Chapter 2
Basics of Electricity and Magnetism

"My direct path to the special theory of relativity was mainly determined by the conviction that the electromotive force induced in a conductor moving in a magnetic field is nothing more than an electric field."

—Albert Einstein, message to the centennial of Albert Michelson's birth, December 19, 1952.

2.1 Introduction

This chapter provides a succinct review of the essential physics of electricity and magnetism that forms the basis for understanding how electric power systems work. Later chapters will use this foundational material to build models of power system components and systems. Electric fields, magnetic fields, and Maxwell's equations are the topics of the three sections of this chapter. Examples are given that illustrate the basic characteristics of core electrical components and electromechanical devices.

2.2 The Electric Field

Coulomb showed that two like point charges repel each other.[1] In fact, the force between two stationary particles of charge q_1 and q_2 a distance r apart in free space is given by the formula

$$F = k\frac{q_1 q_2}{r^2} \tag{2.1}$$

This is *Coulomb's law*. A positive F is repulsion and a negative F is attraction. In SI units, the unit of charge is the coulomb, the unit of distance is the meter, and the

[1]Material in this section can be found more fully discussed in any book on theoretical physics, Example [162].

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unit of force is the newton. In this case, $k = 1/4\pi\varepsilon_0$, where ε_0 is the permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N \cdot m^2$.

A region in space contains an *electrical field* if a charge fixed in it experiences a force. The *electric intensity* **E** at a point in the region is the force exerted on a unit positive charge. Thus, the force **F** on a particle of charge q at a point of electrical intensity **E** is

$$\mathbf{F} = q\mathbf{E}$$

Since the force between two stationary charges depends only on the distance between them, it follows that the electrical intensity is derivable from a potential function; i.e., it is the gradient of a scalar potential function $\varphi(x, y, z)$. The work done in moving a unit charge against the field by an amount $d\mathbf{r}$ is the increase in potential $d\varphi$, $d\varphi = \mathbf{E} \cdot d\mathbf{r}$, or

$$\mathbf{E} = -\nabla\varphi$$

Example 2.1 We will compute the potential V produced by a point charge q. At any distance r from q, from (2.1), we have

$$\mathbf{E} = k\frac{q}{r^2}\frac{\mathbf{r}}{r}$$

so that

$$dV = -k\frac{q}{r^3}\mathbf{r} \cdot d\mathbf{r} = -k\frac{q\,dr}{r^2}$$

Integrating, we obtain

$$V(r) = V_0 + k\frac{q}{r}$$

If we impose the boundary condition $V(0) = 0$, then $V_0 = 0$.

We will be interested in material media that can be classified as *conductors* or *dielectrics*. Conductors contain free electrons which are under the influence of an electric field can flow freely through conductors. So, conductors admit the flow of current. A dielectric is an electrical insulator in that it is highly resistant to current flow. An electrical field applied to a dielectric does cause motion of charges within it. The resultant motion or current is composed of two parts, a negligible conduction current and a displacement current. The neutrally charged atoms or molecules that make up the dielectric typically have the center of positive charge and the center of negative charge displaced. Such an arrangement constitutes an *electrical dipole*. Even if the charge centers are not displaced, the application of an electric field generally induces a displacement. If the dipole consists of a charge $-q$ and a charge q separated by a distance l, then we associate it with a *dipole moment* **p**, a vector of magnitude ql (coulomb meter^2) and direction pointing from $-q$ to q. An applied electric field imposes a force and a torque on the dipole. If the imposed field is constant over the domain of the dipole (which is ordinarily the case at the microscopic scale), then the net force acting on a dipole is negligible and the torque is given by

$$\tau = \mathbf{p} \times \mathbf{E}$$

At the microscopic level, the stretching and twisting of the dipoles that occurs under the influence of the applied electric field alters the potential function defining the electric field within the dielectric, thereby modifying the field. The change in the field is denoted **P**, called the *polarization* and the modified field is denoted **D**, called the *(displacement) electric flux density*, so we have

$$\mathbf{D} = \mathbf{E} + \mathbf{P} \tag{2.2}$$

In isotropic media, the polarization is proportional to the electric field intensity, $\mathbf{P} = \chi\mathbf{E}$ and as a consequence $\mathbf{D} = \varepsilon\mathbf{E}$. χ is known as the *electric susceptibility* and ε as the *permittivity* of the dielectric. In free space $\mathbf{P} = \mathbf{0}$.

2.3 The Magnetic Field

The movement of electrical charge gives rise to a force field known as a *magnetic field*. The magnetic field is characterized by a vector field **B** known as the *magnetic flux density* which has SI units weber/meter^2, equivalently, volt-second/meter^2. Such fields arise on a macroscopic level, as when current flows through a wire, or on an atomic scale, as electron spin in an atom. Consider that a current **I** flows along a differential element $d\mathbf{l}$, then the differential magnetic field produced is given by the Biot–Savart law

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times (\mathbf{r}/r)}{r^2} \tag{2.3}$$

where μ_0 is the permeability of free space, I is the current, $\mathbf{r}$ is the displacement vector from the current element to the field point.

Example 2.2 Current in a Thin Wire. We can use the Biot–Savart law to compute the field produced by a constant current i flowing in a long thin straight wire as shown in Figure 2.1.

In accordance with Figure 2.1, the Biot–Savart law (2.3) can be written

$$\mathbf{B} = \frac{\mu_0 i}{4\pi} \oint \frac{d\mathbf{z} \times \mathbf{r}/r}{r^2}$$

We can express dz and r in terms of ρ and α: $dz = \rho \sec^2 \alpha$, $r = \rho \sec \alpha$, to obtain

$$\mathbf{B} = \frac{\mu_0 i}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\cos\alpha\, d\alpha}{\rho} \mathbf{u}_\varphi = \frac{\mu_0}{2\pi} \frac{i}{\rho} \mathbf{u}_\varphi$$

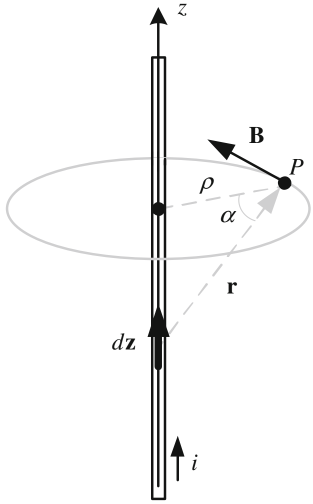


Fig. 2.1 An infinite wire carrying constant current

Some materials have an atomic structure in which the electron spins are aligned, thereby giving rise to a *permanent magnet*. The bar magnet is a familiar example of a magnetic dipole. Magnetic dipoles on atomic or molecular scale are the basic building blocks of all magnetic materials. We associate with a magnetic dipole its *magnetic dipole moment* $\mathbf{m}$, with units ampere-meter^2. When a magnetic dipole is placed in a magnetic field, it experiences a moment

$$\tau = \mathbf{m} \times \mathbf{B}$$

When a magnetic field is applied, the atomic scale magnetic dipoles in the material tend to align with it. Materials can be classified in accordance with degree of alignment produced by the field. Diamagnetic and paramagnetic materials have relatively small alignment, whereas ferromagnetic materials have virtually complete alignment. The dipole alignment in a material gives rise to a macroscopic dipole moment per unit volume, $\mathbf{M}$, called the *magnetization* of the material. So, for example, if each atom in a material has a dipole moment $\mathbf{m}$ and there are N atoms per unit volume, then $\mathbf{M} = N\mathbf{m}$. $\mathbf{M} = 0$ in free space.

The *magnetic field intensity*, $\mathbf{H}$, is defined by the relation

$$\frac{1}{\mu_0}\mathbf{B} = \mathbf{H} + \mathbf{M} \tag{2.4}$$

Notice the similarity of (2.4) with (2.2). For linear materials, the relationship between $\mathbf{H}$ and $\mathbf{M}$ is

$$\mathbf{M} = \chi_m \mathbf{H},$$

where χ_m is the *magnetic susceptibility* of the material. For paramagnetic materials, χ_m is positive and for diamagnetic materials it is negative. Ferromagnetic materials are generally not linear. For linear materials, we have

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi_m) \mathbf{H} = \mu \mathbf{H},$$

where $\mu = \mu_0 (1 + \chi_m)$ is the *permeability* of the material. The ratio μ/μ_0, $\mu_r = (1 + \chi_m)$, is called the *relative permeability*.

2.4 Maxwell's Equations

We will summarize some basic concepts about magnetic and electric fields.

Table 2.1 defines the symbols used in the following discussion.

Four basic equations, called Maxwell's equations, describe the behavior of electromagnetic fields. These include the following:

1. Gauss law describes how charge produces an electrical field,

$$\nabla \cdot \mathbf{D} = \rho \text{ or } \int_S \mathbf{D} \cdot d\mathbf{s} = \int_V \rho dv$$

   This implies that the integral of the electrical flux density over a surface S that encloses a volume V must equal the total charge contained in V,

Table 2.1 Electromagnetic Fields Nomenclature

Symbol	Quantity	Units
$\mathbf{E}$	electric field intensity	volt per meter
$\mathbf{D}$	electric flux density	coulomb per meter^2
$\mathbf{H}$	magnetic field intensity	ampere per meter
$\mathbf{B}$	magnetic flux density	weber per meter^2
Φ	magnetic flux	weber
ρ	electric charge density	coulomb per meter^3
$\mathbf{J}$	current density	ampere per meter^2
$d\mathbf{s}$	differential vector element of surface area with direction perpendicular to surface S	meter^2
dv	differential element of volume enclosed by surface S	meter^3
$d\mathbf{l}$	differential vector element of path length tangential to contour C enclosing surface S	meter

$$\int_S \mathbf{D} \cdot d\mathbf{s} = Q_{enclosed}$$

2. Gauss law for magnetism asserts the absence of magnetic sources

$$\nabla \cdot \mathbf{B} = 0 \text{ or } \int_S \mathbf{B} \cdot d\mathbf{s} = 0,$$

where the surface S again encloses a volume V. The magnetic flux density $\mathbf{B}$ is a vector field defined in three-dimensional space. The integral curves of $\mathbf{B}$ are the "magnetic flux lines" or "magnetic field lines." The integral of magnetic flux density over any closed surface must be zero implies that these lines are closed loops.
The magnetic flux through area S bounded by a closed curve C, Φ, is defined as

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

3. The Maxwell–Faraday equation describes how changing magnetic fields produce electrical fields

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \text{ or } \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s},$$

where S is a surface bounded by the closed curve C. The equation shows how an electric field is produced by varying the magnetic flux passing through a given cross-sectional area. As will be seen, this is the fundamental principle underlying the operation of electric motors and generators.

4. The Ampère–Maxwell law describes how the magnetic fields are produced by currents and changing electrical fields

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \text{ or } \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} + \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{s},$$

where C denotes the closed edge (or boundary) of an open surface S. Define the encircled current

$$I_{encircled} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

If $\mathbf{D}$ is very slowly varying, then the Ampère–Maxwell law reduces to Ampère's law

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{encircled}$$

Another important result is the *Lorentz force equation* which describes the force $\mathbf{F}$ acting on a particle of charge q moving through an electromagnetic field with velocity $\mathbf{v}$

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

In addition, *Ohm's Law* states that the current density in a conductor is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E}$$

where σ is the *conductivity* with units ohms per meter.

Remark 2.3 (*Scalar and Vector Potentials*) Gauss law for magnetism states that the divergence of the magnetic field vanishes, thereby implying that $\mathbf{B}$ can be expressed

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{2.5}$$

where $\mathbf{A}$ is some magnetic vector potential. Then, Faraday's law can be written

$$\nabla \times \left[\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right] = 0$$

But the fact that the curl of a vector vanishes implies that the vector can be expressed as the gradient of a scalar potential, φ. Hence,

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} \tag{2.6}$$

Further substitutions in Maxwell's equations and some algebra lead to partial differential equations for $\mathbf{A}$ and φ [167]:

$$\mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \mu \mathbf{J}$$

$$\mu\varepsilon \frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi = \frac{\rho}{\varepsilon}$$

The implication of this is that the scalar potential φ depends on the charge distribution, whereas the vector potential $\mathbf{A}$ depends on the current density.

Remark 2.4 (*Electromotive Force*) The *electromotive force* (EMF), E, produced by some generating mechanism is the energy per unit charge, i.e., the voltage change, made available by the generating mechanism. The energy required to move a unit charge along a path from point a to point b through an electric field $\mathbf{E}$ is

$$E = \int_a^b \mathbf{F} \cdot d\mathbf{l} = \int_a^b (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

Remark 2.5 (*Magnetomotive Force*) The *magnetomotive force* (MMF), F, plays a role in magnetic circuits similar to that of E in electrical circuits. F is defined by

$$F = \int_a^b \mathbf{H} \cdot d\mathbf{l}$$

In a magnetic circuit comprised of a loop of uniform magnetic material of length l and cross-sectional area A, it is useful to define the *reluctance*, R:

$$R = \frac{l}{\mu_0 \mu_r A}$$

Then

$$F = \oint \mathbf{H} \cdot d\mathbf{l} = R\,\Phi$$

This formula is similar to Ohm's law governing the flow of current through a resistor.

Remark 2.6 (*Continuity of Charge*) Note that the taking the divergence of the Ampère–Maxwell law yields

$$\nabla \cdot \mathbf{J} = -\frac{\partial \nabla \cdot \mathbf{D}}{\partial t}$$

and using Gauss electric field law gives

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \text{ or } \int_S \mathbf{J} \cdot d\mathbf{s} = -\int_V \rho\, dV$$

This of course asserts the principle of continuity (or conservation) of charge.

Example 2.7 Capacitor. A *capacitor* is a device that stores charge. A typical capacitor consists of two conductors separated by a dielectric. The simplest example is the parallel plate capacitor shown in Figure 2.2.

In its uncharged state, both plates have zero charge. The capacitor can be charged using a battery or other means to a charge level Q, in which case one plate becomes positively charged with charge $+Q$ and the other negatively charged with charge $-Q$. The potential difference, ΔV, across the two plates can be obtained by integrating the electric field along a path through the dielectric from the positively to the negatively charged plate. Then, *capacitance*, C, of the device is the ratio of the charge to the potential difference, $C = Q/\Delta V$. The SI unit of capacitance is the farad (F). Thus, one farad is one coulomb per volt.

To compute the capacitance of the capacitor in Figure 2.2, first compute the electric field in the dielectric between the plates. Apply Gauss law

$$\int_S \varepsilon \mathbf{E} \cdot d\mathbf{s} = \int_V \rho\, dv$$

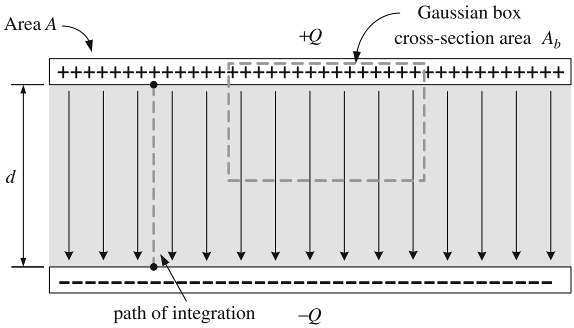


Fig. 2.2 A two-plate capacitor with very large plate area

to the box shown in the figure to find

$$\varepsilon E A_b = \sigma A_b,$$

where σ is the charge per unit area, Q/A, and ε is the permittivity of the dielectric. Thus, $E = \sigma/\varepsilon$. Now integrate from the positive plate to the negative plate along the integration path shown to get the potential difference:

$$\Delta V = -\int_{+}^{-} \mathbf{E} \cdot d\mathbf{l} = E d$$

Thus, the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon A}{d}$$

Example 2.8 Wire Revisited. From Example 2.2, we know that

$$\mathbf{H} = \frac{i}{2\pi\rho}\mathbf{u}_\varphi$$

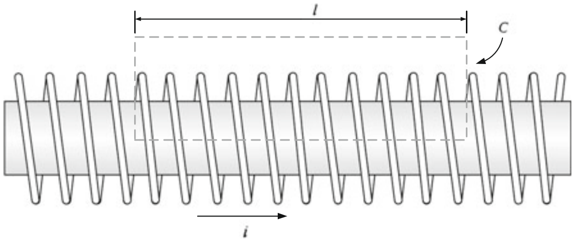
Let us verify Ampère's law. Choose for C a circular path of radius ρ in a plane with z constant.

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_0^{2\pi} \frac{i}{2\pi\rho}\rho d\varphi = i$$

Example 2.9 Infinite Solenoid. Consider an infinite solenoid composed of a tightly wound, thin wire coil with a core as shown in Figure 2.3. The solenoid has n turns per unit length, cross-sectional area, A, length, l and a constant current i passes through it. The core has permeability $\mu = \mu_0\mu_r$.

By symmetry, the induced magnetic field is horizontal, and in view of the winding direction and current flow the field vectors point to the right. We can use Ampère's

Fig. 2.3 Solenoid



law to compute its magnitude. Choose a rectangular contour with horizontal lines outside the coil, one on each side — above and below the coil. Since the net encircled current is zero, the field outside of the coil is zero. To determine the field inside the coil, choose a contour C as shown. Application of Ampère's law yields $Hl = nil$. Consequently, we have

$$\mathbf{H} = \begin{cases} 0 & \text{outside the coil} \\ ni\mathbf{u}_z & \text{inside the coil} \end{cases}$$

Inside the coil, the magnetic flux density is $\mathbf{B} = \mu_0 \mu_r ni\, \mathbf{u_z}$ and the magnetic flux through a cross section is $\Phi = \mu_0 \mu_r i\, A$. The number of loops in a section of length l i nl so the effective area through which $\mathbf{B}$ passes is nlA. Consequently, the effective flux within the coil section is $\lambda = \mu_0 \mu_r nli\, A$. λ is called the *flux linkage*.

Example 2.10 Inductive Loop. A single, perfectly conducting wire loop encircles a core of permeable magnetic material in Figure 2.4.

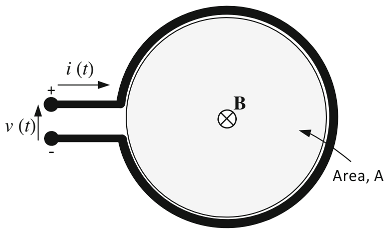
As in the previous example, application of Ampère's law enables computation of the magnetic flux density in the core, $|\mathbf{B}| = \mu i\,(t)$. It follows that the induced back EMF is

$$E = -\frac{d\lambda}{dt}, \quad \lambda = \mu A i\,(t)$$

and so the applied voltage is related to the current in the wire loop by

$$v\,(t) = \mu A \frac{d\,i\,(t)}{dt}$$

Fig. 2.4 A single-wire loop surrounds a magnetic core



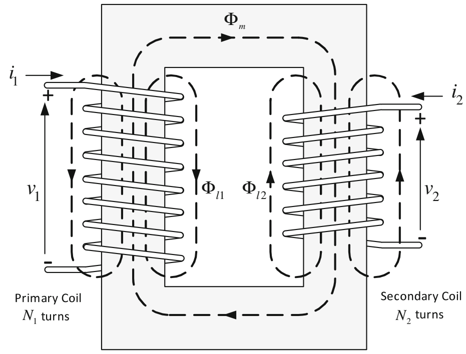


Fig. 2.5 A transformer formed of two coupled coils with a common core

For a tightly wound coil of N loops, the voltage–current relationship is

$$v(t) = \frac{d\,\lambda(t)}{dt} = \mu N A \frac{d\,i(t)}{dt} = L\frac{d\,i(t)}{dt},$$

where L is the *inductance*.

Example 2.11 Transformer. The transformer in Figure 2.5 has a primary coil with N_1 turns and a secondary coil with N_2 turns.

Consider the ideal case, in which the transformer has the following characteristics:

1. no losses
2. zero leakage flux, i.e., $\Phi_{l,1} = \Phi_{l,2} = 0$
3. zero reluctance.

Faraday's law yields

$$v_1 = N_1\frac{d\Phi_m}{dt},\; v_2 = N_2\frac{d\Phi_m}{dt}$$

which implies

$$\frac{v_2}{v_1} = \frac{N_2}{N_1}$$

In addition, zero reluctance implies that the magnetomotive force around a closed loop in the core sums to zero, so that

$$N_1 i_i + N_2 i_2 = 0$$

Thus,

$$i_2 = -\frac{N_1}{N_2} i_1$$

Notice that $v_2 i_2 = -v_1 i_1$ which implies that the instantaneous power entering on the left is the same as that exiting on the right, as expected for this lossless transformer.

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