Chapter 2 Section 1

1. Comparing a measurement method or device to a standard one and, if necessary, working out conversions that will allow the method to produce "correct" (converted) values on average is calibration. Outputs from the measurement device to a "known" or "standard" value permits the analyst to compare what is being recorded to what really is, i.e., an assessment can be made and using calibration, a correction made to the measured value so the result is on average correct.

2. Measurand (x) is the true density (g/cc) of a selected pellet after firing at 1400°C for a selected length of time. The symbol y is the recorded density (g/cc) of a selected pellet after firing at 1400°C for a selected length of time. The term ε is the error from a recorded *y* value and the measurand *x* for a selected pellet after firing at 1400°C for a selected length of time. The term δ is the bias or difference in average recorded value from repeat observations of a single pellet fired for a selected length of time at 1400°C.

3. Assuming constant bias, independent of original density and different length of firing times, implies 5 *x*'s, 5 ε's, 5 *y*'s and one δ. If the constant bias is only for a selected firing time with possibly different original densities, then 5 δ's, one for each of the different firing times.

4. No, should have recorded original density. Without original density, cannot get the difference " after minus before" which reflects firing effect.

5. 1 measurand, 5 y's, 5 ε's and one δ.

6. $\sqrt{3.4}$

7. **a.** .797

b. $\sigma_y = \sqrt{\sigma_x^2 + \sigma_{measurement}^2}$

8. **a.** 5.7

b. $\mu_x + \delta$

Chapter 2 Section 2

1. **a**. $x + \mu_\delta$

b. $\sigma_\delta^2 + \sigma_{device}^2$

c. Part (b) is more important because the square root of (b) is the $\sigma_{R\&R}$.

2. **a.** $x_{1i} - x_{2i} = d_i$, -.1, .9, -1, -1.1, -.9; $\bar{d} = -.44$; estimates $\delta_1 - \delta_2$

b. $s_d = .847$; $estimates \sqrt{\sigma_{device1}^2 + \sigma_{device2}^2}$

c. $\bar{d} \pm t_{4,.05}\, s_d/\sqrt{5}$; (-1.248, .368)

3. **a.** (.733, 7.042). Since 95% C.I. for $\frac{\sigma_1}{\sigma_2}$ includes 1, implies no difference in consistency .

b. (-1.441, .561) using df = 5 because Sattherwaite approx. df is 5.49, rounding down gives 5. Could have used df = min ((5-1), (5-1)) = 4. No difference in bias, $\delta_1 - \delta_2$ doesn't depart from 0 since the confidence interval includes 0.

4. **a.** $s_1 \; estimates \; \sqrt{\sigma_x^2 + \sigma_{device1}^2}$

b. $s_2 \; estimates \; \sqrt{\sigma_x^2 + \sigma_{device2}^2}$

c. $\delta_1 - \delta_2$ (equipment 1 minus equipment 2).

5. The method in problem 3 is better because the variation in the estimate of $\mu_d = \delta_1 - \delta_2$ is smaller.

6. $\bar{y}_1$ estimates $\delta + x_1$ and $\bar{y}_2$ estimates $\delta + x_2$, so $\bar{y}_1 - \bar{y}_2$ estimates $x_1 - x_2$.

7. **a.** The same as in problem 3(b), but this interval now estimates $\mu_{x1} - \mu_{x2}$.

The 95% confidence interval using Satterthwaite df approximation of df = 5 becomes (-1.441, .561). The df truncated from 5.49. Could have used a more conservative df = min ((5-1), (5-1)) = 4.

b. The average density after firing using method 1 for a selected length of time minus that for method 2, i.e., $\mu_{x1} - \mu_{x2}$.

c. No, only one device is considered, device 1, and its bias δ cannot be split out, i.e., $\bar{y}_1 \; estimates \; \mu_{x1} + \delta \; and \; \bar{y}_2 \; estimates \; \mu_{x2} + \delta$.

8. $\bar{y} \pm t_{4;.025}\, s/\sqrt{n}$ becomes (4.711, 6.689) for $\mu_x + \delta$.

9. **a.** 95% C.I. for $\sigma_{measurement}$ (.477, 2.290)

b. $\bar{y} \pm t_{4;.025}\, s/\sqrt{n}$ becomes (4.711, 6.689) for $x + \delta$.

<u>Chapter 2 Section 3</u>

1. **a.** $s_y^2 = 16.3333, n_y = 3;\; s^2 = 3, n = 4;\;\; \hat{\sigma}_x = \sqrt{\max(0, 16.3333 - 3)}$ = 3.6514

b. approximate df = 177.7769/(133.3883 + 3) = 1.3 so let approx. df = 1. 95% confidence interval for σ_x becomes (3.6514 $\sqrt{1/5.024}$, 3.6514 $\sqrt{1/.001}$) or (1.629, 115.469).

2. **a.** $\sqrt{6.66}$ = 2.5807 = $\hat{\sigma}_{repeatability}$, the confidence interval is $\left(\sqrt{3.15},\ \sqrt{22.22}\right)$ or $(1.7748,\ 4.7138)$.

b. $\sqrt{1.92}$ $= 1.38564$ = $\hat{\sigma}_{reproducibility}$, the confidence interval is (0, 3.018)

c. Instrument quality should be addressed, variation operator to operator is less than repeated measurements on same item.

3. **a.** $\sqrt{3.75} = 1.9365 = \hat{\sigma}_{device}$; $(1.3304, 3.5355)$ is the 95% C.I. for σ_{device}.

b. $\sqrt{1.96} = 1.4 = \hat{\sigma}_x$; $(1.9442,\ 2.773)$ is the 95% C.I. for σ_x .

c. No, $\hat{\sigma}_x < \hat{\sigma}_{device}$

Chapter 2 Section 4

1. **a.** m = 4, I = 3, J = 1.

b. R_{11} = 4, R_{21} = 5, R_{31} = 3, $\bar{R} = 4$, $\frac{\bar{R}}{d_2(4)} = \frac{4}{2.059} = 1.9426 = \hat{\sigma}_{device}$

c. $\bar{y}_i = \mu + \alpha_i + \bar{\epsilon}_i$, I = 3, J = 1, m = 4. So, Var $\bar{y}_i$ is $\sigma_\alpha^2 + \frac{\sigma^2}{4}$.

In the context, σ_x^2 is σ_α^2.

$\bar{y}_i{}'s$ are 20.5, 18, 21.25. $\frac{\bar{\Delta}}{d_2(3)} = \frac{3.25}{1.693} = 3.68512$.

$\hat{\sigma}_x = \sqrt{\max(0, [\left(\frac{3.25}{1.693}\right)^2 - \left(\frac{1}{4}\right)(1.9426)^2])}$ = 1.6558.

2. No, only one operator.

3. **a.** I = 1, J = 3, m = 4.

b. $\bar{R} = \frac{17}{3}, d_2(4) = 2.059. So, \frac{\bar{R}}{d_2(4)} = 2.752 = \hat{\sigma}_{repeatability}$.

c. $\bar{y}_i{}'s$ are 20.5, 21, 17.5, $\Delta = 3.5, d_2(3) = 1.693$,

$\hat{\sigma}_{reproducibility} = \sqrt{\max(0, [\left(\frac{3.5}{1.693}\right)^2 - \left(\frac{1}{4}\right)(2.752)^2])} = \sqrt{2.3805} = 1.5428.$

d. No, we only have data from one “x”.

4. **a.** I = 3, J =2, m = 4.

b. $\hat{\sigma}_{repeatability} = \sqrt{.000005} = .0022361.$

$$\hat{\sigma}_{reproducibility} = \sqrt{\max\left(0, \quad \left[\left(\frac{1}{12}\right)(.0000007) + \left(\frac{2}{12}\right)(.0000053) - \left(\frac{1}{4}\right)(.000005)\right]\right)} = \sqrt{0}$$
$= 0.$

$$\hat{\sigma}_{R\&R} = \sqrt{\max\left(0, \quad \left[\left(\frac{1}{12}\right)(.0000007) + \left(\frac{2}{12}\right)(.0000053) + \left(\frac{3}{4}\right)(.000005)\right]\right)}$$

$$= .002181$$

c. (3)(2)(3) = 18 df for 95% C.I. of

$\sigma_{repeatability}$ *becomes* $\left[.0022361\sqrt{\frac{18}{31.526}}\ , .0022316\sqrt{\frac{18}{8.231}}\ \right]$ *or* $[\,.001689, .0033067]$

No confidence interval for $\sigma_{reproducibility}$.

df for confidence interval for $\sigma_{R\&R}$ is $\frac{(.002166)^4}{\frac{1}{16}\left[\frac{(.0000007)^2}{9}+\frac{2(.0000053)^2}{9}+\frac{3(.000005)^2}{6}\right]} =$
18.737 *or truncating gives* 18 *as the* df. The 95% confidence interval for $\sigma_{R\&R}$ becomes

$$[.002181\sqrt{\frac{18}{31.526}}, \qquad .002181\sqrt{\frac{18}{8.231}}\,]\ \ or\ [\,.001648, .0032253].$$

d. $\widehat{GCR} = \frac{6(.002181)}{.2} = .06543,\ 95\%\ confidence\ limits\ for\ GCR\ become$

$\left[\frac{6(.001648)}{.2}\ , \frac{6(.0032253)}{.2}\right]$ *or* $[\,.0494\ .09676]$.

Chapter 2 Section 5

1. **a.** $\hat{y}(x) = .00801 + 1.00104x$

b. $\sqrt{MSE} = \sqrt{.00267054} = .051677$

c. $[.051677\sqrt{\frac{12}{23.337}}\ ,\ .051677\sqrt{\frac{12}{4.404}}\ ]$ *or* $[\,.037056,\ .085303\,]$ is the 95% CI for

$\sigma_{repeatability}$.

2. **a.** $\hat{x} = \frac{6.11 - .00801}{1.00104} = 6.09565$

b. $1.00104 \pm t_{12;.025}$ (.00311) becomes $1.00104 \pm (2.179)(.00311)$ or 95% C.I. for the slope becomes [.99426, 1.00781] , yes, it includes 1.

3. $\hat{y}(8) = 8.0163,$ *the* 95% *prediction interval is* $(7.8997, 8.1329)$.

4. No, y_{new} is outside y's in the data used to model the relationship.

Chapter 2 Section 6

1. **a.**

$\bar{\bar{p}}$	$\bar{\bar{p}}\,(1-\bar{\bar{p}})$	$\overline{\hat{p}(1-\hat{p})}$
0.656250	0.225586	0.224609
0.656250	0.225586	0.220703
0.500000	0.250000	0.24807
0.921875	0.072021	0.069336
0.765625	0.179443	0.174805
0.953125	0.044678	0.043945
0.796875	0.161865	0.155274
0.968750	0.030273	0.029297
0.890625	0.097412	0.094727
0.984375	0.015381	0.014649

b. $\frac{\hat{\sigma}^2_{\text{reproducibility}}}{\hat{\sigma}^2_{R\&R}} = \frac{.0027245}{.130225} = .0209\ or\ 2.09\%$; $Note$: $.130225$ is the average of the $\bar{p}\left(1-\bar{p}\right)$ column and thus equals $\hat{\sigma}^2_{R\&R}$. Also, $\hat{\sigma}^2_{\text{repeatability}} = .1275$ = the average of the $\overline{\hat{p}(1-\hat{p})}$ column.
So, $\hat{\sigma}^2_{\text{reproducibility}} = \hat{\sigma}^2_{R\&R} - \hat{\sigma}^2_{\text{repeatability}} = .130225 - .1275 = .0027245$.

c. $\hat{\sigma}_{\text{reproducibility}} = \sqrt{.0027245} = .052196$

d. $\hat{\sigma}_{\text{repeatability}} = \sqrt{.1275} = .35707$

e. $1-1 \pm 1.645\sqrt{\frac{(2)(.9)(.1)}{16}}\ or\ 0 \pm .1744$ No, the $\hat{p}'s$ are very close for each part.

f. $\hat{p}_{1i} - \hat{p}_{3i} = d_i$; $\bar{d} \pm t_{9;.95}\frac{s_d}{\sqrt{n}}$; $s_d = .0574$; $90\%\ C.I.\ (-.0458, .0209)$.