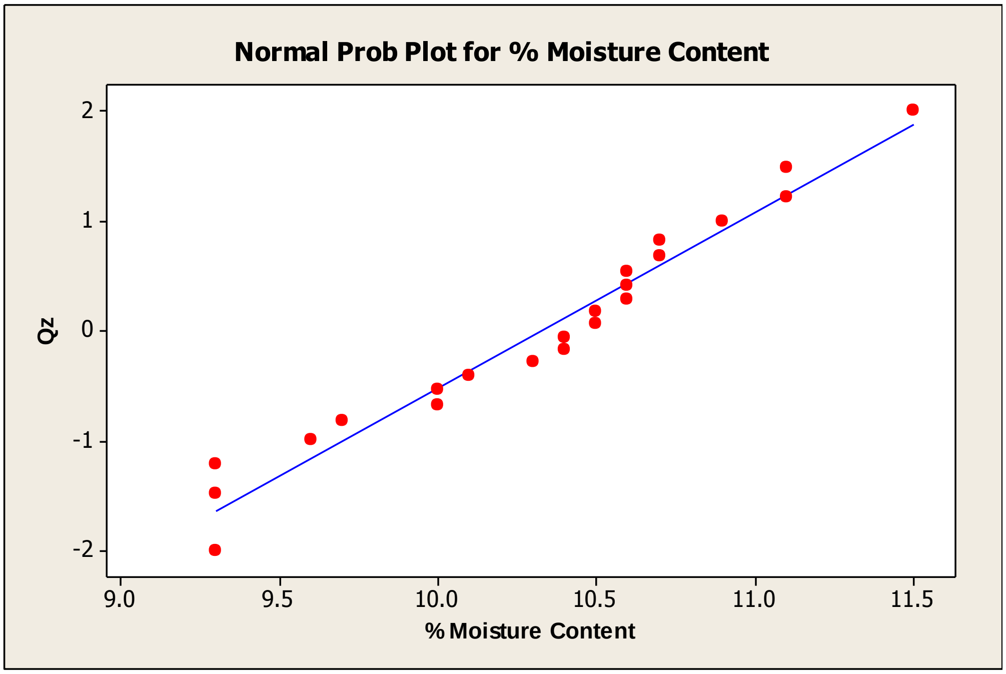
Chapter 4 Section 1

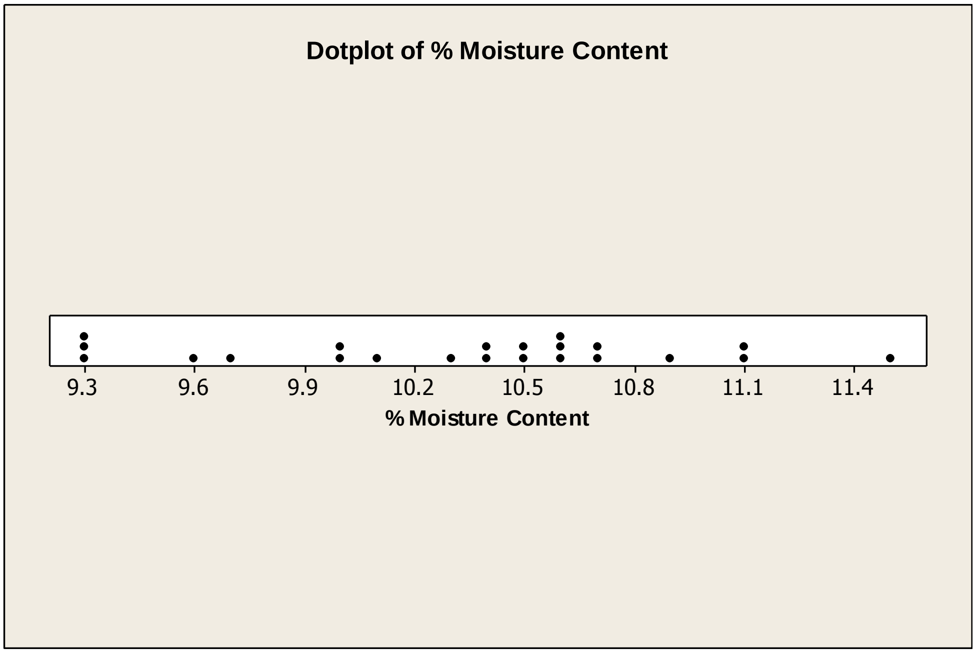
1. **a.**

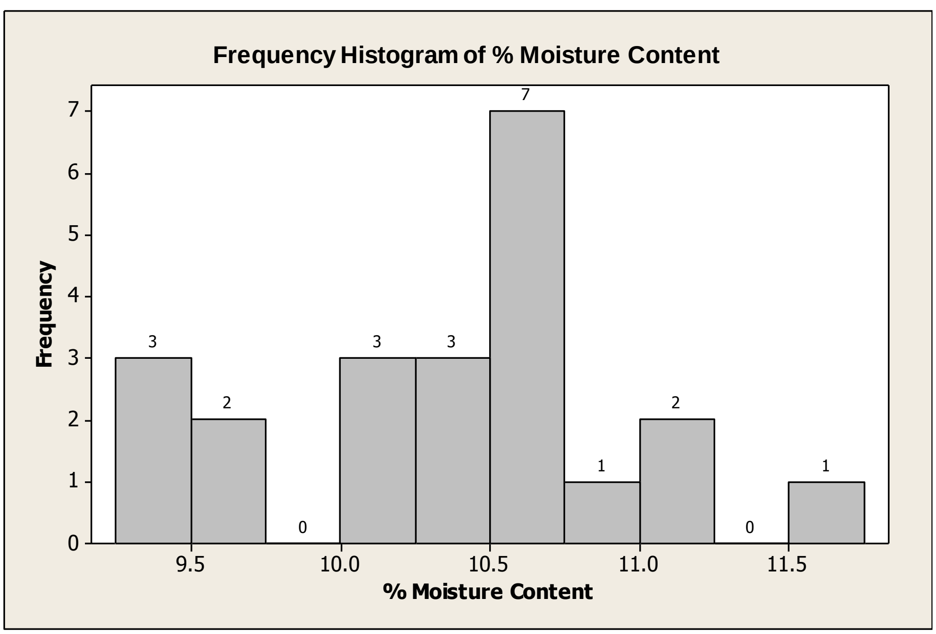


It appears normal distribution is a reasonable model for % moisture content because the plot is approximately linear.

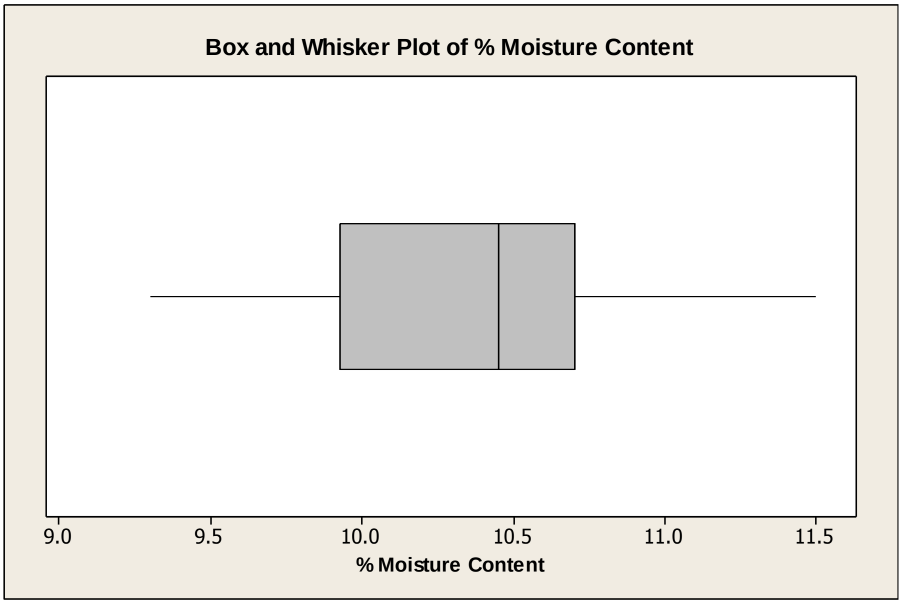
b. The above plot is Qz vs % Moisture content. 1.59614 = slope $\approx \frac{1}{\sigma}$, or $\sigma \approx \frac{1}{1.59614} = .6265.$ $\frac{-\mu}{\sigma} \approx y - intercept$ = -16.4838. So, $\mu \approx 16{,}4737 * .6265 = 10.327.$

Regressing %Moisture Content (y) vs. Qz (x) gives the estimated slope to be .601 which estimates σ and vertical intercept of 10.327 which estimates μ.

c. 
Dotplot of % Moisture Content
9.3
9.6
9.9
10.2
10.5
10.8
11.1
11.4
% Moisture Content

d. 
Frequency Histogram of % Moisture Content
Frequency
0
1
2
3
4
5
6
7
3
2
0
3
3
7
1
2
0
1
9.5
10.0
10.5
11.0
11.5
% Moisture Content

e.



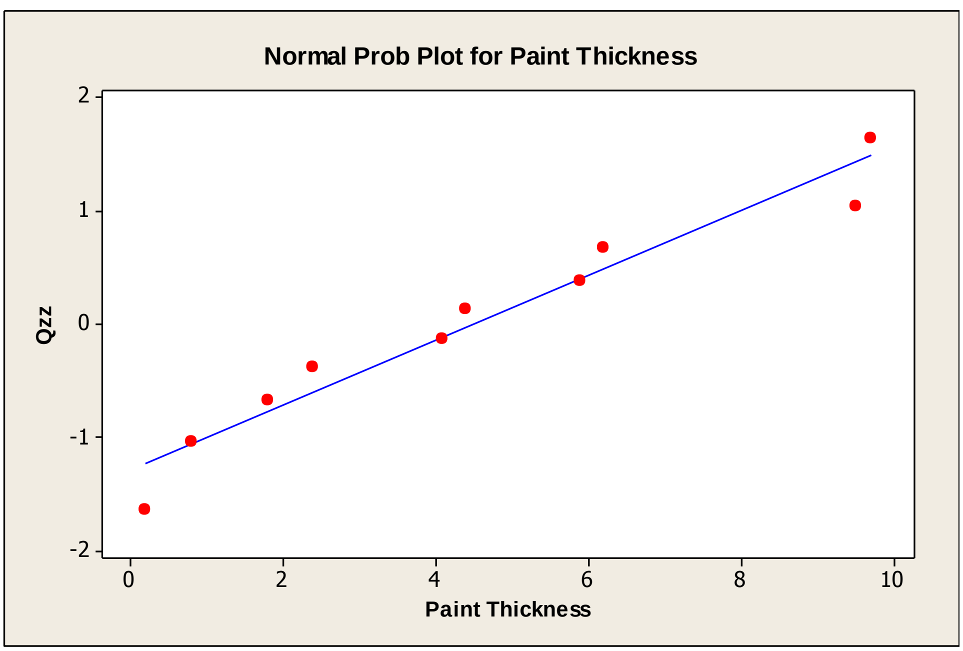
f. Looking at column of (i - .5)/22, 10 is the 25th quantile, 10.45 is the 50th quantile and 10.7 is the 75th quantile.

g. The IQR is 10.7 – 10 = .7

2. a. The time order of measured lots must be recorded.

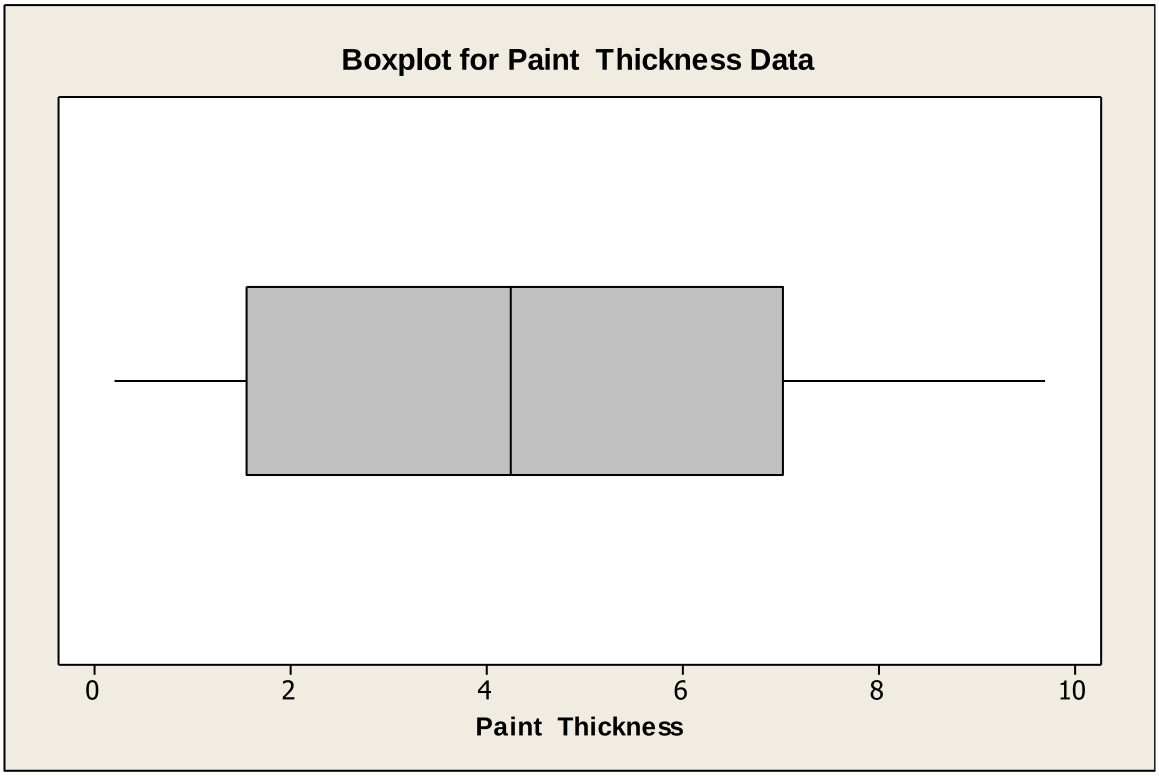
b. The % moisture content must be stable or consistent over time, no trends or cycling over time.

3. a.



A straight line appears to match the plot, implying a normal distribution of paint thickness is a reasonable assumption.

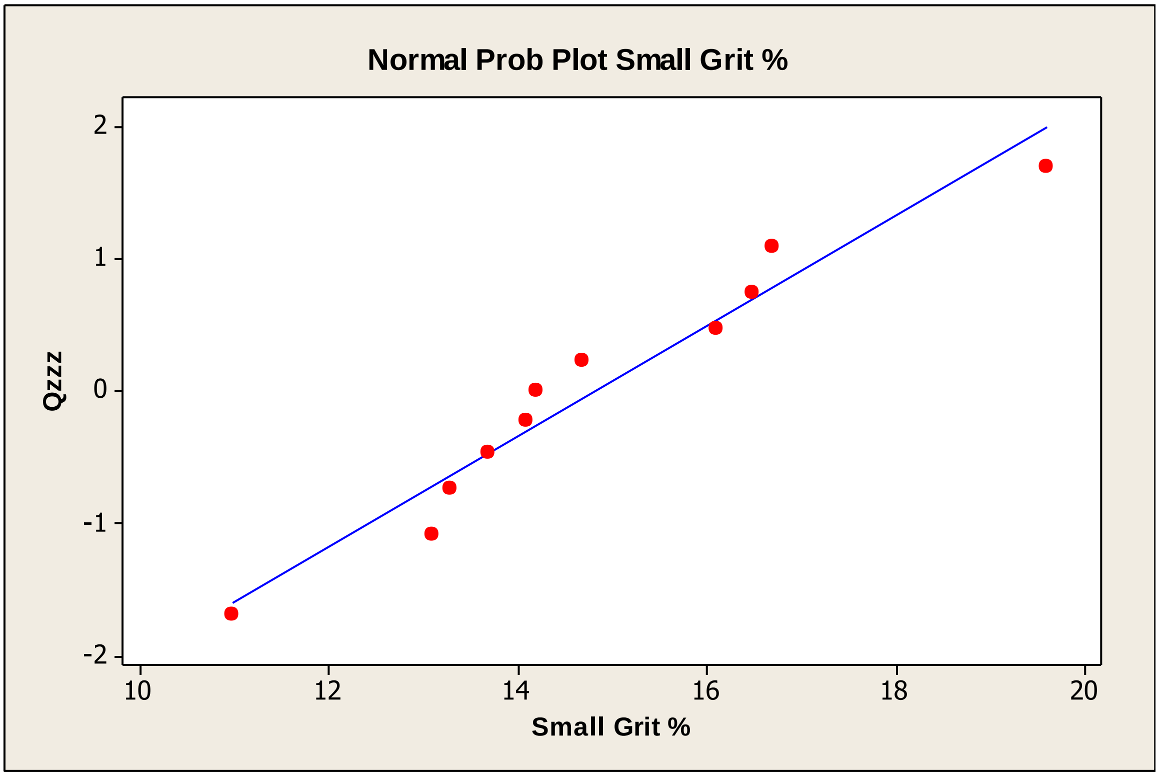
b.



No outliers are detected.

c. 25th quantile is 1.8, 50th quantile is 4.25 and 75th quantile is 6.2.

4. **a.**



No strong departures from linearity, so a normal distribution is a reasonable assumption.

b. $\bar{x} = 14.818$ and $s = 2.299$

c. 17.86 is approx. 90th quantile. .9 is 40% of the distance between .863636 and .954545. So, 17.86 is 40% of the distance from 16.7 to 19.6.

Chapter 4 Section 2

1. **a.** Lower 95% confidence limit = (6)(.61)$\sqrt{21/\chi^2_{21;.975}}$ = 2.82

Upper 95% confidence limit = (6)(.61) $\sqrt{21/\chi^2_{21;.025}}$ = 5.23

b. No, (2.82, 5.232) doesn't set within (0, 3).

c. Estimated C_{pk} = min { (10.327 – 9)/1.83 , (12-10.327)/1.83 }
= min { .7251, .9142 } = .7251. Since this is less than

1.0, we have more than 1% outside specs.

d. $\frac{U-L}{6\hat{\sigma}} = \frac{3}{6(.610)} =$ .82. Even if we could center the process, the "best" quality would be a C_{pk} estimate of .82. Need to reduce variation, even if we center the process.

e. 95% C.I. for C_{pk}; $.7251 \pm 1.96\sqrt{\frac{1}{198} + \frac{(.7251)^2}{44-2}}$ or $.7251 \pm .25979$;

(.4653, .9849).

f. $\left\{ \frac{U-L}{6s}\sqrt{\frac{\chi^2_{lower}}{n-1}}, \frac{U-L}{6s}\sqrt{\frac{\chi^2_{higher}}{n-1}} \right\}$ becomes $\left\{\frac{3}{6(.61)}\sqrt{\frac{10.283}{21}}, \frac{3}{6(.61)}\sqrt{\frac{35.479}{21}}\right\}$

(.57357, 1.0654). 95% C.I. for C_p. Not centered. $\bar{\bar{x}} = 10.327 \neq 10.5$.

2. **a.** Widen spec limits.

b. Same as (a).

3. **a.** $\left\{ \frac{U-L}{6s}\sqrt{\frac{\chi^2_{lower}}{n-1}}, \frac{U-L}{6s}\sqrt{\frac{\chi^2_{higher}}{n-1}} \right\}$ becomes $\left\{\frac{40}{6(6.18)}\sqrt{\frac{3.325}{9}}, \frac{40}{6(6.18)}\sqrt{\frac{16.919}{9}}\right\}$

(.6556, 1.479) 90% C.I. for C_p

b. Estimated C_{pk} =
= min{ (2291.3 – 2280)/(3)(6.18), (2320 – 2291.3)/(3)(6.18) }
= min { .609, 1.548 }
= .609.

90% C.I. for C_{pk}

$.609 \pm 1.645\sqrt{\frac{1}{90} + \frac{(.609)^2}{18}}$ or $.609 \pm .2929$;
(.3161, .9019).

c. If the process can be centered, the quality will improve but still not high enough, the 90% C.I. for C_p in a**.** is not completely above 1.

4. $$\frac{\sqrt{\chi^2_{higher}}}{\sqrt{\chi^2_{lower}}} = \sqrt{45.722/16.047} = 1.688$$

5. **a.** Lower 90% confidence limit for 6σ = (6)(3.35)$\sqrt{9/\chi^2_{9;.95}}$ = 14.659

Upper 90% confidence limit for 6σ = (6)(3.35) $\sqrt{9/\chi^2_{9;.05}}$ = 33.069

b. $\bar{x} = 4.5$, $s = 3.35$; No, $\bar{x} \pm s$ falls completely outside specs.

6. **a.** Estimated C_p = 3.6/6s = .261.

b. Estimated C_{pk} =

= min { (14.818 – 13)/3(2.299), (16.6 – 14.818)/3(2.299) }

= min { .2636, .2584 } = .2584

c. 95% C.I. for $C_{p;}$

$\{\frac{U-L}{6s}\sqrt{\frac{\chi^2_{lower}}{n-1}}, \frac{U-L}{6s}\sqrt{\frac{\chi^2_{higher}}{n-1}}\}$becomes$\{\frac{3.6}{6(2.299)}\sqrt{\frac{3.247}{10}}, \frac{3.6}{6(2.299)}\sqrt{\frac{20.483}{10}}\}$

(.1487 , .3735) 95 % C.I. for C_p

95% C.I. for C_{pk};

$.2584 \pm 1.96\sqrt{\frac{1}{99} + \frac{(.2584)^2}{20}}$ or $.2584 \pm .2272$;

(.0312, .4856).

d. Potential and present not good. Need both centering and reduce “s”.

Chapter 4 Section 3

1. **a.** $10.327 \pm t_{21;.975}(.61)\sqrt{1+\frac{1}{22}}$; $t_{21;.975}$ = 2.080;

10.327 ± 1.2973; $(9.0297, 11.6243)$

b. p = .95; n = 22; $1 - p^n - n(1-p)p^{n-1}$ = 1 - .323533 - .37461 = .30.
30% confident that 95% of additional lots have between 9.3% and 11.5% moisture content.

c. 95% sure the interval contains 95% of the moisture contents
$10.327 \pm 2.7(.61)$; $(8.68, 11.974)$

2. **a.** n = 10; n/(n + 1) = 10/11

b. 95% sure the interval contains 99% of the moisture contents
$2291.3 \pm 4.437(6.18)$; $(2263.879,\ 2318.72)$

c. $2291.3\ \pm t_{9;.975}(6.18)\sqrt{1+\frac{1}{10}}$; $t_{9;.975}$ = 2.262;

$2291.3\ \pm\ 14.6614$; $(2276.638\ ,\ 2305.96)$ contains length of one item with 95% probability.

3. **a.** 95% confidence to contain 99% of paint thicknesses.
$4.5 \pm 4.437(3.35)$; $(0,\ 19.3639)$
b. 9/11 or 81.81%
c. p = .9; n = 10; $1 - p^n - n(1-p)p^{n-1}$ = 1 - .34867 - .38742 = .2639.
26.39% confident that 90% of additional paint thicknesses are between .2 and 9.7.

4. **a.** $14.818\ \pm t_{10;.995}(2.299)\sqrt{1+\frac{1}{11}}$; $t_{10;.995}$ = 3.169;

$14.818 \pm\ 7.609$; $(7.209\ ,\ 22.4275)$ contains lot % of small grit particles from the next lot with 99% prob.
b. 99% confident the interval contains % of small grit particles for 90% of lots.
$14.818 \pm 3.429(2.299)$; $(6.935,\ 22.701)$

c. (a) is a prediction interval; (b) is a tolerance interval

d. 95% confident 95% of all lots have % small grit particles that exceed L. $14.818 - 2.815(2.299)$; $L = 8.436$

Chapter 4 Section 4

1. **a.** $F = kW$. $\mu_F\ \approx\ \mu_k\,\mu_W = .3(10) = 3$.
b. $\sigma_F^2 \approx\ 10^2(.01)^2 + (.3^2)(.04) =\ .01 +\ .0036 =\ .0136$.
$\sigma_F\ \approx\ .1166$.

c. $3 \pm\ 2(.1166)$; $(2.7668,\ 3.2332)$

2. **a.** $x_1 + x_2 + \ldots + x_{200}$; The thickness of each page is a random variable.

b. $\mu_1 + \mu_2 + \cdots + \mu_{200} = 2.$

c. $\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_{200}^2} = \sqrt{(.0001)^2(200)} = .001414.$

d. $1.96\sigma_T = 1.96(.001414) = .00277.\ \ 2 \pm .00277\text{inches}.$
95% of all books have thicknesses within .00277 inches of 2 inches.

3. **a.** $\sigma_u = \sqrt{\sigma_{x1}^2 + \sigma_{x2}^2} = \sqrt{2(.01)^2} = .01414.$

b. D = $\sqrt{(5+u)^2 + v^2}$; $\frac{\partial D}{\partial u} = \frac{1}{2}(5^2 + 10u + u^2 + v^2)^{-\frac{1}{2}}(10 + 2u)$

$\frac{\partial D}{\partial v} = \frac{1}{2}(5^2 + 10u + u^2 + v^2)^{-\frac{1}{2}}(2v).$

$(\frac{\partial D}{\partial u})^2\sigma_u^2 = .25(25+0)^{-1}10^2(.0004)$= .0004.

$(\frac{\partial D}{\partial v})^2\sigma_v^2 = .25(25+0)^{-1}(.0004)(0) = 0.$

$\sigma_D = \sqrt{.0004 + 0} = .02.$

c. 5.0017 $\pm\ t_{19;.95}(.0437)\sqrt{1 + \frac{1}{20}}$; $t_{19;.95}$ = 1.729;

$5.0017 \pm .07742$; $(4.9243\ ,\ 5.079)$ contains next distance with 90% probability.