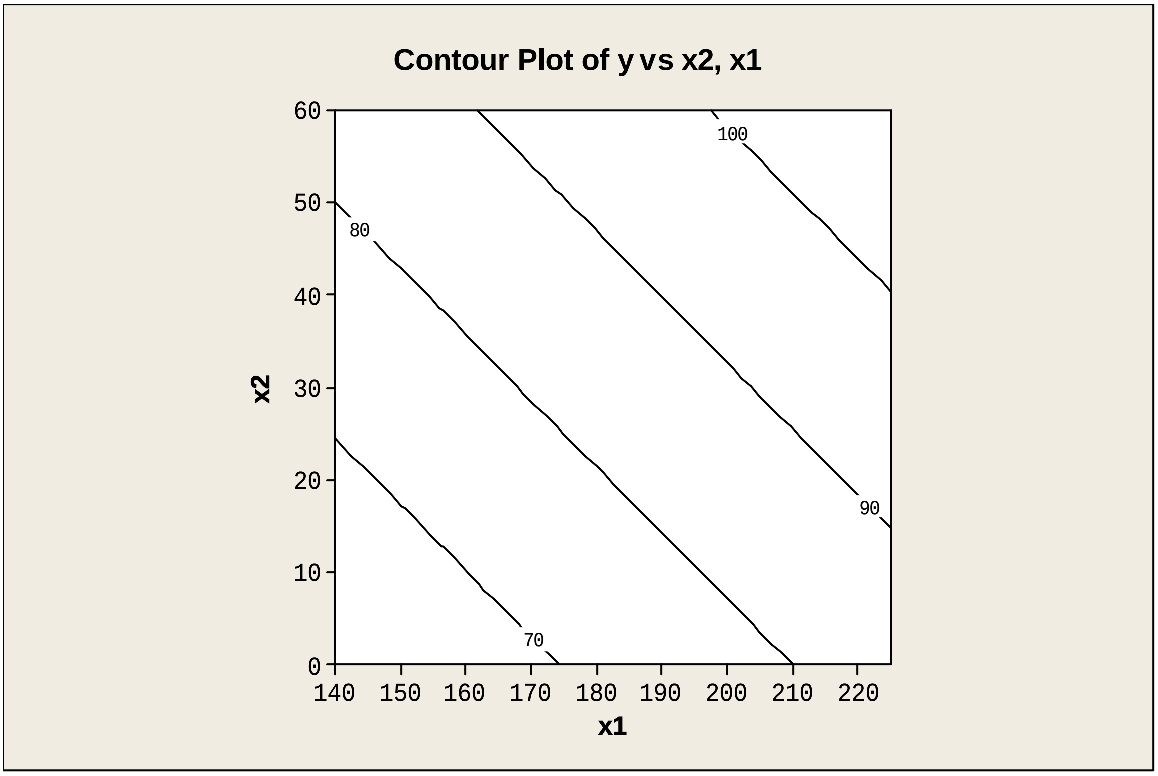
<u>Chapter 6 Section 2</u>

1. **a.**



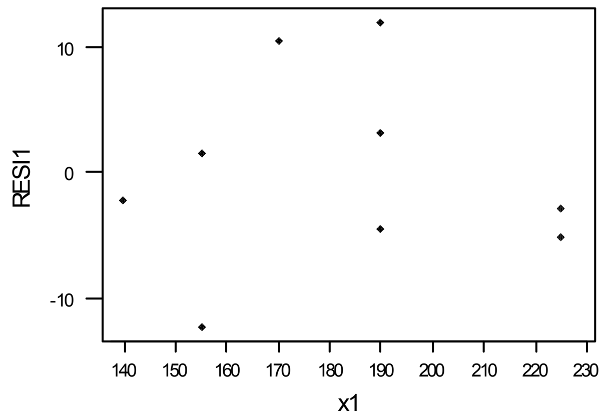
b. $\hat{y}(X_1, X_2) = 21.2816 + .2798X_1 + .3912X_2$

c.

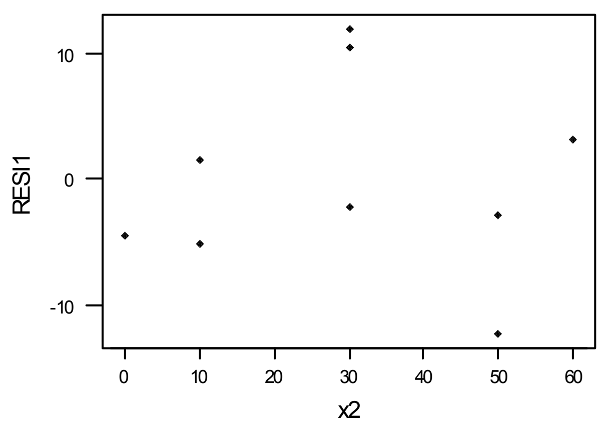


It appears the smallest predicted density occurs for (X_1, X_2) near their simultaneous minimum values over the experimental region, i.e., $X_1 = 155$ and $X_2 = 10$. The largest predicted density appears to be where (X_1, X_2) are simultaneously large within the experimental region, say, $X_1 = 225$, $X_2 = 50$.

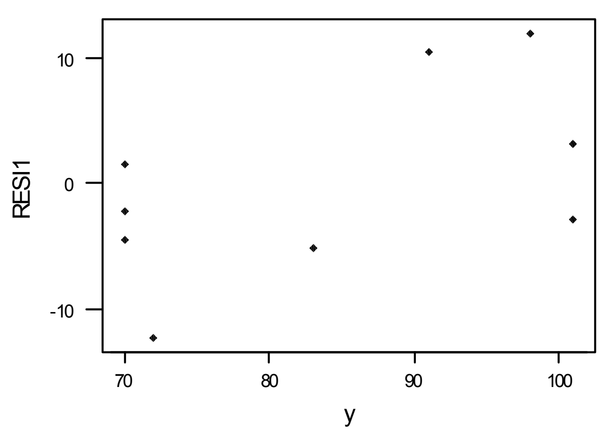
d.



RESI1
10
0
-10
140
150
160
170
180
190
200
210
220
230
x1



RESI1
10
0
-10
0
10
20
30
40
50
60
x2

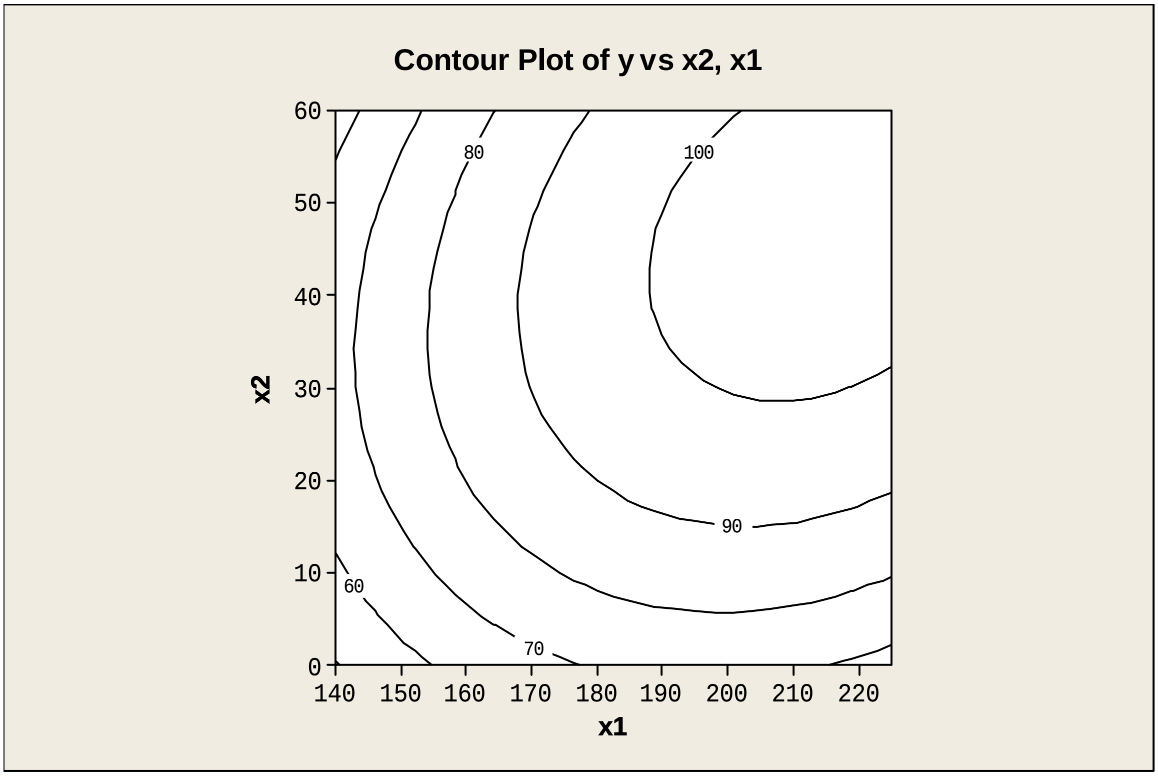


RESI1
10
0
-10
70
80
90
100
y

Only a slight curvature is suggested. Negative, positive, negative trends of residuals vs X_1 or vs X_2 are seen.

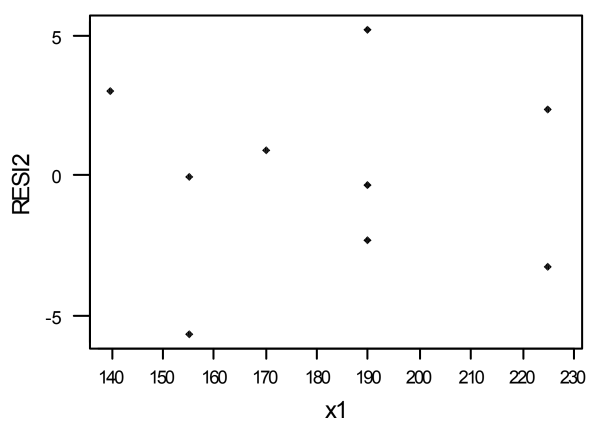
e. $\hat{y}(X_1, X_2) = -206.63 + 2.8424X_1 + .256X_2 + .005714X_1X_2 - .007233X_1^2 - .015842X_2^2$

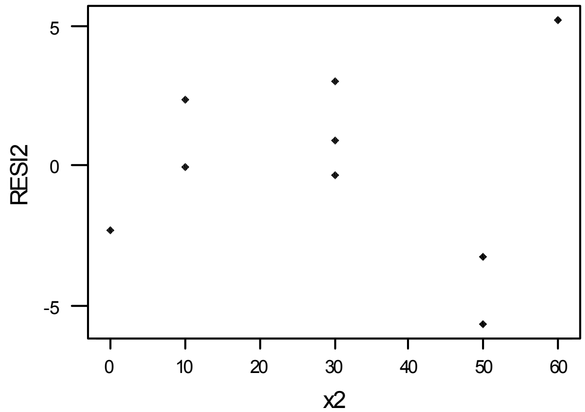
f.

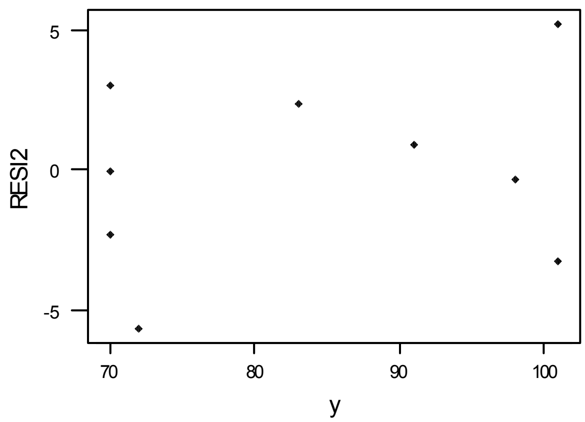


The largest predicted density is for X_1 close to 225 and X_2 close to 50.

g.

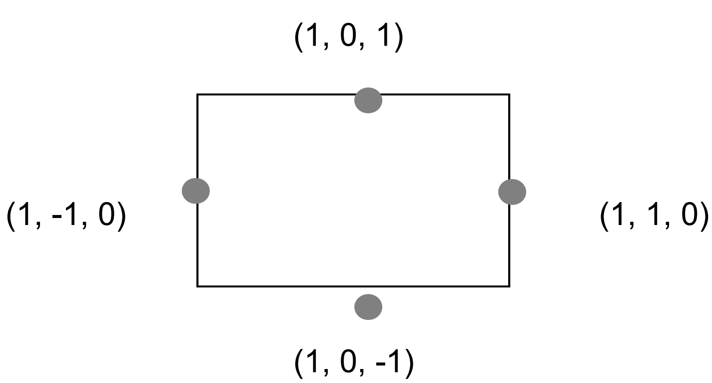






These residual plots do not show much improvement over those in (d). However, the R^2 for the fitted model in (e) is much larger (94.2%) than the R^2 for the fitted model in (b) (69.9%).

2. a. The "front" side of the cube looks like



The first ordinate is 1 = “out of the page”, 0 = “ on page” and -1 = “behind the page”. The 2nd ordinate is “left to right”, i.e. -1, 0, 1. The 3rd ordinate is “top to bottom”, i.e. 1, 0, -1.

Each of the 6 sides of the cube looks like the above sketch. The design points are located in the same relative positions. The 13th design point is the center of the cube at (0, 0, 0). The experimental region is the cube with each corner “sawed” off.

b. The 13 design points do not constitute a central composite design. A 2^3 central composite requires design points at the eight (X_1,X_2,X_3) distinct points such that each X_1, X_2, and X_3 must be 1 or -1. Further, the center point (0, 0, 0) must be included and

(2)(3) = six "star" points

$(\alpha, 0, 0)$, $(-\alpha, 0, 0)$, $(0, \alpha, 0)$, $(0, -\alpha, 0)$, $(0, 0, \alpha)$, $(0, 0, -\alpha)$.

c. Yes, there was replication at the center point (0, 0, 0). Three runs were taken at this point.

d. $\hat{y}(X_1, X_2, X_3) = 25.10 - 8.10X_1 - 15.08X_2 + 2.01X_3$

$R^2 = 80.7\%$, residual plots suggest a model that contains squared terms and cross product terms.

e. $\hat{y}(X_1, X_2, X_3) = 20.1233 - 8.095X_1 - 15.0763X_2 + 2.0062X_3 + 8.275X_1X_2 + .15X_1X_3 - 1.5675X_2X_3 - 1.0679X_1^2 + 8.2696X_2^2 + 2.1196X_3^2$

$R^2 = 99.6\%$, residual plots affirm this fit.

f. Confidence intervals (90% level) for the coefficients of X_2^2, X_3^2, X_2X_3 and X_1X_2 all contain values exclusive of zero. Thus, these terms were helpful to add to the model fitted in (d). Further, the R^2 has increased significantly

g. s = 1.456 using the full quadratic model in (e).