

Chapter 2

A Single-Machine Infinite-Bus Power System Installed with a Power System Stabilizer

2.1 Linearized Model of a Single-Machine Infinite-Bus Power System Installed with a Power System Stabilizer

2.1.1 General Linearized Mathematical Model

2.1.1.1 Full Mathematical Model of a Synchronous Generator

Fundamental equations describing the dynamics of a synchronous generator are the well-known Park's voltage equations. They are given based on a coordinate system consisting of a d-axis (direct axis) fixed on the field winding axis of synchronous generator and a q-axis (quadrature axis). After Park's transformation, three armature phase windings a, b, and c on the stator of synchronous generator are replaced by two equivalent armature phase windings, d and q. Two damper windings on the rotor, D and Q, are permanently short-circuited. Field winding f is DC excited. Park's voltage equations of those five windings have the simplest form as follows:

$$\begin{aligned}\dot{\psi}_d &= \omega_0(v_{td} + r_a i_d + \omega \psi_q) \\ \dot{\psi}_q &= \omega_0(v_{tq} + r_a i_q - \omega \psi_d) \\ \dot{\psi}_f &= \omega_0(v_f - r_f i_f) \\ \dot{\psi}_D &= -\omega_0 r_D i_D \\ \dot{\psi}_Q &= -\omega_0 r_Q i_Q\end{aligned}\tag{2.1}$$

where $\psi_d, \psi_q, \psi_f, \psi_D, \psi_Q, i_d, i_q, i_f, i_D, i_Q, v_{td}, v_{tq}, v_f$, and r_a, r_f, r_D, r_Q are the magnetic flux linkage, current, voltage, and resistance of each corresponding winding, respectively, ω_0 is the synchronous speed, and ω is the rotor speed in per unit (p.u.)

of synchronous generator. Since there is no static coupling between any d-axis winding and q-axis winding, the relationship between the magnetic flux linkage and the current of those windings is as follows:

$$\begin{aligned} \begin{bmatrix} \psi_d \\ \psi_f \\ \psi_D \end{bmatrix} &= \begin{bmatrix} x_d & x_{ad} & x_{ad} \\ x_{ad} & x_f & x_{ad} \\ x_{ad} & x_{ad} & x_D \end{bmatrix} \begin{bmatrix} -i_d \\ i_f \\ i_D \end{bmatrix} \\ \begin{bmatrix} \psi_q \\ \psi_Q \end{bmatrix} &= \begin{bmatrix} x_q & x_{aq} \\ x_{aq} & x_Q \end{bmatrix} \begin{bmatrix} -i_q \\ i_Q \end{bmatrix} \end{aligned} \quad (2.2)$$

where x_d, x_q, x_f, x_D, x_Q are the self-reactance of winding d, q, f, D, and Q, respectively. In Eq. (2.2), it is assumed that the mutual reactance of all windings on a common axis is same, being x_{ad} or x_{aq} respectively.

The rotor motion equation of synchronous generator is as follows:

$$\begin{aligned} \dot{\delta} &= \omega_0(\omega - 1) \\ \dot{\omega} &= \frac{1}{M} [T_m - T_t - D(\omega - 1)] \end{aligned} \quad (2.3)$$

where M is the inertia of the rotor, D the damping coefficient of the rotor motion, δ the rotor angular position of synchronous generator to a reference axis, and T_m and T_t the mechanical torque and electric torque applied on the rotor of generator, respectively.

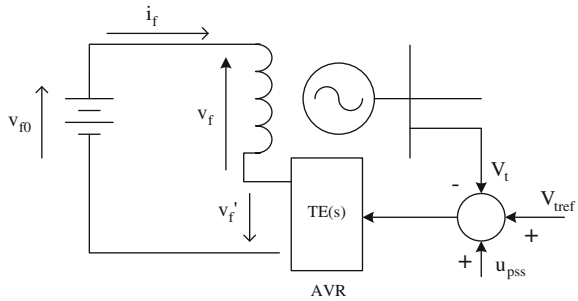
2.1.1.2 Excitation System and the Automatic Voltage Regulator (AVR)

Voltage control in a power system is closely related to the regulation of reactive power flow. The objectives of voltage and reactive power control in the power system are as follows:

1. To maintain the voltage at terminals of all equipment in the power system within acceptable limits. As far as the system voltage profile is concerned, a constraint of system voltage deviations is imposed normally to be within $\pm 5\%$ of the nominal value of voltage.
2. To improve system stability in order to maximize the utilization of the transmission system.
3. To minimize reactive power flow so as to reduce transmission losses.

Power system voltage control has a hierarchy structure with multiple levels. At the primary level, control devices attempt to compensate the rapid and random voltage variations by maintaining their output variables close to the setting reference values. The highest level of voltage control uses global information of power

Fig. 2.1 Arrangement of an AVR



system and is implemented by solving some optimization problems. A synchronous generator can generate or absorb reactive power depending on its excitation, which is controlled by a voltage control device, the automatic voltage regulator (AVR). Figure 2.1 shows simple illustration on the arrangement of the AVR on the synchronous generator.

In Fig. 2.1, $TE(s)$ denotes the transfer function of the AVR. The excitation provided to the field winding of synchronous generator is v_f which is from two sources. One is a constant excitation, v_{f0} , and another is the output from the AVR, v'_f . The AVR measures the terminal voltage of synchronous generator, V_t , and compares it to a reference setting value V_{tref} . The AVR responds to the deviation of terminal voltage to change the excitation of generator and hence regulates the reactive power supply or absorption by generator. By doing so, the terminal voltage of generator is effectively maintained close to the reference setting value. Therefore, the AVR works at the bottom of the hierarchy of voltage control of power system to eliminate voltage variations at generator's terminal. On the other hand, the reference setting value of the AVR is provided by the result of voltage and/or reactive control optimization of the whole power system such that the voltage profile of the system is kept within required constraints and the transmission losses are minimized. Hence, V_{tref} is from the higher level of hierarchy of voltage control of the power system.

Historically, the role of excitation system to improve power system performance has been growing. Early excitation systems were for the maintenance of constant voltage level at generator's terminals and were implemented manually. Gradually, fast-acting AVRs were installed in many generators not only providing a satisfactory voltage control performance, but also considerably improving power system steady-state and transient stability. However, during 1960s to 1970s, it was found that fast-acting AVRs have an adverse effect in providing negative damping to power system oscillations in some occasions. This results in a conflict requirement in the design of the AVRs. Subsequently, the power system stabilizer (PSS) was introduced into excitation control to overcome the problem. Nowadays, it is quite common to have a combination of a fast-acting AVR and a PSS for the excitation control of synchronous generators. Stabilizing signal, u_{pss} , is superimposed on that of the AVR as shown in Fig. 2.1.

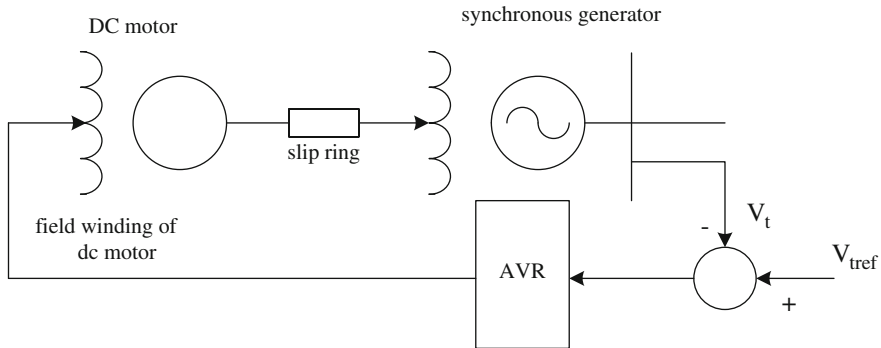


Fig. 2.2 Arrangement of a DC excitation system

Based on the difference of the excitation power sources used, excitation systems can be classified into three major types.

1. DC excitation systems

A DC excitation system uses a DC generator as the source of excitation power to provide field current and is connected to field winding through slip rings. The exciter may be driven by a motor or by the generator itself. The DC excitation system represents the early application of excitation control. Figure 2.2 shows the arrangement of the DC excitation system.

2. AC excitation systems

An AC excitation system uses an AC machine (alternator) as the source of excitation power to provide field current. The AC excitation current is rectified to provide the DC excitation to the synchronous generator. Usually, the AC exciter is on the same shaft of synchronous generator. There are two major types of AC excitation systems, depending on the difference of the arrangement of AC excitation from either the stationary or rotating armature winding of the exciter, as shown in Figs. 2.3 and 2.4, respectively. In a rotating rectifier excitation system, the armature windings of the AC exciter and the diode rectifiers rotate with the synchronous generator field. Thus, the need for slip rings and brushes is eliminated. Hence, such a system is also called a brushless excitation system.

3. Static excitation systems

In a static excitation system, power supply is from the synchronous generator. The DC excitation is provided to the field of generator through slip rings after being rectified. There are three major types of static excitation systems. They are potential-source controlled-rectifier systems, compound-source rectifier systems, and compound-controlled rectified excitation systems. Figure 2.5 shows the arrangement of a potential-source controlled-rectifier excitation system.

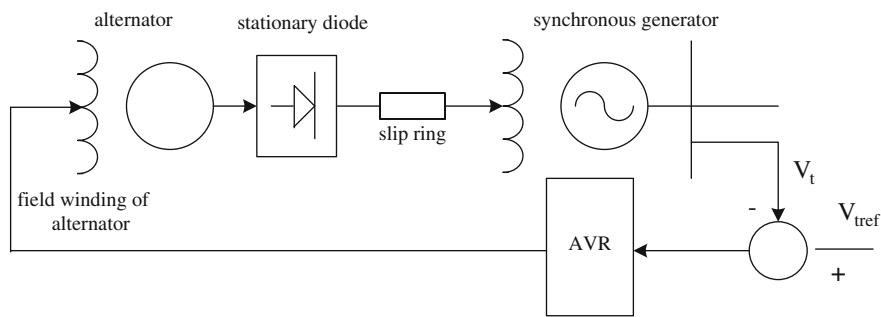


Fig. 2.3 Arrangement of a stationary rectifier excitation system

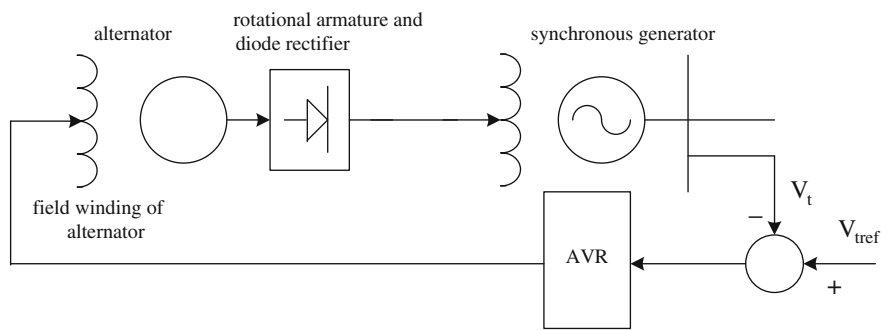


Fig. 2.4 Arrangement of a rotating rectifier excitation system

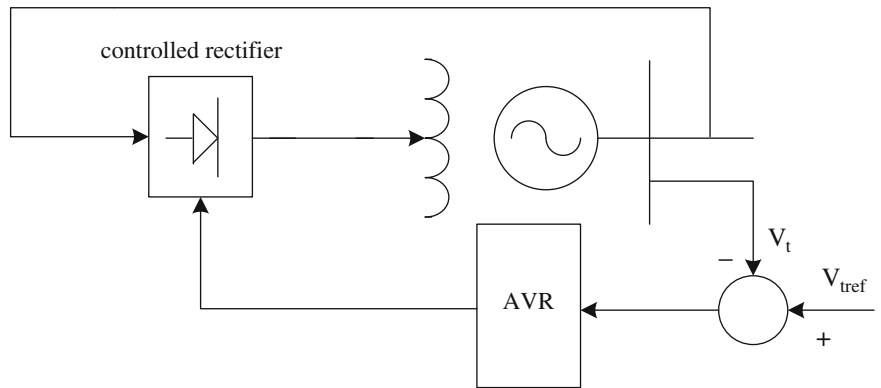


Fig. 2.5 Arrangement of a potential-source controlled-rectifier excitation system

From Fig. 2.1, it can have

$$v_f = v_{f0} + TE(s)(V_{tref} - V_t + u_{pss}) \quad (2.4)$$

where v_{f0} is the constant excitation, u_{pss} the stabilizing signal of the PSS, and V_t and V_{tref} the terminal voltage of generator and its reference setting value, respectively. Various forms of transfer function have been recommended for different types of excitation systems and the AVR. In this book, the following simplest form is used for the purpose of simple presentation,

$$TE(s) = \frac{K_A}{1 + sT_A} \quad (2.5)$$

where K_A is the gain and T_A the time constant of the AVR. Hence, from Eqs. (2.4) and (2.5), mathematical model of the AVR can be written as

$$\begin{aligned} v_f &= v_{f0} + v'_f \\ \dot{v}'_f &= -\frac{1}{T_A} v'_f + \frac{K_A}{T_A} (V_{tref} - V_t + u_{pss}) \end{aligned} \quad (2.6)$$

2.1.1.3 A Single-Machine Infinite-Bus Power System

Figure 2.6 shows the configuration of a power system where a generator sends power to a large network. Capacity of the large network is much greater than that of the generator such that operation of the large network is not affected at all by any changes in the part of the power system on the left-hand side of busbar b in Fig. 2.6. This effectively means that the voltage and frequency at busbar b are constant when the focus of the study is the part of the left-hand side of the power system. Thus, from the point of view of operation of the part of left-hand side of the power system, capacity of the large network is “infinite”. Hence, busbar b is called the “infinite busbar”, and the part of the power system on the left-hand side of busbar b is a “single-machine infinite-bus” power system. The single-machine infinite-bus power system is an approximate representation of a kind of real power systems, where a power plant with a generator or a group of generators are connected by transmission lines to a very large power network.

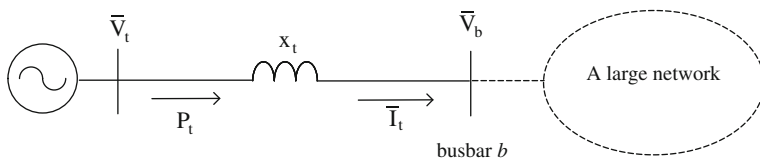
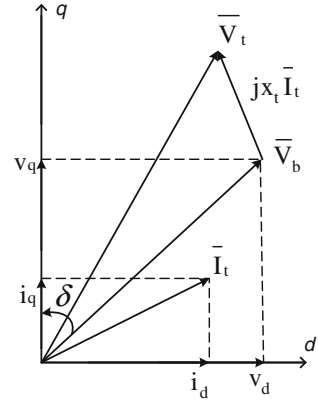


Fig. 2.6 A single-machine infinite-bus power system

Fig. 2.7 d–q coordinate of generator



For the single-machine infinite-bus power system shown in Fig. 2.6, it can have

$$\bar{V}_t = jx_t \bar{I}_t + \bar{V}_b \quad (2.7)$$

In the d–q coordinate of the generator shown in Fig. 2.7, Eq. (2.7) can be written as follows:

$$v_{td} + jv_{tq} = jx_t(i_d + ji_q) + v_d + jv_q \quad (2.8)$$

where v_{td} , v_{tq} and i_d , i_q , v_d , v_q are the d and q components of terminal voltage of generator, $\bar{V}_t$, line current, $\bar{I}_t$, and voltage at the infinite busbar, $\bar{V}_b$, respectively. Comparing the real and imaginary part on the both sides of Eq. (2.8), it can have

$$\begin{aligned} v_{td} &= -x_t i_q + v_d \\ v_{tq} &= x_t i_d + v_q \end{aligned} \quad (2.9)$$

where $v_d = V_b \sin \delta$, $v_q = V_b \cos \delta$ and the terminal voltage of generator is as follows:

$$V_t = \sqrt{v_{td}^2 + v_{tq}^2} \quad (2.10)$$

In per unit, the mechanical and electric torque, T_m and T_t , in Eq. (2.3), is equal to the mechanical power input from the prime mover to the electric power supplied by the generator, respectively, i.e. $T_m = P_m$ and $T_t = P_t$. While P_t is equal to the electric power received at the infinite busbar, that is

$$P_t = v_{td} i_d + v_{tq} i_q = v_d i_d + v_q i_q \quad (2.11)$$

Equations (2.1)–(2.3), (2.6) and (2.9)–(2.11) are the complete dynamic model of the single-machine infinite-bus power system shown in Fig. 2.6 where V_b and P_m are constant.

2.1.1.4 Linearized Model of Single-Machine Infinite-Bus Power System

Linearization of Eqs. (2.1) and (2.2) is as follows:

$$\begin{aligned}
 \Delta \dot{\psi}_d &= \omega_0 (\Delta v_{td} + r_a \Delta i_d + \omega_0 \Delta \psi_q + \psi_{q0} \Delta \omega) \\
 \Delta \dot{\psi}_q &= \omega_0 (\Delta v_{tq} + r_a \Delta i_q - \omega_0 \Delta \psi_d - \psi_{d0} \Delta \omega) \\
 \Delta \dot{\psi}_f &= \omega_0 (\Delta v_f - r_f \Delta i_f) \\
 \Delta \dot{\psi}_D &= -\omega_0 r_D \Delta i_D \\
 \Delta \dot{\psi}_Q &= -\omega_0 r_Q \Delta i_Q
 \end{aligned} \tag{2.12}$$

$$\begin{aligned}
 \begin{bmatrix} \Delta i_d \\ \Delta i_f \\ \Delta i_D \end{bmatrix} &= \begin{bmatrix} x_d & x_{ad} & x_{ad} \\ x_{ad} & x_f & x_{ad} \\ x_{ad} & x_{ad} & x_D \end{bmatrix}^{-1} \begin{bmatrix} \Delta \psi_d \\ \Delta \psi_f \\ \Delta \psi_D \end{bmatrix} \\
 \begin{bmatrix} \Delta i_q \\ \Delta i_Q \end{bmatrix} &= \begin{bmatrix} x_q & x_{aq} \\ x_{aq} & x_Q \end{bmatrix}^{-1} \begin{bmatrix} \Delta \psi_q \\ \Delta \psi_Q \end{bmatrix}
 \end{aligned} \tag{2.13}$$

where prefix, Δ , and subscript 0 are used to denote small increment of a variable (linearized variable) and value of the variable at the power system steady-state operating condition where the linearization is carried out, respectively. This notation will be used throughout this book.

Linearization of Eqs. (2.10) and (2.11) is as follows:

$$\Delta V_t = \frac{v_{td0}}{V_{t0}} \Delta v_{td} + \frac{v_{tq0}}{V_{t0}} \Delta v_{tq} \tag{2.14}$$

$$\Delta P_t = v_{dt0} \Delta i_d + v_{qt0} \Delta i_q + i_{d0} \Delta v_{dt} + i_{q0} \Delta v_{qt} \tag{2.15}$$

By using Eqs. (2.14) and (2.15), linearization of Eqs. (2.3) and (2.6) can be obtained to be

$$\begin{aligned}
 \Delta \dot{\delta} &= \omega_0 \Delta \omega \\
 \Delta \dot{\omega} &= -\frac{1}{M} (\Delta P_t + D \Delta \omega) \\
 &= -\frac{1}{M} (v_{dt0} \Delta i_d + v_{qt0} \Delta i_q + i_{d0} \Delta v_{dt} + i_{q0} \Delta v_{qt} + D \Delta \omega)
 \end{aligned} \tag{2.16}$$

$$\begin{aligned}\Delta v_f &= \Delta v'_f \\ \Delta \dot{v}'_f &= -\frac{1}{T_A} \Delta v'_f + \frac{K_A}{T_A} \left(-\frac{v_{td0}}{V_{t0}} \Delta v_{td} - \frac{v_{tq0}}{V_{t0}} \Delta v_{tq} + \Delta u_{pss} \right)\end{aligned}\quad (2.17)$$

Arranging Eqs. (2.12), (2.13), (2.16), and (2.17) in matrix form with all linearized current variables be cancelled, it can have

$$\begin{aligned}s\Delta \mathbf{X}_{g-dq} &= \mathbf{A}_{g-dq} \Delta \mathbf{X}_{g-dq} + \mathbf{B}_{g-dq} \Delta \mathbf{V}_{dq} + \mathbf{b}_{pss} \Delta u_{pss} \\ \Delta \mathbf{I}_{dq} &= \mathbf{C}_{g-dq} \Delta \mathbf{X}_{g-dq}\end{aligned}\quad (2.18)$$

where

$$\begin{aligned}\Delta \mathbf{X}_{g-dq} &= [\Delta \delta \quad \Delta \omega \quad \Delta v_f \quad \Delta \psi_d \quad \Delta \psi_q \quad \Delta \psi_f \quad \Delta \psi_D \quad \Delta \psi_Q]^T, \\ \Delta \mathbf{V}_{dq} &= [\Delta v_{td} \quad \Delta v_{tq}]^T, \Delta \mathbf{I}_{dq} = [\Delta i_d \quad \Delta i_q]^T\end{aligned}$$

For the single-machine infinite-bus power system, the network voltage equation is Eq. (2.9). Its linearization is as follows:

$$\begin{aligned}\Delta v_{td} &= -x_t \Delta i_q + \Delta v_d = -x_t \Delta i_q + V_b \cos \delta_0 \Delta \delta \\ \Delta v_{tq} &= x_t \Delta i_d + \Delta v_q = x_t \Delta i_d - V_b \sin \delta_0 \Delta \delta\end{aligned}\quad (2.19)$$

In matrix form, the above equation can be written as follows:

$$\Delta \mathbf{V}_{dq} = \mathbf{F}_{dq1} \Delta \mathbf{I}_{dq} + \mathbf{F}_{dq2} \Delta \mathbf{X}_{g-dq}\quad (2.20)$$

where

$$\mathbf{F}_{dq1} = \begin{bmatrix} 0 & -x_t \\ x_t & 0 \end{bmatrix}, \mathbf{F}_{dq2} = \begin{bmatrix} V_b \cos \delta_0 & \mathbf{0} \\ -V_b \sin \delta_0 & \mathbf{0} \end{bmatrix}$$

Substituting Eq. (2.20) into (2.18), state-equation model of the single-machine infinite-bus power system is obtained to be

$$s\Delta \mathbf{X}_{g-dq} = \mathbf{A}_{gc-dq} \Delta \mathbf{X}_{g-dq} + \mathbf{b}_{pss} \Delta u_{pss}\quad (2.21)$$

where $\mathbf{A}_{gc-dq} = \mathbf{A}_{g-dq} + \mathbf{B}_{g-dq} \mathbf{F}_{dq1} \mathbf{C}_{g-dq} + \mathbf{B}_{g-dq} \mathbf{F}_{dq2}$.

2.1.2 Heffron–Phillips Model

2.1.2.1 Simplification

For the study of power system oscillations, full mathematical model of synchronous generator of Eqs. (2.1)–(2.2) can be simplified based on the following considerations:

1. Effect of damper windings is not considered or directly included in the damping coefficient D in the rotor motion equation in Eq. (2.3). Thus, Eq. (2.1) is simplified to be

$$\begin{aligned}\dot{\psi}_d &= \omega_0(v_{td} + r_a i_d + \omega \psi_q) \\ \dot{\psi}_q &= \omega_0(v_{tq} + r_a i_q - \omega \psi_d) \\ \dot{\psi}_f &= \omega_0(v_f - r_f i_f)\end{aligned}\tag{2.22}$$

2. Effect of fast transient and the resistance of d and q armature windings are neglected. Equation (2.22) is further simplified to be

$$\begin{aligned}0 &= v_{td} + \omega \psi_q \\ 0 &= v_{tq} - \omega \psi_d \\ \dot{\psi}_f &= \omega_0(v_f - r_f i_f)\end{aligned}\tag{2.23}$$

3. In small-signal power oscillations, variation of rotor speed is very small, $\omega \approx 1$. Hence, the first two equations in Eq. (2.23) become

$$\begin{aligned}v_{td} &= -\psi_q \\ v_{tq} &= \psi_d\end{aligned}\tag{2.24}$$

To transform the third equation in Eq. (2.23) into a different form, it is defined that

$$E'_q = \frac{x_{ad}}{x_f} \psi_f, \quad E_q = x_{ad} i_f, \quad E_{fd} = \frac{x_{ad} v_f}{r_f}\tag{2.25}$$

where E'_q is called the q-axis transient excitation voltage, E_q the q-axis excitation voltage, and E_{fd} the excitation voltage. Multiplying both sides of the third equation in Eq. (2.23) by $\frac{x_{ad}}{r_f}$, it can have

$$T'_{d0} \dot{E}'_q = E_{fd} - E_q \quad (2.26)$$

where $T'_{d0} = \frac{X_f}{\omega_0 I_f}$, which is the time constant of the field winding.

Equation (2.26) together with Eq. (2.3) forms the simplified third-order model of synchronous generator. Equation (2.2) becomes

$$\begin{aligned} \begin{bmatrix} \psi_d \\ \psi_f \end{bmatrix} &= \begin{bmatrix} x_d & x_{ad} \\ x_{ad} & x_f \end{bmatrix} \begin{bmatrix} -i_d \\ i_f \end{bmatrix} \\ \psi_q &= -x_q i_q \end{aligned} \quad (2.27)$$

From Eqs. (2.24), (2.25), and (2.27), it can have

$$\begin{aligned} v_{td} &= -\psi_q = x_q i_q \\ v_{tq} &= \psi_d = x_{ad} i_f - x_d i_d = E_q - x_d i_d \end{aligned} \quad (2.28)$$

From Eqs. (2.25) and (2.27), it can be obtained that

$$E'_q = \frac{x_{ad}}{x_f} \psi_f = \frac{x_{ad}}{x_f} (x_f i_f - x_{ad} i_d) = E_q - \frac{x_{ad}^2}{x_f} i_d = E_q - (x_d - x'_d) i_d \quad (2.29)$$

where $x'_d = x_d - \frac{x_{ad}^2}{x_f}$, which is called the transient d-axis reactance. Thus, Eq. (2.26) becomes

$$T'_{d0} \dot{E}'_q = E_{fd} - E'_q - (x_d - x'_d) i_d \quad (2.30)$$

2.1.2.2 A Simplified Model of Single-Machine Infinite-Bus Power System

For the single-machine infinite-bus power system shown in Fig. 2.6, from Eqs. (2.9) and (2.28), it can have

$$\begin{aligned} v_{td} &= v_d - x_t i_q = x_q i_q \\ v_{tq} &= v_q + x_t i_d = E_q - x_d i_d = E'_q - x'_d i_d \end{aligned} \quad (2.31)$$

Thus,

$$\begin{aligned} v_d &= (x_t + x_q) i_q = x_{q\Sigma} i_q \\ v_q &= E_q - (x_d + x_t) i_d = E'_q - (x'_d + x_t) i_d = E_q - x_{d\Sigma} i_d = E'_q - x'_{d\Sigma} i_d \end{aligned} \quad (2.32)$$

Fig. 2.8 Equivalent circuit model of single-machine infinite-bus power system

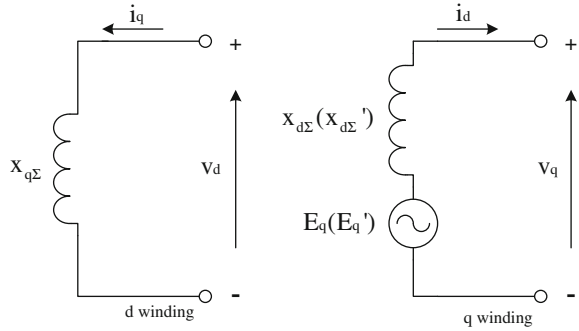
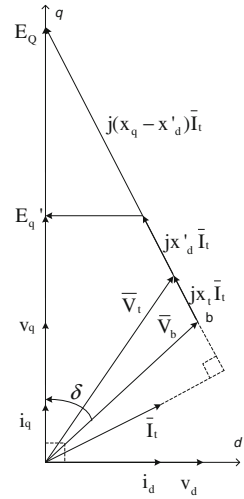


Fig. 2.9 Phasor diagram of single-machine infinite-bus power system



The single-machine infinite-bus power system can be represented by a circuit model of Fig. 2.8. Figure 2.9 shows the phasor diagram of the system on the d-q coordinate.

From Eq. (2.32) or Fig. 2.8, it can have

$$\begin{aligned} i_d &= \frac{E_q' - V_b \cos \delta}{X_{d\Sigma}'} \\ i_q &= \frac{V_b \sin \delta}{X_{q\Sigma}} \end{aligned} \quad (2.33)$$

By substituting Eqs. (2.9) and (2.33) into Eq. (2.11), the electric power supplied by the generator can be expressed as follows:

$$\begin{aligned}
P_t &= V_b \cos \delta \frac{V_b \sin \delta}{x_{q\Sigma}} + V_b \sin \delta \frac{E'_q - V_b \cos \delta}{x'_{d\Sigma}} \\
&= \frac{E'_q V_b}{x'_{d\Sigma}} \sin \delta - \frac{V_b^2 (x_q - x'_d)}{2 x'_{d\Sigma} x_{q\Sigma}} \sin 2\delta
\end{aligned} \tag{2.34}$$

From Eqs. (2.29) and (2.33), it can be obtained that

$$\begin{aligned}
E_q &= E'_q + (x_d - x'_d) i_d = E'_q + (x_d - x'_d) \frac{E'_q - V_b \cos \delta}{x'_{d\Sigma}} \\
&= \frac{E'_q x_{d\Sigma}}{x'_{d\Sigma}} - \frac{(x_d - x'_d) V_b \cos \delta}{x'_{d\Sigma}}
\end{aligned} \tag{2.35}$$

where $x_{d\Sigma} = x_d + x_t$. From Eqs. (2.31) and (2.33), it can have

$$\begin{aligned}
v_{td} &= V_b \sin \delta - x_t i_q = V_b \sin \delta - x_t \frac{V_b \sin \delta}{x_{q\Sigma}} = \frac{x_q V_b \sin \delta}{x_{q\Sigma}}, \\
v_{tq} &= V_b \cos \delta + x_t i_d = V_b \cos \delta + x_t \frac{E'_q - V_b \cos \delta}{x'_{d\Sigma}} = \frac{x_t E'_q}{x'_{d\Sigma}} + \frac{V_b x'_d \cos \delta}{x'_{d\Sigma}}
\end{aligned} \tag{2.36}$$

Hence, the simplified model of single-machine infinite-bus power system is as follows:

$$\begin{aligned}
\dot{\delta} &= \omega_0 (\omega - 1) \\
\dot{\omega} &= \frac{1}{M} [P_m - P_t - D(\omega - 1)] \\
\dot{E}'_q &= \frac{1}{T'_{do}} (-E_q + E_{fd}) \\
\dot{E}'_{fd} &= -\frac{1}{T_A} E'_{fd} + \frac{K_A}{T_A} (V_{tref} - V_t + u_{pss})
\end{aligned} \tag{2.37}$$

where

$$\begin{aligned}
P_t &= \frac{E'_q V_b}{x'_{d\Sigma}} \sin \delta - \frac{V_b^2 (x_q - x'_d)}{2 x'_{d\Sigma} x_{q\Sigma}} \sin 2\delta \\
E_q &= \frac{E'_q x_{d\Sigma}}{x'_{d\Sigma}} - \frac{(x_d - x'_d) V_b \cos \delta}{x'_{d\Sigma}} \\
E_{fd} &= E_{fd0} + E'_{fd} \\
v_{td} &= \frac{x_q V_b \sin \delta}{x_{q\Sigma}}, \quad v_{tq} = \frac{x_t E'_q}{x'_{d\Sigma}} + \frac{V_b x'_d \cos \delta}{x'_{d\Sigma}}, \quad V_t = \sqrt{v_{td}^2 + v_{tq}^2}
\end{aligned} \tag{2.38}$$

The model is a group of 4 first-order differential equations plus 6 algebraic equations.

2.1.2.3 Heffron–Phillips Model [1–3]

By linearizing Eqs. (2.37) and (2.38) at an operating point of power system, where $V_t = V_{t0}$, $V_{td} = V_{td0}$, $V_{tq} = V_{tq0}$, $\delta = \delta_0$, $\omega_0 = 1$, $E'_q = E'_{q0}$, $E_{fd} = E_{fd0}$, it can have

$$\begin{aligned}\Delta\dot{\delta} &= \omega_0 \Delta\omega \\ \Delta\dot{\omega} &= \frac{1}{M} (-\Delta P_t - D\Delta\omega) \\ \Delta\dot{E}'_q &= \frac{1}{T'_{do}} (-\Delta E_q + \Delta E'_{fd}) \\ \Delta\dot{E}'_{fd} &= -\frac{1}{T_A} \Delta E'_{fd} - \frac{K_A}{T_A} (\Delta V_t - \Delta u_{pss})\end{aligned}\tag{2.39}$$

$$\begin{aligned}\Delta P_t &= K_1 \Delta\delta + K_2 \Delta E'_q \\ \Delta E_q &= K_3 \Delta E'_q + K_4 \Delta\delta \\ \Delta V_t &= K_5 \Delta\delta + K_6 \Delta E'_q\end{aligned}\tag{2.40}$$

where

$$\begin{aligned}K_1 &= \frac{E'_{q0} V_b X'_{d\Sigma}}{\cos \delta_0} \delta_0 - \frac{V_b^2 (X_q - X'_d)}{X'_{d\Sigma} X_{q\Sigma}} \cos 2\delta_0 \\ K_2 &= \frac{V_b}{X'_{d\Sigma}} \sin \delta_0 \\ K_3 &= \frac{X_{d\Sigma}}{X'_{d\Sigma}} \\ K_4 &= \frac{(X_d - X'_d) V_b \sin \delta_0}{X'_{d\Sigma}} \\ K_5 &= \frac{V_{td0} X_q V_b \cos \delta_0}{V_{t0} X_{q\Sigma}} - \frac{V_{tq0} V_{b0} X'_q \sin \delta_0}{V_{t0} X'_{d\Sigma}} \\ K_6 &= \frac{V_{tq0} X_t}{V_{t0} X'_{d\Sigma}}\end{aligned}$$

Substituting Eq. (2.40) into Eq. (2.39), it can be obtained that

$$\begin{aligned}\Delta\dot{\delta} &= \omega_0 \Delta\omega \\ \Delta\dot{\omega} &= \frac{1}{M} (-K_1 \Delta\delta - K_2 \Delta E'_q - D\Delta\omega) \\ \Delta\dot{E}'_q &= \frac{1}{T'_{do}} (-K_3 \Delta E'_q - K_4 \Delta\delta + \Delta E'_{fd}) \\ \Delta\dot{E}'_{fd} &= -\frac{1}{T_A} \Delta E'_{fd} - \frac{K_A}{T_A} (K_5 \Delta\delta + K_6 \Delta E'_q - \Delta u_{pss})\end{aligned}\tag{2.41}$$

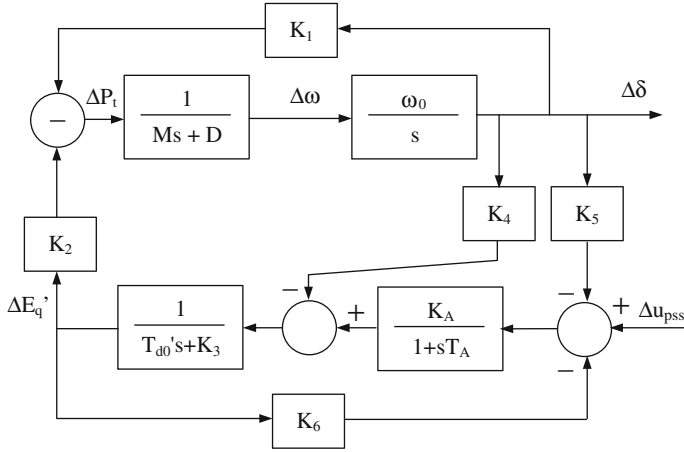


Fig. 2.10 Heffron–Phillips model of a single-machine infinite-bus power system

Equation (2.41) is the so-called Heffron–Phillips model of single-machine infinite-bus power system, which is shown in Fig. 2.10.

The Heffron–Phillips model can be written in the form of state-space representation of Eq. (2.21) where

$$\Delta \mathbf{X}_{g-dq} = \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix}, \mathbf{A}_{gc-dq} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ -\frac{K_1}{M} & -\frac{D}{M} & -\frac{K_2}{M} & 0 \\ -\frac{K_4}{T'_{do}} & 0 & -\frac{K_3}{T'_{do}} & \frac{1}{T'_{do}} \\ -\frac{K_A K_5}{T_A} & 0 & -\frac{K_A K_6}{T_A} & -\frac{1}{T_A} \end{bmatrix}, \quad (2.42)$$

$$\mathbf{b}_{pss} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_A}{T_A} \end{bmatrix}$$

2.2 Modal Analysis

2.2.1 Basis of Modal Analysis Theory

2.2.1.1 Modal Decomposition

State-space representation of a linear system is as follows:

$$\begin{aligned} s\mathbf{X} &= \mathbf{A}_0\mathbf{X} + \mathbf{b}_0u \\ y &= \mathbf{c}_0^T\mathbf{X} \\ u &= H(s)y \end{aligned} \quad (2.43)$$

where $\mathbf{A}_0$, $\mathbf{b}_0$ and $\mathbf{c}_0^T$ is the state matrix, control vector, and output vector of open-loop system, respectively, and $H(s)$ is the transfer function of feedback controller. Transfer function of open-loop system is as follow:

$$G(s) = \frac{y}{u} = \mathbf{c}_0^T(s\mathbf{I} - \mathbf{A}_0)^{-1}\mathbf{b}_0 \quad (2.44)$$

The system is shown in Fig. 2.11. Transfer function of closed-loop system is as follows:

$$T(s) = \frac{y}{w} = \frac{G(s)}{1 - G(s)H(s)} \quad (2.45)$$

Eigen solution is one of the basic techniques in the modal analysis, involving the computation of eigenvalues and eigenvectors of state matrix, $\mathbf{A}_0$. An eigenvalue of matrix $\mathbf{A}_0$, λ , is a scalar parameter, which satisfies the following equation

$$\mathbf{A}_0\mathbf{v} = \lambda\mathbf{v} \quad (2.46)$$

with a non-trivial solution ($\mathbf{v} \neq \mathbf{0}$).

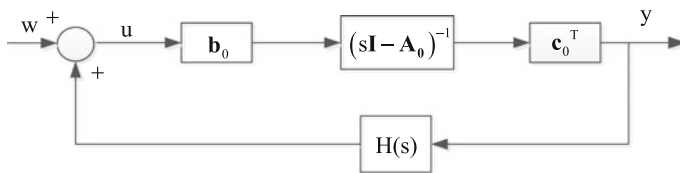


Fig. 2.11 Block diagram of a closed-loop control system

Obviously, Eq. (2.46) can be written in the following form

$$(\mathbf{A}_0 - \lambda \mathbf{I})\mathbf{v} = 0 \quad (2.47)$$

where $\mathbf{I}$ is an unity matrix. In order for Eq. (2.47) to have the non-trivial solution, it should have

$$|\mathbf{A}_0 - \lambda \mathbf{I}| = 0 \quad (2.48)$$

Equation (2.48) is the following polynomial equation if $\mathbf{A}_0$ is an $M \times M$ matrix

$$(-1)^M \lambda^M + a_{M-1} \lambda^{M-1} + \dots + a_1 \lambda + a_0 = 0 \quad (2.49)$$

which is called the characteristic equation of state matrix $\mathbf{A}_0$. The characteristic equation should have M solutions; that is, matrix $\mathbf{A}_0$ has M eigenvalues, if the dimension of matrix is M .

For the i th eigenvalue of matrix $\mathbf{A}_0$, λ_i , if a nonzero vector $\mathbf{v}_i$ satisfies the equation

$$\mathbf{A} \mathbf{v}_i = \lambda_i \mathbf{v}_i, \quad i = 1, 2, \dots, M \quad (2.50)$$

$\mathbf{v}_i$ is called the right eigenvector of matrix $\mathbf{A}$ associated with λ_i . Equation (2.50) can be arranged as follows:

$$\mathbf{A}[\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_M] = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_M] \begin{bmatrix} \lambda_1 & 0 & \mathbf{0} & 0 \\ 0 & \lambda_2 & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ 0 & 0 & \mathbf{0} & \lambda_M \end{bmatrix} \quad (2.51)$$

That is

$$\mathbf{V}^{-1} \mathbf{A} \mathbf{V} = \mathbf{\Lambda} \quad (2.52)$$

where

$$\mathbf{V} = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_M], \quad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \mathbf{0} & 0 \\ 0 & \lambda_2 & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ 0 & 0 & \mathbf{0} & \lambda_M \end{bmatrix}$$

Denote

$$\mathbf{V}^{-1} = \begin{bmatrix} \mathbf{w}_1^T \\ \mathbf{w}_2^T \\ \vdots \\ \mathbf{w}_M^T \end{bmatrix} = [\mathbf{w}_1 \quad \mathbf{w}_2 \quad \dots \quad \mathbf{w}_M]^T = \mathbf{W}^T \quad (2.53)$$

From Eqs. (2.52) and (2.53), obviously it can have

$$\mathbf{w}_i^T \mathbf{A} = \mathbf{w}_i^T \lambda_i, \quad i = 1, 2, \dots, M \quad (2.54)$$

Hence, $\mathbf{w}_i^T$ is called the left eigenvector corresponding to eigenvalue λ_i .
If a new state variable vector $\mathbf{z}$ is introduced and defined to be

$$\mathbf{X} = \mathbf{VZ} \quad (2.55)$$

From Eqs. (2.43), (2.52), and (2.53), it can be obtained that

$$\begin{aligned} s\mathbf{Z} &= \mathbf{A}\mathbf{Z} + \mathbf{W}^T \mathbf{b}_0 u \\ y &= \mathbf{c}_0^T \mathbf{VZ} \end{aligned} \quad (2.56)$$

That is

$$\begin{aligned} sz_i &= \lambda_i z_i + \mathbf{w}_i^T \mathbf{b}_0 u \quad i = 1, 2, \dots, M \\ y &= \mathbf{c}_0^T \sum_{i=1}^M \mathbf{v}_i z_i \end{aligned} \quad (2.57)$$

According to Eq. (2.57), the system can also be shown in Fig. 2.12. This is the modal decomposition representation of state-space model of open-loop system.

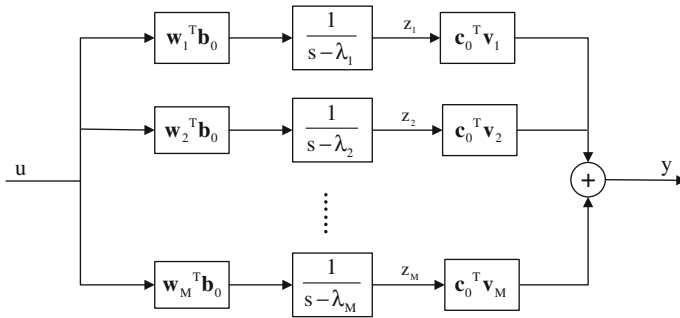


Fig. 2.12 Block diagram of modal decomposition representation of state-space model of open-loop system

2.2.1.2 Stability of Open-Loop System and Closed-Loop System

Considering the open-loop system when $u = 0$, the state-space representation of Eq. (2.57) is as follows:

$$s z_i = \lambda_i z_i \quad (2.58)$$

Solution of Eq. (2.58) is as follows:

$$z_i(t) = z_i(0)e^{\lambda_i t}, \quad i = 1, 2, \dots, M \quad (2.59)$$

where $z_i(0)$ is the initial value of state variable $z_i(t)$, $i = 1, 2, \dots, M$. From Eqs. (2.55) and (2.59), it can have

$$\mathbf{X} = \mathbf{V} \begin{bmatrix} z_1(0)e^{\lambda_1 t} \\ z_2(0)e^{\lambda_2 t} \\ \vdots \\ z_n(0)e^{\lambda_M t} \end{bmatrix} \quad (2.60)$$

Hence, time response of the k th state variable of the system, $x_k(t)$, $i = 1, 2, \dots, M$, is as follows:

$$x_k(t) = v_{k1}z_1(0)e^{\lambda_1 t} + v_{k2}z_2(0)e^{\lambda_2 t} + \dots + v_{kM}z_M(0)e^{\lambda_M t} = \sum_{i=1}^M v_{ki}z_i(0)e^{\lambda_i t} \quad (2.61)$$

Obviously, the time response of system state variables is decided by the eigenvalues of state matrix $\mathbf{A}_0$. If there is one or more eigenvalues on the right-hand half of the complex plane (the real part of eigenvalue is equal to or greater than zero), the system is unstable. If all the eigenvalues of $\mathbf{A}_0$ are on the left-hand side of the complex plane, the system is stable. Hence, eigenvalues of $\mathbf{A}_0$ determine the system stability. They often are called the modes of the system. If a pair of eigenvalues are conjugate complex number, i.e. $\tilde{\lambda}_{i,i+1} = \xi_i \pm j\omega_i$, the corresponding component in the time response of the k th state variable of the system should be

$$\bar{v}_{ki}z_i(0)e^{\tilde{\lambda}_i t} = \bar{v}_{ki}z_i(0)e^{(\xi_i + j\omega_i)t} = \bar{v}_{ki}z_i(0)e^{\xi_i t}[\cos \omega_i t + j \sin \omega_i t] \quad (2.62)$$

The component is oscillatory in respect of time. The oscillation angular frequency is ω_i . The decaying and increasing of the oscillation are determined by the

real part of the mode ξ_i . The pair of conjugate eigenvalues of state matrix $\mathbf{A}_o$ are often called the oscillation mode of the system.

The oscillation frequency f_i (Hz) and damping ζ_i associated with $\bar{\lambda}_{i,i+1} = \xi_i \pm j\omega_i$ are normally defined as follows:

$$f_i = \frac{\omega_i}{2\pi}, \quad \zeta_i = -\frac{\xi_i}{\sqrt{\xi_i^2 + \omega_i^2}} \quad (2.63)$$

From Eq. (2.59), it can be seen that $z_i(t)$, $i = 1, 2, \dots, M$ is related only with the i th mode of the system λ_i . Hence, $z_i(t)$, $i = 1, 2, \dots, M$ often is seen as the i th mode of the system. Equation (2.57) is often called the modal decomposition of state-space representation.

From Eq. (2.61), it can also be seen that the magnitude of v_{ki} measures how much the i th mode λ_i contributes to the k th state variable $x_k(t)$. Thus, $|v_{ki}|$ is a kind of measurement of the “observability” of the i th mode in the k th state variable.

On the basis of above discussion, from Fig. 2.12, it can be seen that $\mathbf{w}_i^T \mathbf{b}_o$ is the weight on how much the control signal u affects the i th mode of the open-loop system, the so-called controllability index, whereas $\mathbf{c}_o^T \mathbf{v}_i$ is the weight on how much the i th mode is observed in the system output, which is called the observability index. The product of controllability and observability index is called the residue. That is

$$R_i = \mathbf{w}_i^T \mathbf{b}_o \mathbf{c}_o^T \mathbf{v}_i \quad (2.64)$$

From Eqs. (2.53) and (2.55), it can have

$$\mathbf{Z} = \mathbf{V}^{-1} \mathbf{X} = \mathbf{W}^T \mathbf{X} \quad (2.65)$$

or

$$z_i(t) = w_{1i}x_1(t) + w_{2i}x_2(t) + \dots + w_{Mi}x_M(t) \quad (2.66)$$

w_{ki} is the i th row k th column element of matrix $\mathbf{W}$. Equation (2.66) indicates that the magnitude of w_{ki} measures the influence of the k th state variable $x_k(t)$ on the i th state variable $z_i(t)$, or the i th mode λ_i of the system. It is a kind of measurement of “controllability” of the k th state variable on the i th mode.

Let the realization of the transfer function of feedback controller $H(s)$ be

$$\begin{aligned} s\mathbf{X}_f &= \mathbf{A}_f \mathbf{X}_f + \mathbf{b}_f y \\ u &= \mathbf{c}_f^T \mathbf{X}_f \end{aligned} \quad (2.67)$$

That is $H(s) = \mathbf{c}_f^T(s\mathbf{I} - \mathbf{A}_f)^{-1}\mathbf{b}_f$. Thus, from Eqs. (2.43) and (2.67), the state-space representation of closed-loop system can be obtained to be

$$\begin{bmatrix} s\mathbf{X} \\ s\mathbf{X}_f \end{bmatrix} = \begin{bmatrix} \mathbf{A}_o & \mathbf{b}_o\mathbf{c}_f^T \\ \mathbf{b}_f\mathbf{c}_o^T & \mathbf{A}_f \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{X}_f \end{bmatrix} = \mathbf{A}_c \begin{bmatrix} \mathbf{X} \\ \mathbf{X}_f \end{bmatrix} \quad (2.68)$$

where $\mathbf{A}_c$ is the state matrix of closed-loop system. Obviously, based on the discussion above, eigenvalues of $\mathbf{A}_c$ or modes of closed-loop system determine the stability of closed-loop system.

From Eq. (2.68), it can be obtained that

$$\mathbf{A}_c = \mathbf{A}_o + H(s)\mathbf{b}_o\mathbf{c}_o^T \quad (2.69)$$

Denote a variable parameter of feedback controller as α . Thus, state matrix is a function of the parameter. Influence of the parameter on the i th mode of closed-loop system can be calculated by use of the following equation

$$\frac{\partial \lambda_i}{\partial \alpha} = \mathbf{w}_i^T \frac{\partial \mathbf{A}_c(\alpha)}{\partial \alpha} \mathbf{v}_i = \frac{\partial H(\lambda_i, \alpha)}{\partial \alpha} \mathbf{w}_i^T \mathbf{b}_o \mathbf{c}_o^T \mathbf{v}_i = R_i \frac{\partial H(\lambda_i, \alpha)}{\partial \alpha} \quad (2.70)$$

Hence, the residue measures how much the mode of closed-loop system is affected by the parameter of the controller.

2.2.2 Applications of Modal Analysis

2.2.2.1 Modal Analysis for the AVR

Consider the simple case of a single-machine infinite-bus power system expressed by the Heffron–Phillips model shown in Fig. 2.10 without the PSS installed ($\Delta u_{\text{pss}} = 0$). The upper part can be considered as the open-loop system and lower part the feedback controller. Thus, Eq. (2.41) can be rearranged as follows:

$$\begin{aligned} \begin{bmatrix} s\Delta\delta \\ s\Delta\omega \end{bmatrix} &= \begin{bmatrix} 0 & \omega_o \\ -\frac{K_t}{M} & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{K_2}{M} \end{bmatrix} \Delta E'_q \\ \Delta\delta &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} \\ \Delta E'_q &= F_{\text{delta}}(s)\Delta\delta \end{aligned} \quad (2.71)$$

In the above state-space model of the power system, the input to the open-loop system is $\Delta E'_q$, the output is $\Delta\delta$, and the transfer function of feedback controller is $F_{\text{delta}}(s)$. Obviously, the state-space realization of $F_{\text{delta}}(s)$ is as follows:

$$\begin{aligned} \begin{bmatrix} s\Delta E'_q \\ s\Delta E'_{fd} \end{bmatrix} &= \begin{bmatrix} -\frac{K_3}{T_{do}} & \frac{1}{T_{do}} \\ -\frac{K_A K_6}{T_A} & -\frac{1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} + \begin{bmatrix} -\frac{K_4}{T_{do}} \\ -\frac{K_A K_5}{T_A} \end{bmatrix} \Delta\delta \\ \Delta E'_q &= [1 \quad 0] \begin{bmatrix} \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} \end{aligned} \quad (2.72)$$

According to Eq. (2.48), the modes of open-loop system can be found by solving the following characteristic equation

$$\left| \begin{bmatrix} 0 & \omega_o \\ -\frac{K_1}{M} & -\frac{D}{M} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = \left| \begin{bmatrix} -\lambda & \omega_o \\ -\frac{K_1}{M} & -\frac{D}{M} - \lambda \end{bmatrix} \right| = \lambda^2 + \frac{D}{M}\lambda + \frac{K_1}{M}\omega_o = 0 \quad (2.73)$$

Solution of Eq. (2.73) gives the oscillation mode of the single-machine infinite-bus power system when $\Delta E'_q = 0$, that is the case when the generator is modelled as a constant voltage source. The oscillation mode is as follows:

$$\bar{\lambda}_{1,2} = \frac{1}{2} \left[-\frac{D}{M} \pm \sqrt{\left(\frac{D}{M}\right)^2 - \frac{4\omega_o K_1}{M}} \right] = \xi_o \pm j\omega_{\text{NOF}} \quad (2.74)$$

The oscillation mode is related to the rotor motion of generator, i.e. state variables $\Delta\delta$ and $\Delta\omega$. It is often called the electromechanical oscillation mode of the power system.

From Eq. (2.71), it can have

$$\begin{bmatrix} s\Delta\delta \\ s\Delta\omega \end{bmatrix} = \begin{bmatrix} 0 & \omega_o \\ -\frac{K_1}{M} - \frac{K_2}{M}F_{\text{delta}}(s) & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} = \mathbf{A}_c \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} \quad (2.75)$$

where $\mathbf{A}_c$ is the state matrix of closed-loop system.

From Eq. (2.72), it can be obtained that

$$\begin{aligned}
 F_{\text{delta}}(s) &= [1 \quad 0] \begin{bmatrix} s + \frac{K_3}{T'_{\text{do}}} & -\frac{1}{T'_{\text{do}}} \\ \frac{K_A K_6}{T_A} & s + \frac{1}{T_A} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{K_4}{T'_{\text{do}}} \\ -\frac{K_A K_5}{T_A} \end{bmatrix} \\
 &= \frac{1}{\left(s + \frac{K_3}{T'_{\text{do}}}\right)\left(s + \frac{1}{T_A}\right) + \frac{K_A K_6}{T_A} \frac{1}{T'_{\text{do}}}} [1 \quad 0] \begin{bmatrix} s + \frac{1}{T_A} & \frac{1}{T'_{\text{do}}} \\ -\frac{K_A K_6}{T_A} & s + \frac{K_3}{T'_{\text{do}}} \end{bmatrix} \begin{bmatrix} -\frac{K_4}{T'_{\text{do}}} \\ -\frac{K_A K_5}{T_A} \end{bmatrix} \\
 &= \frac{T'_{\text{do}} T_A}{(s T'_{\text{do}} + K_3)(T_A s + 1) + K_A K_6} \begin{bmatrix} s + \frac{1}{T_A} & \frac{1}{T'_{\text{do}}} \\ -\frac{K_A K_6}{T_A} & s + \frac{K_3}{T'_{\text{do}}} \end{bmatrix} \begin{bmatrix} -\frac{K_4}{T'_{\text{do}}} \\ -\frac{K_A K_5}{T_A} \end{bmatrix} \\
 &= -\frac{(T_A s + 1)K_4 + K_A K_5}{(s T'_{\text{do}} + K_3)(T_A s + 1) + K_A K_6}
 \end{aligned} \tag{2.76}$$

Hence, according to Eq. (2.70), it can have

$$\frac{\partial \bar{\lambda}_i}{\partial \alpha} = -R_i \frac{\partial \frac{(T_A \bar{\lambda}_i + 1)K_4 + K_A K_5}{(\bar{\lambda}_i T'_{\text{do}} + K_3)(T_A \bar{\lambda}_i + 1) + K_A K_6}}{\partial \alpha} \tag{2.77}$$

where $\alpha = K_A$ or T_A . By using Eq. (2.77), effect of the AVR on the damping of electromechanical oscillation, i.e. power oscillation, can be examined.

2.2.2.2 Modal Analysis for the PSS

The general linearized model of the single-machine infinite-bus power system with the PSS installed is Eq. (2.21) which can be rearranged as follows:

$$\begin{aligned}
 \begin{bmatrix} s\Delta\delta \\ s\Delta\omega \\ s\Delta\mathbf{x}_{3-8} \end{bmatrix} &= \begin{bmatrix} 0 & \omega_0 & \mathbf{0} \\ \frac{-a_{21}}{M} & \frac{-a_{22}}{M} & \frac{-a_{23-8}^T}{M} \\ \mathbf{a}_{13-8} & \mathbf{a}_{23-8} & \mathbf{A}_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta\mathbf{x}_{3-8} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathbf{b}_{\text{pss}-3} \end{bmatrix} \Delta u_{\text{pss}} \\
 y = \Delta\omega &= [0 \quad 1 \quad \mathbf{0}] \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta\mathbf{x}_{3-8} \end{bmatrix} \\
 \Delta u_{\text{pss}} &= T_{\text{pss}}(s)\Delta\omega
 \end{aligned} \tag{2.78}$$

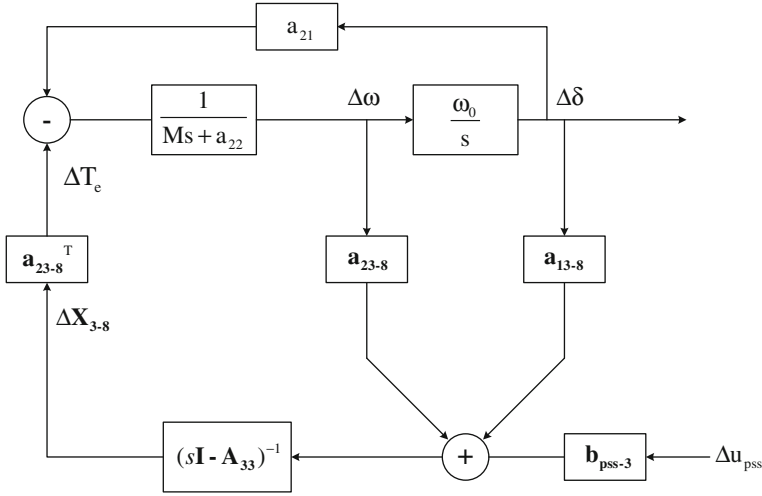


Fig. 2.13 General linearized model of single-machine infinite-bus power system with PSS installed

The model is shown in Fig. 2.13.

Let the oscillation mode of the system without the PSS installed be $\bar{\lambda}_i$ and the corresponding left and right eigenvector be

$$\mathbf{w}_i^T = [\mathbf{w}_{i1} \quad \mathbf{w}_{i2} \quad \mathbf{w}_{i3}^T], \mathbf{v}_i = \begin{bmatrix} \mathbf{v}_{i1} \\ \mathbf{v}_{i2} \\ \mathbf{v}_{i3} \end{bmatrix} \quad (2.79)$$

That is

$$[\mathbf{w}_{i1} \quad \mathbf{w}_{i2} \quad \mathbf{w}_{i3}^T] \begin{bmatrix} 0 & \omega_0 & \mathbf{0} \\ \frac{-\mathbf{a}_{21}}{M} & \frac{-\mathbf{a}_{22}}{M} & \frac{-\mathbf{a}_{23-8}^T}{M} \\ \mathbf{a}_{13-8} & \mathbf{a}_{23-8} & \mathbf{A}_{33} \end{bmatrix} = \bar{\lambda}_i [\mathbf{w}_{i1} \quad \mathbf{w}_{i2} \quad \mathbf{w}_{i3}^T] \quad (2.80)$$

It can have

$$\begin{aligned} -\bar{\mathbf{w}}_{i2} \frac{\mathbf{a}_{21}}{M} + \bar{\mathbf{w}}_{i3}^T \mathbf{a}_{13-8} &= \bar{\lambda}_i \bar{\mathbf{w}}_{i1} \\ \omega_0 \bar{\mathbf{w}}_{i1} - \bar{\mathbf{w}}_{i2} \frac{\mathbf{a}_{22}}{M} + \bar{\mathbf{w}}_{i3}^T \mathbf{a}_{23-8} &= \bar{\lambda}_i \bar{\mathbf{w}}_{i2} \\ -\bar{\mathbf{w}}_{i2} \frac{\mathbf{a}_{23-8}^T}{M} + \bar{\mathbf{w}}_{i3}^T \mathbf{A}_{33} &= \bar{\lambda}_i \bar{\mathbf{w}}_{i3}^T \end{aligned} \quad (2.81)$$

Hence,

$$\bar{\mathbf{w}}_{i3}^T = -\bar{w}_{i2} \frac{\mathbf{a}_{23-8}^T}{M} (\bar{\lambda}_i \mathbf{I} - \mathbf{A}_{33})^{-1} \quad (2.82)$$

From Eqs. (2.64), (2.78), and (2.82), the residue can be obtained to be

$$\begin{aligned} \mathbf{R}_i &= \begin{bmatrix} w_{i1} & w_{i2} & \bar{\mathbf{w}}_{i3}^T \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \mathbf{b}_{pss-3} \end{bmatrix} \begin{bmatrix} 0 & 1 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \bar{v}_{i1} \\ \bar{v}_{i2} \\ \bar{\mathbf{v}}_{i3} \end{bmatrix} \\ &= \mathbf{w}_{i3}^T \mathbf{b}_{pss-3} \bar{v}_{i2} \\ &= -\bar{w}_{i2} \bar{v}_{i2} \frac{\mathbf{a}_{23-8}^T}{M} (\bar{\lambda}_i \mathbf{I} - \mathbf{A}_{33})^{-1} \mathbf{b}_{pss-3} \end{aligned} \quad (2.83)$$

2.2.2.3 Design of PSS by Pole Assignment

If the feedback signal and transfer function of the PSS to be designed is y and $T_{pss}(s)$ respectively, Eq. (2.78) can be written more generally as follows:

$$\begin{aligned} s\mathbf{X} &= \mathbf{A}\mathbf{X} + \mathbf{b}\Delta u_{pss} \\ y &= \mathbf{c}^T \mathbf{X} \\ \Delta u_{pss} &= T_{pss}(s)y \end{aligned} \quad (2.84)$$

Transfer function of open-loop system is as follows:

$$G(s) = \mathbf{c}^T (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{b} \quad (2.85)$$

Characteristic equation of closed-loop control system is as follows:

$$1 + G(s)T_{pss}(s) = 0 \quad (2.86)$$

If design of the PSS is to assign the electromechanical oscillation mode of the single-machine infinite-bus power system to a target position, $\bar{\lambda}_c = -\xi_c \pm j\omega_c$, $\bar{\lambda}_c$ must be the solution of the characteristic equation of closed-loop control system of Eq. (2.86). Hence, it should have

$$1 + G(\bar{\lambda}_c)T_{pss}(\bar{\lambda}_c) = 0 \quad (2.87)$$

By separating the real and imaginary part of the above equation, two equations will be obtained which can be used to determine two parameters of the transfer function of PSS. If the transfer function of PSS adopts the following format of a lead-lag block,

$$T_{\text{pss}}(s) = K_{\text{pss}} \frac{(1 + s\alpha T)^2}{(1 + sT)^2} \quad (2.88)$$

With a predetermined T , parameters of the PSS, K_{pss} and α , can be set according to Eq. (2.87), thus completing the design of PSS via the pole assignment.

2.3 Damping Torque Analysis

2.3.1 Damping Torque and Synchronizing Torque

2.3.1.1 Damping Torque and Synchronizing Torque Derived from Heffron–Phillips Model

The damping torque analysis (DTA) was firstly introduced on the basis of the Heffron–Phillips model for a single-machine infinite-bus power system to examine the effect of excitation control, such as the AVR, on power system small-signal stability [1–3]. It was developed based on the understanding that the dynamic of the electromechanical oscillation loop of a generator decides the damping of power oscillations in the single-machine infinite-bus power system.

In the Heffron–Phillips model shown in Fig. 2.10, the upper part obviously is the linearized rotor motion equation and lower part is formed from the mathematical description of dynamic of the field winding of generator and the AVR. Figure 2.14 shows the upper part of the model which is called the electromechanical oscillation loop. Signal ΔT_e from the lower part in the Heffron–Phillips model is obviously an electric torque. Hence, from Fig. 2.14, it can have

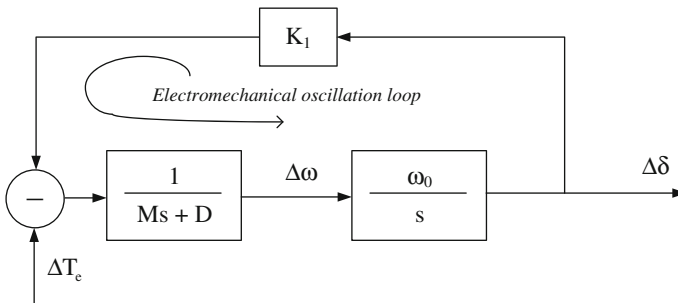


Fig. 2.14 Electromechanical oscillation loop of generator

$$s^2 \Delta\delta + \frac{D}{M} s \Delta\delta + \frac{\omega_0 K_1}{M} \Delta\delta + \frac{\omega_0}{M} \Delta T_e = 0 \quad (2.89)$$

If firstly the contribution from the lower part of Heffron–Phillips model, ΔT_e , is not considered, the electromechanical oscillation loop of generator shown in Fig. 2.14 is described by the following second-order differential equation

$$s^2 \Delta\delta + \frac{D}{M} s \Delta\delta + \frac{\omega_0 K_1}{M} \Delta\delta = 0 \quad (2.90)$$

Equation (2.90) in fact is the linearized model of the single-machine infinite-bus power system when the dynamic of exciter and the AVR are not considered. This is the case when the generator is modelled only by the rotor motion equation in Eq. (2.3).

Solution of Eq. (2.90) is as follows:

$$\Delta\delta(t) = a_0 e^{-\frac{D}{2M}t} \cos \omega_{\text{NOF}} t + b_0 \quad (2.91)$$

where a_0 and b_0 are two constants and $\omega_{\text{NOF}} = \frac{1}{2} \sqrt{\left(\frac{D}{M}\right)^2 - \frac{4\omega_0 K_1}{M}}$.

Equation (2.91) describes the behaviour of rotor motion, i.e. the acceleration and deceleration to store or release electric power. Hence, it determines the variations of active power supplied by the generator during dynamic transient (electromechanical transient), when the power system is subject to small disturbances. If $\frac{D}{2M}$ is small or negative, a poorly damped or magnitude-increasing power oscillations occur. This is the electromechanical oscillation associated with the rotor motion of synchronous generator, i.e. the power system low-frequency oscillation.

In Eq. (2.91), ω_{NOF} is called the angular frequency of natural oscillation. The angular oscillation frequency, ω_s , of power oscillation in the single-machine infinite-bus power system is normally very close to the angular frequency of natural oscillation. Equation (2.91) indicates that the damping of power oscillation of the single-machine infinite-bus power system is determined by the coefficient of the first-order derivative in the second-order differential equation in Eq. (2.91) $\frac{D}{M}$.

At the angular oscillation frequency ω_s , the electric torque contributed from the lower part of Heffron–Phillips model can be decomposed into two components

$$\Delta T_e = T_d \Delta\omega + T_s \Delta\delta \quad (2.92)$$

Equation (2.89) becomes

$$s^2\Delta\delta + \left(\frac{D}{M} + \frac{T_d}{M\omega_0}\right)s\Delta\delta + \left(\frac{\omega_0 K_1}{M} + \frac{\omega_0 T_s}{M}\right)\Delta\delta = 0 \quad (2.93)$$

Obviously, from the discussion on Eq. (2.91), it is easy to understand that the component in the decomposition of ΔT_e , $T_d\Delta\omega$, contributes to the damping of power oscillation. This component is called the damping torque. In Eq. (2.92), $T_s\Delta\delta$ is called the synchronizing torque.

2.3.1.2 Electric Torque Contributed from the PSS

From Fig. 2.10, it can be seen that the electric torque contributed from the lower part of Heffron–Phillips model is as follows:

$$\Delta T_{et} = F_{\delta}(s)\Delta\delta + F_{pss}(s)\Delta u_{pss} \quad (2.94)$$

where $F_{\delta}(s)$ and $F_{pss}(s)$ are the transfer function from $\Delta\delta$ and Δu_{pss} respectively, to form the electric torque contribution to the electromechanical oscillation loop of generator.

The electric torque contribution from the PSS is as follows:

$$\Delta T_{pss} = F_{pss}(s)\Delta u_{pss} \quad (2.95)$$

Figure 2.15 shows that the PSS contributes the electric torque, ΔT_{pss} , to the electromechanical oscillation loop of generator. Obviously, $F_{pss}(s)$ is the transfer function of forward path from the stabilizing signal of the PSS to the

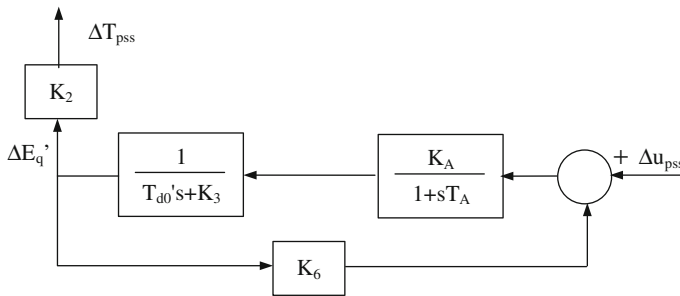


Fig. 2.15 Forward path of stabilizing signal of the PSS

electromechanical oscillation loop of generator in the Heffron–Phillips model. From Fig. 2.15, it can be obtained that

$$F_{\text{pss}}(s) = K_2 \frac{\frac{1}{K_3 + sT'_{d0}} \frac{K_A}{1 + sT_A}}{1 + K_6 \frac{1}{K_3 + sT'_{d0}} \frac{K_A}{1 + sT_A}} = K_2 \frac{K_A}{(K_3 + sT'_{d0})(1 + sT_A) + K_6 K_A} \quad (2.96)$$

Hence, at the angular oscillation frequency, ω_s , the electric torque provided by the PSS to the electromechanical oscillation loop is as follows:

$$\Delta T_{\text{pss}} = \bar{F}_{\text{pss}}(j\omega_s) \Delta u_{\text{pss}}$$

For example, if the PSS is a pure-gain controller and takes the deviation of rotor speed of generator as the feedback signal, i.e. $\Delta u_{\text{pss}} = K_{\text{pss}} \Delta \omega$, the electric torque contributed from the PSS to the electromechanical oscillation loop of generator is as follows:

$$\Delta T_{\text{pss}} = K_{\text{pss}} \bar{F}_{\text{pss}}(j\omega_s) \Delta \omega = K_{\text{pss}} \text{Re}[\bar{F}_{\text{pss}}(j\omega_s)] \Delta \omega + j K_{\text{pss}} \text{Im}[\bar{F}_{\text{pss}}(j\omega_s)] \Delta \omega \quad (2.97)$$

where $\text{Re}[\bar{F}_{\text{pss}}(j\omega_s)]$ and $\text{Im}[\bar{F}_{\text{pss}}(j\omega_s)]$ denote the real and imaginary part of $\bar{F}_{\text{pss}}(j\omega_s)$, respectively (this notation will be used throughout the book). From the first equation in Eq. (2.41), it can have $s\Delta\delta = \omega_0 \Delta\omega$, i.e.

$$\Delta\omega = \frac{j\omega_s}{\omega_0} \Delta\delta \quad (2.98)$$

By substituting Eqs. (2.98) into Eq. (2.97), it can be obtained that

$$\Delta T_{\text{pss}} = K_{\text{pss}} \text{Re}[\bar{F}_{\text{pss}}(j\omega_s)] \Delta\omega - \frac{\omega_s}{\omega_0} K_{\text{pss}} \text{Im}[\bar{F}_{\text{pss}}(j\omega_s)] \Delta\delta \quad (2.99)$$

Hence, the damping torque supplied by the PSS is $K_{\text{pss}} \text{Re}[\bar{F}_{\text{pss}}(j\omega_s)] \Delta\omega$, and the synchronizing torque is $-\frac{\omega_s}{\omega_0} K_{\text{pss}} \text{Im}[\bar{F}_{\text{pss}}(j\omega_s)] \Delta\delta$.

2.3.1.3 Damping Torque and Synchronizing Torque Derived from the General Linearized Model

In Eq. (2.78) or Fig. 2.13, denote $\Delta T_e = -\mathbf{a}_{23-8}^T \Delta \mathbf{x}_{3-8}$. From Eq. (2.78) or Fig. 2.13, it can have

$$s^2 \Delta\delta + \frac{a_{22}}{M} s \Delta\delta + \omega_0 \frac{a_{21}}{M} \Delta\delta + \Delta T_e = 0 \quad (2.100)$$

Taking the similar procedure of DTA presented above in Sects. 2.3.1.1 and 2.3.1.2, transfer function of the forward path of stabilizing signal of the PSS can be obtained to be

$$\mathbf{F}_{\text{pss}}(s) = -\mathbf{a}_{23-8}^T (s\mathbf{I} - \mathbf{A}_{33})^{-1} \mathbf{b}_{\text{pss}-3} \quad (2.101)$$

At the angular oscillation frequency, ω_s , the electric torque contribution from the PSS is as follows:

$$\Delta T_{\text{pss}} = -\mathbf{a}_{23-8}^T (j\omega_s \mathbf{I} - \mathbf{A}_{33})^{-1} \mathbf{b}_{\text{pss}-3} \Delta u_{\text{pss}} \quad (2.102)$$

The electric torque can be decomposed into the damping and synchronizing torque. The damping torque contribution from the PSS determines its effect on the damping of power oscillation.

Assume that the installation of PSS brings about a change of damping coefficient $D_{\text{pss}} \Delta \omega$ in the electromechanical oscillation loop of generator. The state-space representation of power system with the PSS installed can be equivalently written as follows:

$$\begin{bmatrix} s\Delta\delta \\ s\Delta\omega \\ \Delta\mathbf{x}_{3-8} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 \\ -\frac{\mathbf{a}_{21}}{M} & -\frac{\mathbf{a}_{22} + D_{\text{pss}}}{M} & -\frac{\mathbf{a}_{23-8}^T}{M} \\ \mathbf{a}_{13-8} & \mathbf{a}_{23-8} & \mathbf{A}_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta\mathbf{x}_{3-8} \end{bmatrix} \quad (2.103)$$

From Eqs. (2.70) and (2.103), it can have

$$\begin{aligned} \frac{\partial \bar{\lambda}_i}{\partial D_{\text{pss}}} &= \bar{\mathbf{w}}_i^T \frac{\partial \mathbf{A}_c}{\partial D_{\text{pss}}} \bar{\mathbf{v}}_i \\ &= [\bar{w}_{i1} \quad \bar{w}_{i2} \quad \bar{\mathbf{w}}_{i3}^T] \frac{\partial}{\partial D_{\text{pss}}} \begin{bmatrix} 0 & \omega_0 & 0 \\ -\frac{\mathbf{a}_{21}}{M} & -\frac{\mathbf{a}_{22} + D_{\text{pss}}}{M} & -\frac{\mathbf{a}_{23-8}^T}{M} \\ \mathbf{a}_{13-8} & \mathbf{a}_{23-8} & \mathbf{A}_{33} \end{bmatrix} \begin{bmatrix} \bar{v}_{i1} \\ \bar{v}_{i2} \\ \bar{\mathbf{v}}_{i3} \end{bmatrix} \\ &= -\frac{\bar{w}_{i2} \bar{v}_{i2}}{M} \end{aligned} \quad (2.104)$$

From Eqs. (2.83), (2.102), and (2.104), it can be seen that the residue in fact measures the effect of the PSS on the electromechanical oscillation mode of the system. At the complex frequency $\bar{\lambda}_i$, it is equal to the forward path of the PSS multiplied by the sensitivity of the mode to the damping torque contribution.

2.3.2 Damping Torque Analysis and Design of PSS by Phase Compensation

2.3.2.1 Theoretical Basis of the Damping Torque Analysis

This section explains the theoretical basis of the damping torque analysis by use of the Phillips–Heffron model as follows.

Firstly, the effect of PSS is not considered, i.e. $\Delta u_{\text{pss}} = 0$. From Fig. 2.10, it can have

$$\begin{aligned} (Ms^2 + Ds + \omega_0 K_1) \Delta \delta(s) &= -\omega_0 \Delta T(s) \\ \Delta T(s) &= F_{\text{delta}}(s) \Delta \delta(s) \end{aligned} \quad (2.105)$$

where $F_{\text{delta}}(s)$ is the transfer function from $\Delta \delta(s)$ to $\Delta T(s)$. Combining two equations above gives

$$[Ms^2 + Ds + \omega_0 K_1 + \omega_0 F_{\text{delta}}(s)] \Delta \delta(s) = 0 \quad (2.106)$$

Thus, characteristic equation of the system is as follows:

$$Ms^2 + Ds + \omega_0 K_1 + \omega_0 F_{\text{delta}}(s) = 0 \quad (2.107)$$

Solutions of the characteristic equation are the eigenvalues of state matrix of system model given by Eq. (2.42). One of the pair of complex solutions is called the electromechanical oscillation mode. Its real part defines the damping of power oscillation. Denote the mode as $\bar{\lambda}_s = \xi_s + j\omega_s$. In the complex frequency domain, it should have

$$M\bar{\lambda}_s^2 + D\bar{\lambda}_s + \omega_0 K_1 + \omega_0 \bar{F}_{\text{delta}}(\bar{\lambda}_s) = 0 \quad (2.108)$$

The second equation in Eq. (2.105) expressed in the complex frequency domain is as follows:

$$\Delta T(\bar{\lambda}_s) = \bar{F}_{\text{delta}}(\bar{\lambda}_s) \Delta \delta(\bar{\lambda}_s) \quad (2.109)$$

Also in the complex frequency domain, the first equation in Eq. (2.41) becomes

$$\Delta \omega(\bar{\lambda}_s) = \frac{\xi_s + j\omega_s}{\omega_0} \Delta \delta(\bar{\lambda}_s) = \frac{\xi_s}{\omega_0} \Delta \delta(\bar{\lambda}_s) + j \frac{\omega_s}{\omega_0} \Delta \delta(\bar{\lambda}_s) \quad (2.110)$$

Let the electric torque defined by Eq. (2.109) be decomposed as follows:

$$\Delta T(\bar{\lambda}_s) = T_{s1} \Delta \delta(\bar{\lambda}_s) + T_{d1} \Delta \omega(\bar{\lambda}_s) \quad (2.111)$$

From Eqs. (2.109), (2.110), and (2.111), it can be obtained that

$$\bar{F}_{\text{delta}}(\bar{\lambda}_s)\Delta\delta(\bar{\lambda}_s) = T_{s1}\Delta\delta(\bar{\lambda}_s) + T_{d1}\frac{\xi_s}{\omega_0}\Delta\delta(\bar{\lambda}_s) + jT_{d1}\frac{\omega_s}{\omega_0}\Delta\delta(\bar{\lambda}_s) \quad (2.112)$$

That is

$$\bar{F}_{\text{delta}}(\bar{\lambda}_s) = T_{s1} + T_{d1}\frac{\xi_s}{\omega_0} + jT_{d1}\frac{\omega_s}{\omega_0} \quad (2.113)$$

From Eq. (2.113), it can have

$$\begin{cases} T_{d1} = \frac{\omega_0}{\omega_s} \text{Im}[\bar{F}_{\text{delta}}(\bar{\lambda}_s)] \\ T_{s1} = \text{Re}[\bar{F}_{\text{delta}}(\bar{\lambda}_s)] - \frac{T_{d1}\xi_s}{\omega_0} \end{cases} \quad (2.114)$$

The above derivation indicates that in the complex frequency domain, the electric torque can be decomposed into damping and synchronizing torque according to Eq. (2.111). Substituting Eqs. (2.111) into (2.108), it can have

$$\begin{aligned} (M\bar{\lambda}_s^2 + D\bar{\lambda}_s + \omega_0 K_1)\Delta\delta(\bar{\lambda}_s) &= -\omega_0 T_{s1}\Delta\delta(\bar{\lambda}_s) - \omega_0 T_{d1}\Delta\omega(\bar{\lambda}_s) \\ &= -\omega_0 T_{s1}\Delta\delta(\bar{\lambda}_s) - T_{d1}\bar{\lambda}_s\Delta\delta(\bar{\lambda}_s) \end{aligned} \quad (2.115)$$

Thus,

$$M\bar{\lambda}_s^2 + (D + T_{d1})\bar{\lambda}_s + \omega_0 K_1 + \omega_0 T_{s1} = 0 \quad (2.116)$$

Solution of the above equation is as follows:

$$\xi_s = -\frac{D + T_{d1}}{2M} \quad (2.117)$$

Equation (2.117) indicates that the damping torque affects the real part of electromechanical oscillation mode, i.e. the damping of power oscillation.

2.3.2.2 Graphical Explanation of the Damping Torque Analysis

Equations (2.16) and (2.39) include the following linearized rotor motion equation of generator

$$\begin{aligned} \Delta \dot{\delta} &= \omega_0 \Delta\omega \\ \Delta \dot{\omega} &= -\frac{1}{M}(\Delta P_t + D\Delta\omega) \end{aligned} \quad (2.118)$$

Without referring to any linearized model of single-machine infinite-bus power system, assume ΔP_t being comprised of contributions from $\Delta\delta$ and stabilizing signal of the PSS, Δu_{pss} . That is to let

$$\begin{aligned}\Delta P_t &= \Delta P_t(\Delta\delta) + \Delta P_t(\Delta u_{pss}) \\ \Delta P_t(\Delta\delta) &= F_{\text{delta}}(s)\Delta\delta, \Delta P_t(\Delta u_{pss}) = F_{\text{pss}}(s)\Delta u_{pss}\end{aligned}\quad (2.119)$$

At a known angular oscillation frequency ω_s , if ΔP_t is decomposed in the complex frequency domain, then

$$\begin{aligned}\Delta P_t(\Delta\delta) &= \bar{F}_{\text{delta}}(j\omega_s)\Delta\delta = C_{\text{delta}}\Delta\delta + D_{\text{delta}}\Delta\omega, \\ \Delta P_t(\Delta u_{pss}) &= \bar{F}_{\text{pss}}(j\omega_s)\Delta u_{pss} = C_{\text{pss}}\Delta\delta + D_{\text{pss}}\Delta\omega\end{aligned}\quad (2.120)$$

Obviously, $D_{\text{delta}}\Delta\omega$ and $D_{\text{pss}}\Delta\omega$ are the damping torque contributed to the electromechanical oscillation loop of generator from $\Delta\delta$ and Δu_{pss} , respectively. Function of the damping torque component in suppressing the power oscillation can be explained graphically by the use of the linearized equal-area criterion as follows.

Without affecting the conclusions of following discussion, it is assumed that in Eq. (2.120), $D_{\text{delta}} = 0$. When there is no PSS installed ($\Delta P_t(\Delta u_{pss}) = 0$),

$$\Delta P_t = \Delta P_t(\Delta\delta) = C_{\text{delta}}\Delta\delta \quad (2.121)$$

The linearized $\Delta P_t - \Delta\delta$ curve is line a–f as shown in Fig. 2.16. In Fig. 2.16, the operating point of system at steady state is d (P_{t0}, δ_0) and it moves to point a (P_{t1}, δ_1) after the system is subject to a small disturbance. Hence, when the operating point moves down from the initial point a (P_{t1}, δ_1) along line a–f, it will stop at point f ($P_{t2'}, \delta_{2'}$) with area a–d–c being equal to area d–g–f. Obviously, in this case, $|P_{t1} - P_{t0}| = |P_{t2'} - P_{t0}|$, $|\delta_1 - \delta_0| = |\delta_{2'} - \delta_0|$, power oscillation is of unchanged magnitude and not damped at all.

When the PSS is installed to provide a pure positive damping torque,

$$\Delta P_t(\Delta u_{pss}) = D_{\text{pss}}\Delta\omega \quad (\text{assuming } D_{\text{pss}} > 0) \quad (2.122)$$

From Eqs. (2.120), (2.121), and (2.122), it should have

$$\Delta P_t = \Delta P_t(\Delta\delta) + \Delta P_t(\Delta u_{pss}) = C_{\text{delta}}\Delta\delta + D_{\text{pss}}\Delta\omega, \quad (2.123)$$

When the operating point moves down from point a (P_{t1}, δ_1), power angle of generator decreases and thus $\Delta\omega < 0$. $D_{\text{pss}}\Delta\omega < 0$ is added on $C_{\text{delta}}\Delta\delta$ as shown in Eq. (2.123). Hence, the operating point should move below line a–f along curve $\Delta P_t = C_{\text{delta}}\Delta\delta + D_{\text{pss}}\Delta\omega$. When the operating point stops moving, $\Delta\omega = 0$. Thus, it should stop on line a–f at point c (P_{t2}, δ_2) with area A_1 being equal to area A_2 .

$$\Delta u_{\text{PSS}} = T_{\text{PSS}}(s)\Delta\omega \quad (2.124)$$

From Eqs. (2.95) or (2.102) and above equation, it can have

$$\Delta T_{\text{PSS}} = F_{\text{PSS}}(s)T_{\text{PSS}}(s)\Delta\omega \quad (2.125)$$

At the angular oscillation frequency, ω_s , the decomposition of the electric torque contributed by the PSS is as follows:

$$\begin{aligned} \Delta T_{\text{PSS}} &= \bar{F}_{\text{PSS}}(j\omega_s)\bar{T}_{\text{PSS}}(j\omega_s)\Delta\omega \\ &= \text{Re}[\bar{F}_{\text{PSS}}(j\omega_s)\bar{T}_{\text{PSS}}(j\omega_s)]\Delta\omega + j\text{Im}[\bar{F}_{\text{PSS}}(j\omega_s)\bar{T}_{\text{PSS}}(j\omega_s)]\Delta\omega \\ &= \text{Re}[\bar{F}_{\text{PSS}}(j\omega_s)\bar{T}_{\text{PSS}}(j\omega_s)]\Delta\omega - \frac{\omega_s}{\omega_0}\text{Im}[\bar{F}_{\text{PSS}}(j\omega_s)\bar{T}_{\text{PSS}}(j\omega_s)]\Delta\delta \\ &= T_{\text{PSSd}}\Delta\omega + T_{\text{PSSs}}\Delta\delta \end{aligned} \quad (2.126)$$

The damping and synchronizing torque provided by the PSS is $T_{\text{PSSd}}\Delta\omega$ and $T_{\text{PSSs}}\Delta\delta$, respectively. In order to achieve the most efficient design, ideally the PSS should provide only the damping torque, that is,

$$\Delta T_{\text{PSS}} = D_{\text{PSS}}\Delta\omega, \quad D_{\text{PSS}} > 0 \quad (2.127)$$

where D_{PSS} is the coefficient of the damping torque which needs to be provided by the PSS. Hence, from Eqs. (2.126) and (2.127), it can be seen that design of the PSS should satisfy that

$$D_{\text{PSS}} = \bar{F}_{\text{PSS}}(j\omega_s)\bar{T}_{\text{PSS}}(j\omega_s) \quad (2.128)$$

According to Eq. (2.128), design of the PSS should set the phase of the PSS, $\angle\bar{T}_{\text{PSS}}(j\omega_s)$, to be equal to the minus phase of the forward path, $\angle\bar{F}_{\text{PSS}}(j\omega_s)$, that is to design the PSS such that it can compensate the phase lag of the forward path and ensure it to provide a pure positive damping torque. Hence, the method to design the PSS based on Eq. (2.128) is called the phase compensation method.

If it is denoted that

$$\bar{F}_{\text{PSS}}(j\omega_s) = |F_{\text{PSS}}|\angle\phi, \quad \bar{T}_{\text{PSS}}(j\omega_s) = |T_{\text{PSS}}|\angle\gamma \quad (2.129)$$

The phase compensation method requires

$$\begin{aligned} T_{\text{PSSd}} &= |F_{\text{PSS}}T_{\text{PSS}}|\cos(\phi + \gamma) = D_{\text{PSS}} \\ T_{\text{PSSs}} &= |F_{\text{PSS}}T_{\text{PSS}}|\sin(\phi + \gamma) = 0 \end{aligned} \quad (2.130)$$

This can be achieved by setting

$$\gamma = -\phi, \quad |T_{\text{pss}}| = \frac{D_{\text{pss}}}{|F_{\text{pss}}|} \quad (2.131)$$

Often the PSS is constructed as a lead-lag block with its main part of transfer function to be

$$T_{\text{pss}}(s) = K_{\text{pss}} \frac{(1+sT_2)(1+sT_4)}{(1+sT_1)(1+sT_3)} = K_{\text{pss1}} \frac{(1+sT_2)}{(1+sT_1)} K_{\text{pss2}} \frac{(1+sT_4)}{(1+sT_3)} \quad (2.132)$$

where $K_{\text{pss}} = K_{\text{pss1}} K_{\text{pss2}}$. Parameters of the PSS then can be set to satisfy

$$\begin{aligned} K_{\text{pss1}} \frac{(1+j\omega_s T_2)}{(1+j\omega_s T_1)} &= \frac{D_{\text{pss}}}{|F_{\text{pss}}|} \angle -\frac{\phi}{2} \\ K_{\text{pss2}} \frac{(1+j\omega_s T_4)}{(1+j\omega_s T_3)} &= 1.0 \angle -\frac{\phi}{2} \end{aligned} \quad (2.133)$$

for the PSS to provide a positive damping torque $D_{\text{pss}} \Delta\omega$.

With the PSS installed, from Fig. 2.10, it can have

$$\begin{aligned} (Ms^2 + Ds + \omega_0 K_1) \Delta\delta(s) &= -\omega_0 \Delta T_{\text{delta}}(s) - \omega_0 \Delta T_{\text{pss}}(s) \\ \Delta T_{\text{delta}}(s) &= F_{\text{delta}}(s) \Delta\delta(s) \\ \Delta T_{\text{pss}}(s) &= F_{\text{pss}}(s) T_{\text{pss}}(s) \Delta\omega(s) \end{aligned} \quad (2.134)$$

Let $\bar{\lambda}_c = \xi_c + j\omega_c$ be the electromechanical oscillation mode of the closed-loop system with the PSS installed. According to the DTA discussed above in Sect. 2.3.2.1, in the complex frequency domain, it can have

$$\begin{aligned} \Delta T_{\text{delta}}(\bar{\lambda}_c) &= \bar{F}_{\text{delta}}(\bar{\lambda}_c) \Delta\delta(\bar{\lambda}_c) = C_1 \Delta\delta(\bar{\lambda}_c) + D_1 \Delta\omega(\bar{\lambda}_c) \\ \Delta T_{\text{pss}}(\bar{\lambda}_c) &= \bar{F}_{\text{pss}}(\bar{\lambda}_c) \bar{T}_{\text{pss}}(\bar{\lambda}_c) \Delta\omega(\bar{\lambda}_c) = C_{\text{pss}} \Delta\delta(\bar{\lambda}_c) + D_{\text{pss}} \Delta\omega(\bar{\lambda}_c) \end{aligned} \quad (2.135)$$

Taking the similar procedure of discussion in Sect. 2.3.2.1, it can easily prove that $D_{\text{pss}} \Delta\omega(\bar{\lambda}_c)$ affects the damping of the electromechanical oscillation mode. Hence, the PSS can be designed to satisfy

$$F_{\text{pss}}(\bar{\lambda}_c) T_{\text{pss}}(\bar{\lambda}_c) = D_{\text{pss}} \quad (2.136)$$

When the PSS is being designed, if $\bar{\lambda}_c = \xi_c + j\omega_c$ is given, the PSS can be designed by using the phase compensation method defined by Eq. (2.136) to move the electromechanical oscillation mode strictly to the position at $\bar{\lambda}_c = \xi_c + j\omega_c$ in the complex plane. If only the amount of damping torque provision $D_{\text{pss}} \Delta\omega$, $D_{\text{pss}} > 0$ is given, the PSS can also be designed by the phase compensation method introduced above from Eqs. (2.126) to (2.133).

2.4 Examples

2.4.1 Linearized Mathematical Models of an Example Power System

2.4.1.1 Linearized Mathematical Model with Full Model of Generator Used

Parameters (in p.u.) of an example single-machine infinite-bus power system are as follows:

1. Generator:

$$x_d = 1.18, x_q = 0.78, x_{ad} = 1.0, x_{aq} = 0.6, x_D = 1.11, x_Q = 0.73, x_f = 1.13, \\ r_a = 0.005, r_f = 0.00075, r_D = 0.002, r_Q = 0.04, M = 7 \text{ s}, D = 0, T'_{d0} = 5 \text{ s}$$

2. AVR: $K_A = 100, T_A = 0.01 \text{ s}$

3. Transmission line: $x_t = 0.15$

4. Steady-state operating point: $P_{t0} = 0.5, V_{t0} = 1.05, V_{b0} = 1.0$

At the steady-state operating point, the complex power received at the infinite busbar is as follows:

$$\bar{V}_{b0} \left(\frac{\bar{V}_{t0} - \bar{V}_{b0}}{jx_t} \right)^* = P_{t0} + jQ_{b0}$$

where Q_{b0} is the reactive power received at the infinite busbar and * denotes the conjugate of a complex phasor. By choosing $\bar{V}_{b0}$ as the reference phasor, that is, $\bar{V}_{b0} = \bar{V}_{b0}^* = V_{b0}$, from the above equation, it can be obtained that

$$Q_{b0} = \frac{V_{b0}}{x_t} \left[\sqrt{V_{t0}^2 - \left(\frac{x_t P_{t0}}{V_{b0}} \right)^2} - V_{b0} \right] = 0.3155 \text{ p.u.}$$

In the above, calculation is in p.u. Throughout the following calculation in p.u., p.u. is omitted.

The line current is as follows:

$$\bar{I}_{t0} = \frac{P_{t0} - jQ_{b0}}{V_{b0}} = 0.5 - j0.3155$$

Terminal voltage of generator is :

$$\bar{V}_{t0} = jx_t \bar{I}_{t0} + V_{b0} = 1.0473 + j0.075$$

At steady state, $i_{D0} = i_{Q0} = 0$, $\omega_0 = 1$, Eqs. (2.1) and (2.2) become

$$\begin{aligned} 0 &= v_{td0} + r_a i_{d0} + \psi_{q0} \\ 0 &= v_{tq0} + r_a i_{q0} - \psi_{d0} \\ 0 &= v_{f0} - r_f i_{f0} \\ \begin{bmatrix} \psi_{d0} \\ \psi_{f0} \end{bmatrix} &= \begin{bmatrix} x_d & x_{ad} \\ x_{ad} & x_f \end{bmatrix} \begin{bmatrix} -i_{d0} \\ i_{f0} \end{bmatrix} \\ \psi_{q0} &= -x_q i_{q0} \end{aligned}$$

Thus, according to Eq. (2.28)

$$\begin{aligned} v_{td0} &= -r_a i_{d0} - \psi_{q0} = -r_a i_{d0} + x_q i_{q0} \\ v_{tq0} &= -r_a i_{q0} + \psi_{d0} = -r_a i_{q0} + x_{ad} i_{f0} - x_d i_{d0} = -r_a i_{q0} + E_{q0} - x_d i_{d0} \end{aligned}$$

Hence, in d-q coordinate of generator,

$$\begin{aligned} \bar{V}_{t0} &= V_{td0} + jV_{tq0} = -r_a i_{d0} + x_q i_{q0} + j(-r_a i_{q0} + E_{q0} - x_d i_{d0}) \\ &= -r_a (i_{d0} + j i_{q0}) + x_q i_{q0} - j x_q i_{d0} + j(E_{q0} - x_d i_{d0} + x_q i_{d0}) \end{aligned}$$

Let an imaginary electromotive force (EMF) be

$$E_{Q0} = E_{q0} - x_d i_{d0} + x_q i_{d0} = E_{q0} - (x_d - x_q) i_{d0}$$

Thus,

$$\bar{V}_{t0} = -(r_a + jx_q)(i_{d0} + j i_{q0}) + \bar{E}_{Q0}$$

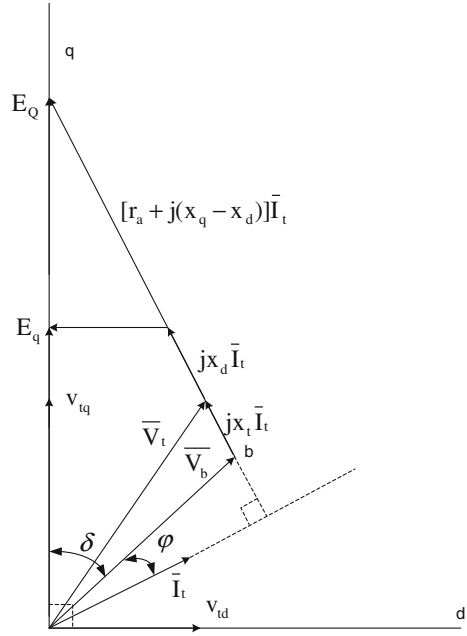
Thus, the q-axis of generator can be found by calculating the imaginary EMF $\bar{E}_Q$

$$\bar{E}_{Q0} = \bar{V}_{t0} + (r_a + jx_q) \bar{I}_{t0} = 1.2959 + j0.4634 = 1.38 \angle 19.68^\circ$$

Hence, $\delta_0 = 19.68^\circ$. From the above equation about E_{q0} and E_{Q0} , it can have

$$\begin{aligned} E_{q0} &= E_{Q0} - (x_q - x_d) i_{d0} = 1.1506 \\ i_{f0} &= \frac{E_{q0}}{x_{ad}} = 1.5624 \end{aligned}$$

Fig. 2.17 Phasor diagram for computing initial values of state variables



Above computation can be shown by the phasor diagram of Fig. 2.17 where subscript 0 is omitted. From Fig. 2.17, it can be seen that

$$i_{d0} = I_{t0} \sin(\delta_0 + \phi_0) = 0.4654, i_{q0} = I_{t0} \cos(\delta_0 + \phi_0) = 0.3646$$

$$v_{d0} = V_{b0} \sin \delta_0 = 0.3368, v_{q0} = V_{b0} \cos \delta_0 = 0.9416$$

Because

$$\bar{V}_{t0} = v_{td0} + jv_{tq0} = jx_t\bar{I}_{t0} + V_{b0} = jx_t(i_{d0} + j\dot{i}_{q0}) + v_{d0} + jv_{q0}$$

thus

$$v_{td0} = v_{d0} - x_t i_{q0} = 0.2820$$

$$v_{tq0} = x_t i_{d0} + v_{q0} = 1.0114$$

From Eq. (2.23), it can have

$$i_{f0} = \frac{E_{q0}}{x_{ad}} = 1.5624$$

Because $i_{D0} = 0, i_{Q0} = 0$, thus according to Eq. (2.2), it can be obtained that

$$\begin{bmatrix} \psi_{d0} \\ \psi_{f0} \\ \psi_{D0} \end{bmatrix} = \begin{bmatrix} x_d & x_{ad} & x_{ad} \\ x_{ad} & x_f & x_{ad} \\ x_{ad} & x_{ad} & x_D \end{bmatrix} \begin{bmatrix} -i_{d0} \\ i_{f0} \\ i_{D0} \end{bmatrix} = \begin{bmatrix} 1.0132 \\ 1.3001 \\ 1.0970 \end{bmatrix}$$

$$\begin{bmatrix} \psi_{q0} \\ \psi_{Q0} \end{bmatrix} = \begin{bmatrix} x_q & x_{aq} \\ x_{aq} & x_Q \end{bmatrix} \begin{bmatrix} -i_{q0} \\ i_{Q0} \end{bmatrix} = \begin{bmatrix} -0.2844 \\ -0.2187 \end{bmatrix}$$

Denote

$$\begin{bmatrix} x_d & x_{ad} & x_{ad} \\ x_{ad} & x_f & x_{ad} \\ x_{ad} & x_{ad} & x_D \end{bmatrix}^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} -4.2331 & 1.8311 & 2.1640 \\ -1.8311 & 5.1570 & -2.9963 \\ -2.1640 & -2.9963 & 5.5498 \end{bmatrix}$$

$$\begin{bmatrix} x_q & x_{aq} \\ x_{aq} & x_Q \end{bmatrix}^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{13} & b_{14} \end{bmatrix} = \begin{bmatrix} -3.6842 & 2.8653 \\ -2.8653 & 3.7249 \end{bmatrix}$$

From Eqs. (2.12) and (2.13), it can have

$$\mathbf{A}_{g-dq} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{D}{M} & 0 & -\frac{a_{11}V_{td0}}{M} & -\frac{b_{11}V_{u0}}{M} & -\frac{a_{12}V_{td0}}{M} & -\frac{a_{13}V_{u0}}{M} & -\frac{b_{12}V_{u0}}{M} \\ 0 & 0 & -\frac{1}{T_A} & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_0\psi_{q0} & 0 & a_{11}\omega_0r_a & \omega_0\omega & a_{12}\omega_0r_a & a_{13}\omega_0r_a & 0 \\ 0 & \omega_0\psi_{d0} & 0 & -\omega_0\omega & b_{11}\omega_0r_a & 0 & 0 & b_{12}\omega_0r_a \\ 0 & 0 & \omega_0 & -a_{21}\omega_0r_f & 0 & -a_{22}\omega_0r_f & -a_{23}\omega_0r_f & 0 \\ 0 & 0 & 0 & -a_{31}\omega_0r_D & 0 & -a_{32}\omega_0r_D & -a_{33}\omega_0r_D & 0 \\ 0 & 0 & 0 & 0 & -b_{21}\omega_0r_Q & 0 & 0 & -b_{22}\omega_0r_Q \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 314.16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1706 & 0.5037 & -0.0738 & -0.0872 & -0.4140 \\ 0 & 0 & -100 & 0 & 0 & 0 & 0 & 0 \\ 0 & -89.3377 & 0 & -6.6494 & 314.16 & 2.8762 & 3.3992 & 0 \\ 0 & 318.3167 & 0 & -314.16 & -5.4760 & 0 & 0 & 4.5008 \\ 0 & 0 & 314.16 & 0.4314 & 0 & -1.2151 & 0.7060 & 0 \\ 0 & 0 & 0 & 1.35797 & 0 & 1.8826 & -3.4871 & 0 \\ 0 & 0 & 0 & 0 & 36.0068 & 0 & 0 & -46.8088 \end{bmatrix}$$

$$\begin{aligned}
\mathbf{B}_{g-dq} &= \begin{bmatrix} 0 & 0 \\ -\frac{\dot{I}_{d0}}{M} & -\frac{\dot{I}_{q0}}{M} \\ -\frac{K_A V_{td0}}{T_A V_{t0}} & -\frac{K_A V_{tq0}}{T_A V_{t0}} \\ \omega_0 & 0 \\ 0 & \omega_0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -0.0665 & -0.0521 \\ -2686.1 & -9632.5 \\ 314.1593 & 0 \\ 0 & 314.1593 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
\mathbf{C}_{g-dq} &= \begin{bmatrix} 0 & 0 & 0 & a_{11} & 0 & a_{12} & a_{13} & 0 \\ 0 & 0 & 0 & 0 & b_{11} & 0 & 0 & b_{12} \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 & -4.2331 & 0 & 1.8331 & 2.1640 & 0 \\ 0 & 0 & 0 & 0 & -3.4862 & 0 & 0 & 2.8653 \end{bmatrix} \\
\mathbf{F}_{dq1} &= \begin{bmatrix} 0 & -x_t \\ x_t & 0 \end{bmatrix} = \begin{bmatrix} 0 & -0.15 \\ 0.15 & 0 \end{bmatrix} \\
\mathbf{F}_{dq2} &= \begin{bmatrix} V_b \cos \delta_0 & \mathbf{0} \\ -V_b \sin \delta_0 & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 0.9416 & \mathbf{0} \\ -0.3367 & \mathbf{0} \end{bmatrix}
\end{aligned}$$

Thus, state matrix and control vector of linearized state-space model are obtained to be

$$\begin{aligned}
\mathbf{A}_{gc-dq} &= \mathbf{A}_{g-dq} + \mathbf{B}_{g-dq} \mathbf{F}_{dq1} \mathbf{C}_{g-dq} + \mathbf{B}_{g-dq} \mathbf{F}_{dq2} \\
&= \begin{bmatrix} 0 & 314.16 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.045 & 0 & 0 & 0.20 & 0.47 & -0.09 & -0.104 & -0.38 \\ 714.3 & 0 & -100 & 6116.3 & -1404.6 & -2646 & -3127 & 1154.5 \\ 295.8 & -89.34 & 0 & -6.64 & 478.4 & 2.88 & 3.39 & -135.03 \\ -105.78 & -318.3 & 0 & -513.64 & -5.48 & 86.29 & 101.9 & 4.50 \\ 0 & 0 & 314.2 & 0.43 & 0 & -1.22 & 0.70 & 0 \\ 0 & 0 & 0 & 1.36 & 0 & 1.88 & -3.48 & 0 \\ 0 & 0 & 0 & 0 & 36.0 & 0 & 0 & -46.8 \end{bmatrix} \\
\mathbf{b}_{pss} &= \begin{bmatrix} 0 \\ 0 \\ \frac{K_A}{T_A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{aligned}$$

Eigenvalues of above state matrix are computed to be

$$\begin{aligned}\bar{\lambda}_1, \bar{\lambda}_2 &= -24.14 \pm j971.56 \\ \bar{\lambda}_3, \bar{\lambda}_4 &= -36.33 \pm j359.63 \\ \lambda_5 &= -35.57 \\ \bar{\lambda}_6, \bar{\lambda}_7 &= -0.71 \pm j8.44 \\ \lambda_8 &= -5.71\end{aligned}$$

According to Eq. (2.63), the oscillation frequency f_i (Hz) and damping ζ_i for the electromechanical oscillation mode, $\bar{\lambda}_6$ and $\bar{\lambda}_7$, are as follows:

$$\begin{aligned}f_i &= \frac{\omega_i}{2\pi} = 1.34 \text{ Hz} \\ \zeta_i &= -\frac{\xi_i}{\sqrt{\xi_i^2 + \omega_i^2}} = 0.0838, \quad i = 6, 7\end{aligned}$$

2.4.1.2 Heffron–Phillips Model of Example Power System

With D and Q damping winding of generator being ignored, from the given parameters of above example power system and results of calculation, it can have

$$\begin{aligned}x'_d &= x_d - \frac{x_{ad}^2}{x_f} = 0.2951 \\ E'_{q0} &= E_{Q0} - (x_q - x'_d)i_{td0} = 1.1506\end{aligned}$$

From Eq. (2.40), it can be obtained that

$$\begin{aligned}K_1 &= 1.5248, K_2 = 0.7602, K_3 = 2.9885, \\ K_4 &= 0.6727, K_5 = -0.0027, K_6 = 0.3245\end{aligned}$$

State equation of (2.41) can be obtained to be

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{E}'_q \\ \Delta \dot{E}'_{fd} \end{bmatrix} = \begin{bmatrix} 0 & 314.16 & 0 & 0 \\ -0.2178 & 0 & -0.1086 & 0 \\ -0.1346 & 0 & -0.5978 & 0.2 \\ 26.4729 & 0 & -3245 & -100 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10000 \end{bmatrix} \Delta u_{pss}$$

Eigenvalues of state matrix are as follows:

$$\lambda_1 = -92.9741, \bar{\lambda}_{2,3} = -0.0114 \pm j8.2610, \lambda_4 = -7.6008$$

According to Eq. (2.63), oscillation frequency f_i (Hz) and damping ζ_i for the electromechanical oscillation mode, $\bar{\lambda}_{2,3}$, are as follows:

$$f_i = \frac{\omega_i}{2\pi} = 1.315 \text{ Hz}$$

$$\zeta_i = -\frac{\xi_i}{\sqrt{\xi_i^2 + \omega_i^2}} = 0.0014, \quad i = 2, 3$$

2.4.2 Modal Analysis of Example Power System

Heffron–Phillips model established above is used to demonstrate the modal analysis of example power system in this section.

2.4.2.1 Modal Decomposition and Stability of Example Power System

Right eigenvectors corresponding to each of eigenvalues of state matrix of Heffron–Phillips model are calculated to be

$$\mathbf{v}_1 = \begin{bmatrix} -8.4781 \times 10^{-6} \\ 2.5091 \times 10^{-6} \\ 0.0022 \\ -1 \end{bmatrix}, \bar{\mathbf{v}}_2 = \begin{bmatrix} 0.9014 \\ j0.0237 \\ -0.0050 + j0.0050 \\ 0.3868 - j0.1930 \end{bmatrix}$$

$$\bar{\mathbf{v}}_3 = \begin{bmatrix} 0.9014 \\ -j0.0237 \\ -0.0050 - j0.0050 \\ 0.3868 + j0.1930 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} -0.0077 \\ 0.0002 \\ 0.0284 \\ -0.9996 \end{bmatrix}$$

Hence,

$$\bar{\mathbf{V}} = [\mathbf{v}_1 \quad \bar{\mathbf{v}}_2 \quad \bar{\mathbf{v}}_3 \quad \mathbf{v}_4]$$

$$= \begin{bmatrix} -8.4781 \times 10^{-6} & 0.9014 & 0.9014 & -0.0077 \\ 2.5091 \times 10^{-6} & j0.0237 & -j0.0237 & 0.0002 \\ 0.0022 & -0.0050 + j0.0050 & -0.0050 - j0.0050 & 0.0284 \\ -1 & 0.3868 - j0.1930 & 0.3868 + j0.1930 & -0.9996 \end{bmatrix}$$

From Eq. (2.53),

$$\begin{aligned}\bar{\mathbf{W}}^T &= [\mathbf{w}_1 \quad \bar{\mathbf{w}}_2 \quad \bar{\mathbf{w}}_3 \quad \mathbf{w}_4]^T = \bar{\mathbf{V}}^{-1} \\ &= \begin{bmatrix} 0.2524 & 0.5554 - j0.0001 & 0.5554 + j0.0001 & 0.1773 \\ -0.8529 & -0.0312 - j21.1226 & -0.0312 + j21.1226 & -7.3269 \\ -38.0214 & 0.1626 + j0.1475 & 0.1626 - j0.1475 & 38.2206 \\ -1.0823 & 0.0003 + j0.0003 & 0.0003 - j0.0003 & 0.0827 \end{bmatrix}\end{aligned}$$

Modal decomposition thus is obtained to be

$$s\mathbf{z}_i = \lambda_i \mathbf{z}_i + \mathbf{w}_i^T \mathbf{b}_0 \mathbf{u}, \quad i = 1, 2, 3, 4$$

Without considering the PSS ($\Delta u_{\text{pss}} = 0$), solution of modal decomposition is obtained to be

$$\mathbf{z}_i(t) = \mathbf{z}_i(0)e^{\lambda_i t}, \quad i = 1, 2, 3, 4$$

From Eq. (2.55), it can have

$$\mathbf{Z}(0) = \mathbf{V}^{-1} \mathbf{X}(0) = \mathbf{W}^T \mathbf{X}(0)$$

Time response of state variables can be written as follows:

$$\mathbf{x}_k(t) = \sum_{i=1}^4 \mathbf{v}_{ki} \mathbf{z}_i(0) e^{\lambda_i t} = \sum_{i=1}^4 \mathbf{v}_{ki} \mathbf{z}_i(0) e^{\xi_i t} [\cos \omega_i t + j \sin \omega_i t]$$

It can be seen that when time approaches infinity ($t \rightarrow \infty$),

$$\begin{aligned}\lim_{t \rightarrow \infty} \mathbf{x}_k(t) &= \lim_{t \rightarrow \infty} \sum_{i=1}^4 \mathbf{v}_{ki} \mathbf{z}_i(0) e^{\lambda_i t} = \lim_{t \rightarrow \infty} \sum_{i=1}^4 \mathbf{v}_{ki} \mathbf{z}_i(0) e^{\xi_i t} [\cos \omega_i t + j \sin \omega_i t] \\ &= \lim_{t \rightarrow \infty} \{e^{-92.9741t} \mathbf{v}_{k1} \mathbf{z}_1(0) + e^{-0.0114t} \mathbf{v}_{k2} \mathbf{z}_2(0) [\cos(8.2610t) + j \sin(8.2610t)] \\ &\quad + e^{-0.0114t} \mathbf{v}_{k3} \mathbf{z}_3(0) [\cos(8.2610t) - j \sin(8.2610t)] + e^{-7.6008t} \mathbf{v}_{k4} \mathbf{z}_4(0)\} = 0\end{aligned}$$

Since $\mathbf{X}(t) = [\Delta \delta(t) \quad \Delta \omega(t) \quad \Delta E'_q(t) \quad \Delta E'_{fd}(t)]^T$ and

$$\begin{aligned}\Delta \delta(t) &= \delta(t) - \delta_0, \Delta \omega(t) = \omega(t) - \omega_0, \\ \Delta E'_q(t) &= E'_q(t) - E'_{q0}(t), \Delta E'_{fd}(t) = E'_{fd}(t) - E'_{fd0}(t)\end{aligned}$$

$\lim_{t \rightarrow \infty} x_k(t) = 0$, $k = 1, 2, 3, 4$ should give

$$\lim_{t \rightarrow \infty} \delta(t) = \delta_0, \lim_{t \rightarrow \infty} \omega(t) = \omega_0, \lim_{t \rightarrow \infty} E'_q(t) = E'_{q0}(t), \lim_{t \rightarrow \infty} E'_{fd}(t) = E'_{fd0}(t)$$

All the state variables return to their initial points $\mathbf{X}(0)$, the equilibrium point of the system. Hence, the system is stable in terms of small-signal stability.

2.4.2.2 Modal Analysis of the AVR

State matrix, control vector, and output vector of Eq. (2.71) are as follows:

$$\begin{aligned} \mathbf{A}_0 &= \begin{bmatrix} 0 & w_0 \\ -\frac{K_1}{M} & -\frac{D}{M} \end{bmatrix} = \begin{bmatrix} 0 & 314.16 \\ -0.2178 & 0 \end{bmatrix} \\ \mathbf{b}_0 &= \begin{bmatrix} 0 \\ -\frac{K_2}{M} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1086 \end{bmatrix} \quad \mathbf{c}_0^T = [1 \quad 0] \end{aligned}$$

Its eigenvalues are calculated to be $\bar{\lambda}_{1,2} = \pm j8.2725$ with the corresponding eigenvectors to be

$$\bar{\mathbf{v}}_1 = \begin{bmatrix} 0.9997 \\ j0.0263 \end{bmatrix}, \bar{\mathbf{v}}_2 = \begin{bmatrix} 0.9997 \\ -j0.0263 \end{bmatrix}$$

Thus

$$\begin{aligned} \bar{\mathbf{W}}^T &= [\bar{\mathbf{w}}_1 \quad \bar{\mathbf{w}}_2]^T = \bar{\mathbf{V}}^{-1} = [\bar{\mathbf{v}}_1 \quad \bar{\mathbf{v}}_2]^{-1} \\ &= \begin{bmatrix} 0.9997 & 0.9997 \\ j0.0263 & -j0.0263 \end{bmatrix}^{-1} = \begin{bmatrix} 0.5002 & 0.5002 \\ -j18.9947 & j18.9947 \end{bmatrix}^T \end{aligned}$$

Hence, the controllability and observability index, respectively, can be obtained to be

$$\begin{aligned} \bar{\mathbf{w}}_1^T \mathbf{b}_0 &= [0.5002 \quad -j18.9914] \begin{bmatrix} 0 \\ -0.1086 \end{bmatrix} = j2.0628 \\ \bar{\mathbf{w}}_2^T \mathbf{b}_0 &= [0.5002 \quad j18.9914] \begin{bmatrix} 0 \\ -0.1086 \end{bmatrix} = -j2.0628 \\ \mathbf{c}_0^T \bar{\mathbf{v}}_1 &= [1 \quad 0] \begin{bmatrix} 0.9997 \\ j0.0263 \end{bmatrix} = 0.9997 \\ \mathbf{c}_0^T \bar{\mathbf{v}}_2 &= [1 \quad 0] \begin{bmatrix} 0.9997 \\ -j0.0263 \end{bmatrix} = 0.9997 \end{aligned}$$

The residue is as follows:

$$\begin{aligned}\bar{R}_1 &= \bar{w}_1^T b_0 * c_0^T v_1 = j2.0622 \\ \bar{R}_2 &= \bar{w}_2^T b_0 * c_0^T v_2 = -j2.0622\end{aligned}$$

From Eq. (2.77), when $\alpha = K_A$, it can have

$$\begin{aligned}\frac{\partial \bar{\lambda}_i}{\partial K_A} &= -R_i \frac{\partial \frac{(T_A \bar{\lambda}_i + 1)K_4 + K_A K_5}{(\bar{\lambda}_i T'_{do} + K_3)(T_A \bar{\lambda}_i + 1) + K_A K_6}}{\partial K_A} \\ &= -R_i \frac{K_5 [(\bar{\lambda}_i T'_{do} + K_3)(T_A \bar{\lambda}_i + 1) + K_A K_6] - [(T_A \bar{\lambda}_i + 1)K_4 + K_A K_5]K_6}{[(\bar{\lambda}_i T'_{do} + K_3)(T_A \bar{\lambda}_i + 1) + K_A K_6]^2} \\ &= 0.00018158 \pm j0.00005839 \quad i = 1, 2\end{aligned}$$

When $\alpha = T_A$ it can have

$$\begin{aligned}\frac{\partial \bar{\lambda}_i}{\partial T_A} &= -R_i \frac{\partial \frac{(T_A \bar{\lambda}_i + 1)K_4 + K_A K_5}{(\bar{\lambda}_i T'_{do} + K_3)(T_A \bar{\lambda}_i + 1) + K_A K_6}}{\partial T_A} \\ &= -R_i \frac{\bar{\lambda}_i K_4 [(\bar{\lambda}_i T'_{do} + K_3)(T_A \bar{\lambda}_i + 1) + K_A K_6] - [(T_A \bar{\lambda}_i + 1)K_4 + K_A K_5](\bar{\lambda}_i T'_{do} + K_3)\bar{\lambda}_i}{[(\bar{\lambda}_i T'_{do} + K_3)(T_A \bar{\lambda}_i + 1) + K_A K_6]^2} \\ &= 0.0357 \pm j0.1533 \quad i = 1, 2\end{aligned}$$

Above results indicate that increase of the gain value and time constant will move the electromechanical oscillation mode towards the right on the complex plane and hence is detrimental to the small-signal angular stability of the power system. It has been well known that the fast-acting high-gain AVR may be detrimental to the damping of power system electromechanical oscillation modes. This means that increase of gain value of the AVR could move the oscillation mode to the right. However, increase of the time constant (slower action of the AVR) should not.

In order to further clarify the results of derivative of the oscillation mode in respect to the parameters of the AVR obtained above, Fig. 2.18 presents the loci of movement of the electromechanical oscillation mode on the complex plane with the change of gain value and time constant of the AVR. In Fig. 2.18, K_A increases from $K_A = 50$ to $K_A = 100$ with $T_A = 0.01$ and T_A increases from $T_A = 0.01$ to $T_A = 0.1$ with $K_A = 100$. From Fig. 2.18, it can be seen that although at the point $K_A = 100, T_A = 0.01$ where the derivatives are calculated, the trend of loci movement is towards the right with the increase of the gain value and time constant, and the oscillation mode in fact moves towards left when the time constant of the AVR increases. The trend of the loci with the increase of the time constant of the AVR actually changes the direction at the point $K_A = 100, T_A = 0.01$.

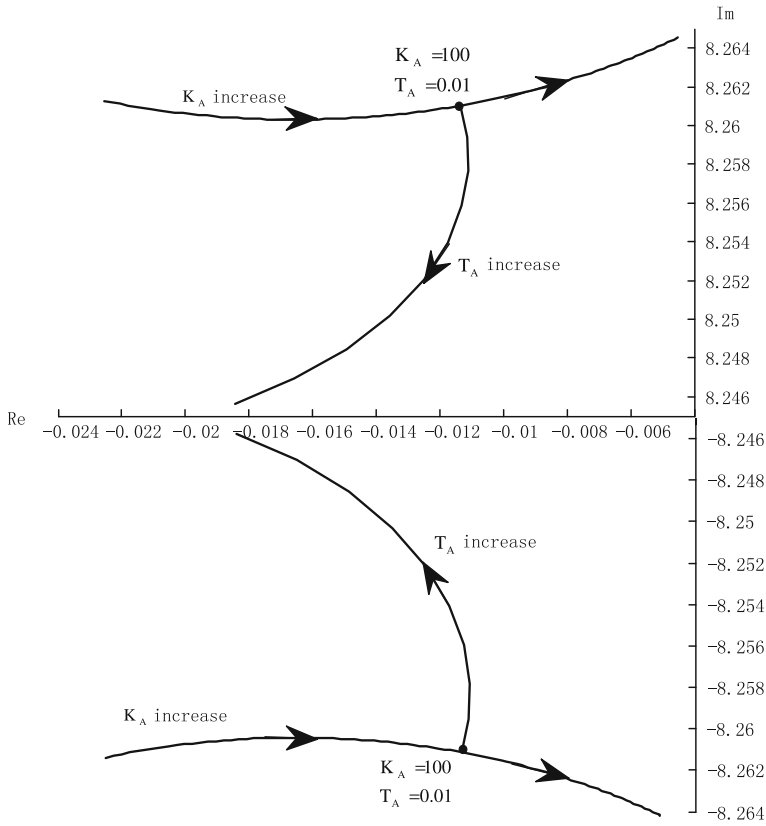


Fig. 2.18 Loci of the movement of the oscillation mode on the complex plane with the changes of parameters of the AVR

2.4.2.3 Design of the PSS by Pole Assignment for the Example Power System

A PSS can be installed to increase the damping of example power system by assigning the electromechanical oscillation mode, $\bar{\lambda}_{2,3} = -0.0114 \pm j8.2611$, to a target position on the complex plane, $\bar{\lambda}_c = -0.9 \pm j8.2611$, which is damping over 10 %.

Let the transfer function of the PSS be (see Eq. (2.88)) with $T = 0.1$ s

$$T_{\text{pss}}(s) = K_{\text{pss}} \frac{(1 + s\alpha T)^2}{(1 + sT)^2}$$

State-space model of example power system with the PSS to be installed is

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{E}'_q \\ \Delta \dot{E}'_{fd} \end{bmatrix} = \begin{bmatrix} 0 & 314.16 & 0 & 0 \\ -0.2178 & 0 & -0.1086 & 0 \\ -0.1346 & 0 & -0.5978 & 0.2 \\ 26.4729 & 0 & -3245 & -100 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10000 \end{bmatrix} \Delta u_{\text{pss}}$$

$$\Delta \omega = [0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix}$$

$$\Delta u_{\text{pss}} = T_{\text{pss}}(s) \Delta \omega$$

Thus, the transfer function of open-loop system can be obtained to be

$$G(s) = \mathbf{c}^T (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{b} = \frac{-238.8s}{1.1s^4 + 110.6s^3 + 854.5s^2 + 7563s + 53020}$$

From the characteristic equation of closed-loop control system of Eq. (2.85), for the electromechanical oscillation mode $\bar{\lambda}_c$, it should have

$$T_{\text{pss}}(\bar{\lambda}_c) = \frac{1}{G(\bar{\lambda}_c)} = \frac{1}{0.0753 - j0.0996}$$

That is

$$K_{\text{pss}} \frac{(1 + \bar{\lambda}_c \alpha T)^2}{(1 + \bar{\lambda}_c T)^2} = \frac{1}{0.0753 - j0.0996}$$

By solving the above equations, parameters of the PSS can be obtained to be

$$\begin{cases} K_{\text{pss}} = 2.6151 \\ \alpha = 2.4256 \end{cases}$$

In order to establish the state-space model of closed-loop system with the PSS installed, let

$$\Delta x_1 = \frac{(1 + s\alpha T)}{(1 + sT)} \Delta \omega$$

$$\Delta u_{\text{pss}} = K_{\text{pss}} \frac{(1 + s\alpha T)}{(1 + sT)} \Delta x_1$$

Thus, state equation of the PSS is as follows:

$$\begin{aligned}
 s\Delta x_1 &= \frac{1}{T}\Delta\omega + \alpha \cdot s\Delta\omega - \frac{1}{T}\Delta x_1 \\
 &= -\frac{K_1}{M}\alpha\Delta\delta + \left(\frac{1}{T} - \frac{D}{M}\alpha\right)\Delta\omega - \frac{K_2}{M}\alpha\Delta E'_q - \frac{1}{T}\Delta x_1 \\
 s\Delta u_{\text{pss}} &= \frac{1}{T}K_{\text{pss}}\Delta x_1 - \frac{1}{T}\Delta u_{\text{pss}} + K_{\text{pss}}\alpha \cdot s\Delta x_1 \\
 &= -K_{\text{pss}}\frac{K_1}{M}\alpha^2\Delta\delta + K_{\text{pss}}\alpha\left(\frac{1}{T} - \frac{D}{M}\alpha\right)\Delta\omega - K_{\text{pss}}\alpha^2\frac{K_2}{M}\Delta E'_q \\
 &\quad + (1 - \alpha)\frac{1}{T}K_{\text{pss}}\Delta x_1 - \frac{1}{T}\Delta u_{\text{pss}}
 \end{aligned}$$

By writing the state equation of open-loop system and the PSS together, the state-space model of closed-loop system is obtained to be

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{E}'_q \\ \Delta \dot{E}'_{fd} \\ \Delta \dot{x}_1 \\ \Delta \dot{u}_{\text{pss}} \end{bmatrix} = \begin{bmatrix} 0 & 314.159 & 0 & 0 & 0 & 0 \\ -0.218 & 0 & -0.109 & 0 & 0 & 0 \\ -0.134 & 0 & -0.597 & 0.200 & 0 & 0 \\ 26.582 & 0 & -3245.044 & -100 & 0 & 10000 \\ -0.528 & 10 & -0.263 & 0 & -10 & 0 \\ 3.352 & -63.432 & 1.671 & 0 & 37.281 & -10 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'_q \\ \Delta E'_{fd} \\ \Delta x_1 \\ \Delta u_{\text{pss}} \end{bmatrix}$$

Eigenvalues of state matrix are calculated to be

$$\begin{aligned}
 \bar{\lambda}_1 &= -93.4535 \\
 \bar{\lambda}_{2,3} &= -0.8995 \pm j5.2621 \\
 \bar{\lambda}_{4,5} &= -9.4136 \pm j5.0918 \\
 \bar{\lambda}_6 &= -6.5176
 \end{aligned}$$

Hence, the electromechanical oscillation mode is successfully assigned to the target position.

Figure 2.19 shows the simulation result of example power system without and with the PSS installed. At 1.0 s of simulation, a three-phase to-earth short circuit occurred on the transmission line which was cleared in 100 ms. From Fig. 2.19, it can be seen that the low-frequency oscillation is damped effectively by the PSS designed by use of the method of pole assignment.

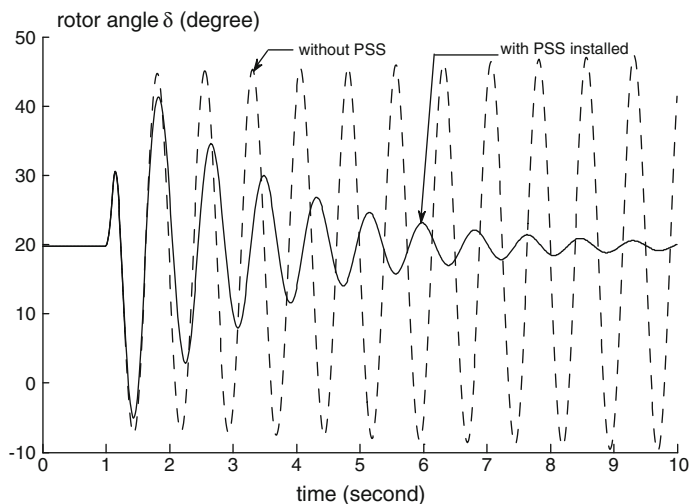


Fig. 2.19 Simulation result of example power system without and with PSS installed

2.4.3 Damping Torque Analysis of Example Power System

Heffron–Phillips model established is again used in this section to demonstrate the DTA of example power system.

2.4.3.1 Damping Torque Provided by the AVR in the Example Power System

From Fig. 2.10, it can be seen that the electric torque provided by the AVR to the electromechanical oscillation loop of generator is as follows:

$$\begin{aligned}\Delta T_{\text{avr}} &= K_2 \frac{-K_5 \frac{1}{sT'_{\text{do}} + K_3} \frac{K_A}{sT_A + 1}}{1 + \frac{K_6}{sT'_{\text{do}} + K_3} \frac{K_A}{sT_A + 1}} \Delta\delta \\ &= -\frac{K_2 K_5 K_A}{K_6 K_A + (sT'_{\text{do}} + K_3)(sT_A + 1)} \Delta\delta = F_{\text{avr}}(s) \Delta\delta\end{aligned}$$

At the complex frequency of the electromechanical oscillation with $\bar{\lambda}_s = \xi_s + j\omega_s = \bar{\lambda}_1 = -0.0114 + j8.2610$, let the decomposition of the electric torque provided by the AVR be

$$\Delta T_{\text{avr}}(\bar{\lambda}_s) = T_{\text{savr}} \Delta \delta(\bar{\lambda}_s) + T_{\text{davr}} \Delta \omega(\bar{\lambda}_s)$$

Because

$$\Delta \omega(\bar{\lambda}_s) = \frac{\xi_s + j\omega_s}{\omega_0} \Delta \delta(\bar{\lambda}_s) = \frac{\xi_s}{\omega_0} \Delta \delta(\bar{\lambda}_s) + j \frac{\omega_s}{\omega_0} \Delta \delta(\bar{\lambda}_s)$$

thus

$$\Delta T_{\text{avr}}(\bar{\lambda}_s) = T_{\text{savr}} \Delta \delta(\bar{\lambda}_s) + T_{\text{davr}} \frac{\xi_s}{\omega_0} \Delta \delta(\bar{\lambda}_s) + j T_{\text{davr}} \frac{\omega_s}{\omega_0} \Delta \delta(\bar{\lambda}_s)$$

That is

$$T_{\text{davr}} \frac{\omega_s}{\omega_0} = \text{Im} F_{\text{avr}}(\bar{\lambda}_s)$$

Since

$$\begin{aligned} F_{\text{avr}}(\bar{\lambda}_s) &= - \frac{K_2 K_5 K_A}{K_6 K_A + (\bar{\lambda}_s T'_{d0} + K_3)(\bar{\lambda}_s T_A + 1)} \\ &= - \frac{K_2 K_5 K_A}{K_6 K_A + K_3 + (K_3 T_A + T'_{d0})\bar{\lambda}_s + T'_{d0} T_A \bar{\lambda}_s^2} \\ &= - \frac{K_2 K_5 K_A}{K_6 K_A + K_3 + (K_3 T_A + T'_{d0})(\xi_s + j\omega_s) + T'_{d0} T_A (\xi_s + j\omega_s)^2} \\ &= - \frac{K_2 K_5 K_A}{a + jb} = - \frac{K_2 K_5 K_A}{a^2 + b^2} (a - jb) = -0.0024 + j0.0031 \end{aligned}$$

where

$$a = K_6 K_A + K_3 + (K_3 T_A + T'_{d0})\xi_s + T'_{d0} T_A (\xi_s^2 - \omega_s^2) = 31.9695$$

$$b = (K_3 T_A + T'_{d0})\omega_s + 2T'_{d0} T_A \xi_s \omega_s = 41.5426$$

thus it can be obtained

$$\begin{aligned} T_{\text{davr}} &= \frac{\omega_0}{\omega_s} \text{Im} F_{\text{avr}}(\bar{\lambda}_s) = \frac{\omega_0}{\omega_s} \frac{K_2 K_5 K_A}{a^2 + b^2} b \\ &= 0.0988 \end{aligned}$$

Since

$$\begin{aligned}\frac{\partial a}{\partial K_A} &= K_6 = 0.3245, & \frac{\partial a}{\partial T_A} &= K_3 \xi_s + T'_{d0} (\xi_s^2 - \omega_s^2) = -341.2561 \\ \frac{\partial b}{\partial K_A} &= 0, & \frac{\partial b}{\partial T_A} &= K_3 \omega_s + 2T'_{d0} \xi_s \omega_s = 23.7488\end{aligned}$$

sensitivity of the damping torque provided by the AVR to its parameters can be obtained to be

$$\begin{aligned}\frac{\partial T_{davr}}{\partial K_A} &= \frac{\omega_0}{\omega_s} \frac{K_2 K_5 \left[\left(b + K_A \frac{\partial b}{\partial K_A} \right) (a^2 + b^2) - K_A b \left(2a \frac{\partial a}{\partial K_A} + 2b \frac{\partial b}{\partial K_A} \right) \right]}{(a^2 + b^2)^2} = -0.0003 \\ \frac{\partial T_{davr}}{\partial T_A} &= \frac{\omega_0}{\omega_s} \frac{K_2 K_5 K_A \left[\frac{\partial b}{\partial T_A} (a^2 + b^2) - b \left(2a \frac{\partial a}{\partial T_A} + 2b \frac{\partial b}{\partial T_A} \right) \right]}{(a^2 + b^2)^2} = -0.9055\end{aligned}$$

The above results indicate that (1) with the increase of the AVR gain, less damping torque will be provided by the AVR, detrimental to the system small-signal angular stability and (2) with the increase of the AVR time constant, less damping torque will be provided by the AVR, also detrimental to the damping of low-frequency power oscillations.

2.4.3.2 Design of PSS Installed in the Example Power System by the Phase Compensation Method

The PSS to be designed is to provide a damping torque $\Delta T_{pss} = D_{pss} \Delta \omega$, $D_{pss} = 15$. From Eq. (2.96), the forward path of stabilizing signal of the PSS can be obtained to be ($j\omega_s = j8.44$)

$$\begin{aligned}F_{pss}(j\omega_s) &= K_2 \frac{K_A}{(K_3 + j\omega_s T'_{d0})(1 + j\omega_s T_A) + K_6 K_A} \\ &= 0.8598 - j1.1451 = 1.4320 \angle -53.0989^\circ\end{aligned}$$

The PSS adopts the deviation of rotor speed of generator as the feedback signal, and its transfer function is as follows:

$$T_{pss}(s) = K_1 \frac{(1 + sT_2)}{(1 + sT_1)} K_2 \frac{(1 + sT_4)}{(1 + sT_3)} \quad \text{with } T_1 = 0.09 \text{ s}, T_3 = 0.09 \text{ s}$$

According to Eq. (2.133), parameters of the PSS are set to compensate the phase of the forward path and thus obtained to be

$$K_{\text{pss}} = K_{\text{pss1}} K_{\text{pss2}} = 3.2271, T_2 = 0.2405 \text{ s}, T_4 = 0.2405 \text{ s}$$

Let

$$\begin{aligned}\Delta x_1 &= K_{\text{pss2}} \frac{(1 + sT_4)}{(1 + sT_3)} \Delta \omega = (9.3706 + j4.6819) \Delta \omega \\ \Delta u_{\text{pss}} &= K_{\text{pss1}} \frac{(1 + sT_2)}{(1 + sT_1)} \Delta x_1 = (0.8946 + j0.4470) \Delta x_1\end{aligned}$$

Thus, state-space realization of the PSS is as follows:

$$\begin{aligned}s\Delta x_1 &= -\frac{1}{T_3} \Delta x_1 + \frac{K_{\text{pss2}}}{T_3} (T_4 s \Delta \omega + \Delta \omega) \\ &= -\frac{1}{T_3} \Delta x_1 + \frac{K_{\text{pss2}}}{T_3} \left[\frac{T_4}{M} (-K_1 \Delta \delta - K_2 \Delta E'_q - D \Delta \omega) + \Delta \omega \right] \\ &= -\frac{K_{\text{pss2}} T_4 K_1}{T_3 M} \Delta \delta + \frac{K_{\text{pss2}}}{T_3} \left(1 - \frac{T_4}{M} D \right) \Delta \omega - \frac{K_{\text{pss2}} T_4 K_2}{T_3 M} \Delta E'_q - \frac{1}{T_3} \Delta x_1 \\ &= -0.3230 \Delta \delta + 6.1672 \Delta \omega - 0.1224 \Delta E'_q - 11.1111 \Delta x_1 \\ s\Delta u_{\text{pss}} &= -\frac{1}{T_1} \Delta u_{\text{pss}} + \left(\frac{K_{\text{pss1}}}{T_1} - \frac{T_2}{T_3} \frac{K_{\text{pss1}}}{T_1} \right) \Delta x_1 + \frac{K_{\text{pss1}} T_2}{T_1} \left[-\frac{K_{\text{pss2}} T_4 K_1}{T_3 M} \Delta \delta \right. \\ &\quad \left. - \frac{K_{\text{pss2}} T_4 K_2}{T_3 M} \Delta E'_q + \frac{K_{\text{pss2}}}{T_3} \left(1 - \frac{T_4}{M} D \right) \Delta \omega \right] \\ &= -\frac{K_{\text{pss}} T_2 T_4 K_1}{T_1 T_3 M} \Delta \delta + \frac{K_{\text{pss}} T_2}{T_1 T_3} \left(1 - \frac{T_4}{M} D \right) \Delta \omega - \frac{K_{\text{pss}} T_2 T_4 K_2}{T_1 T_3 M} \Delta E'_q \\ &\quad + \left(\frac{K_{\text{pss1}}}{T_1} - \frac{T_2}{T_3} \frac{K_{\text{pss1}}}{T_1} \right) \Delta x_1 - \frac{1}{T_1} \Delta u_{\text{pss}} \\ &= -5.0182 \Delta \delta + 95.8026 \Delta \omega - 2.5018 \Delta E'_q - 108.0008 \Delta x_1 - 11.1111 \Delta u_{\text{pss}}\end{aligned}$$

By writing the state equation of open-loop system and the PSS together, state matrix of closed-loop system is obtained to be

$$\mathbf{A} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 & 0 & 0 \\ -\frac{K_1}{M} & -\frac{D}{M} & -\frac{K_2}{M} & 0 & 0 & 0 \\ -\frac{K_4}{T_{d0}} & 0 & -\frac{K_3}{T_{d0}} & -\frac{1}{T_{d0}} & 0 & 0 \\ -\frac{K_A K_5}{T_A} & 0 & -\frac{K_A K_6}{T_A} & -\frac{K_A}{T_A} & 0 & \frac{K_A}{T_A} \\ -\frac{K_{PSS2} T_4 K_1}{T_3 M} & \frac{K_{PSS2}}{T_3} \left(1 - \frac{T_4}{T_3 M}\right) & -\frac{K_{PSS2} T_4 K_2}{T_3 M} & 0 & -\frac{1}{T_3} & 0 \\ -\frac{T_2 T_4 K_1}{T_1 T_3 M} K_{PSS} & \frac{T_2}{T_1 T_3} \left(1 - \frac{T_4 D}{M}\right) K_{PSS} & -\frac{T_2 T_4 K_2}{T_1 T_3 M} K_{PSS} & 0 & \frac{K_{PSS1}}{T_1} \left(1 - \frac{T_2}{T_3}\right) & -\frac{1}{T_1} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 314.16 & 0 & 0 & 0 & 0 \\ -0.2178 & 0 & -0.1086 & 0 & 0 & 0 \\ -0.1345 & 0 & -0.5977 & 0.2 & 0 & 0 \\ 26.5821 & 0 & 3245 & -100 & 0 & 10000 \\ -0.3230 & 6.1672 & -0.1224 & 0 & -11.1111 & 0 \\ -5.0182 & 95.8026 & -2.5018 & 0 & -108.0008 & -11.1111 \end{bmatrix}$$

Eigenvalues of state matrix are calculated to be

$$\lambda_1 = -93.6838$$

$$\bar{\lambda}_{2,3} = -1.2125 \pm j8.0051$$

$$\bar{\lambda}_{4,5} = -8.7171 \pm j6.2080$$

$$\lambda_6 = -9.2771$$

Hence, damping of the electromechanical oscillation mode is successfully increased by the PSS designed above via the phase compensation method.

Figure 2.20 shows the simulation result of the example power system without and with the PSS installed. At 1.0 s of the simulation, a three-phase to-earth short

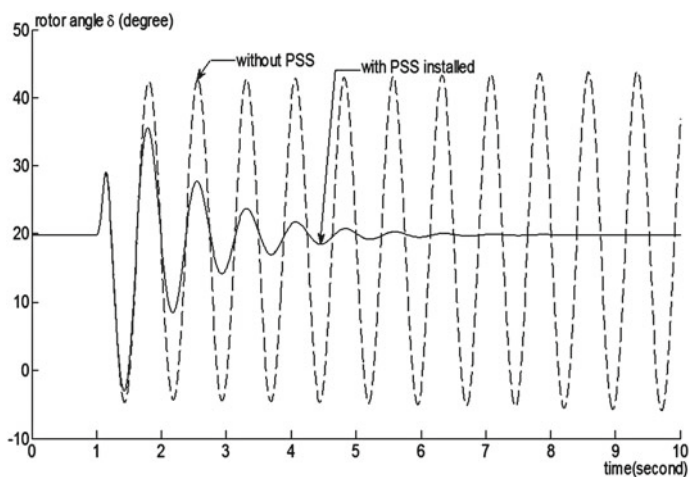


Fig. 2.20 Simulation result of power system without and with the PSS designed by use of the phase compensation method

circuit occurred on the transmission line which was cleared in 100 ms. From Fig. 2.20, it can be seen that the low-frequency oscillation is damped effectively by the PSS designed by use of the phase compensation method.

2.4.3.3 Theoretical Basis and Graphical Explanation of the Damping Torque Analysis

From Fig. 2.10 and Eq. (2.105), it can be obtained that

$$\begin{aligned} F_{\text{delta}}(s) &= K_2 \left(\frac{-K_4 \frac{1}{sT'_{\text{do}} + K_3}}{1 + \frac{K_6}{sT'_{\text{do}} + K_3} \frac{K_A}{sT_A + 1}} + \frac{-K_5 \frac{1}{sT'_{\text{do}} + K_3} \frac{K_A}{sT_A + 1}}{1 + \frac{K_6}{sT'_{\text{do}} + K_3} \frac{K_A}{sT_A + 1}} \right) \\ &= -\frac{K_2 [K_4 (sT_A + 1) + K_5 K_A]}{K_6 K_A + (sT'_{\text{do}} + K_3)(sT_A + 1)} \end{aligned}$$

At the complex frequency of electromechanical oscillation with $\bar{\lambda}_s = \bar{\xi}_s + j\omega_s = \bar{\lambda}_1 = -0.0114 + j8.2610$,

$$\bar{F}_{\text{delta}}(\bar{\lambda}_s) = -0.0042 + j0.0042$$

According to Eq. (2.114)

$$\begin{cases} T_{\text{d1}} = \frac{\omega_0}{\omega_s} \text{Im}[\bar{F}_{\text{delta}}(\bar{\lambda}_s)] = 0.1591 \\ T_{\text{s1}} = \text{Re}[\bar{F}_{\text{delta}}(\bar{\lambda}_s)] - \frac{T_{\text{d1}} \bar{\xi}_s}{\omega_0} = -0.0042 \end{cases}$$

Substituting the above result into Eq. (2.116), it can have

$$7\bar{\lambda}_s^2 + 0.1553\bar{\lambda}_s + 477.7128 = 0$$

Solution of above equation is $\bar{\lambda}_s = -0.0114 + j8.2610$. It is the electromechanical oscillation mode of example power system without the PSS being installed.

From Fig. 2.10, it can have

$$\Delta P_t = K_1 \Delta \delta + F_{\text{delta}}(\bar{\lambda}_s) \Delta \delta = K_1 \Delta \delta + T_{\text{s1}} \Delta \delta + T_{\text{d1}} \Delta \omega = 1.5207 \Delta \delta + 0.1553 \Delta \omega$$

This is the first equation in Eq. (2.120). Figure 2.21 presents the $P_t - \delta$ curve from simulation. At 1 s of simulation, the mechanical power input to the generator

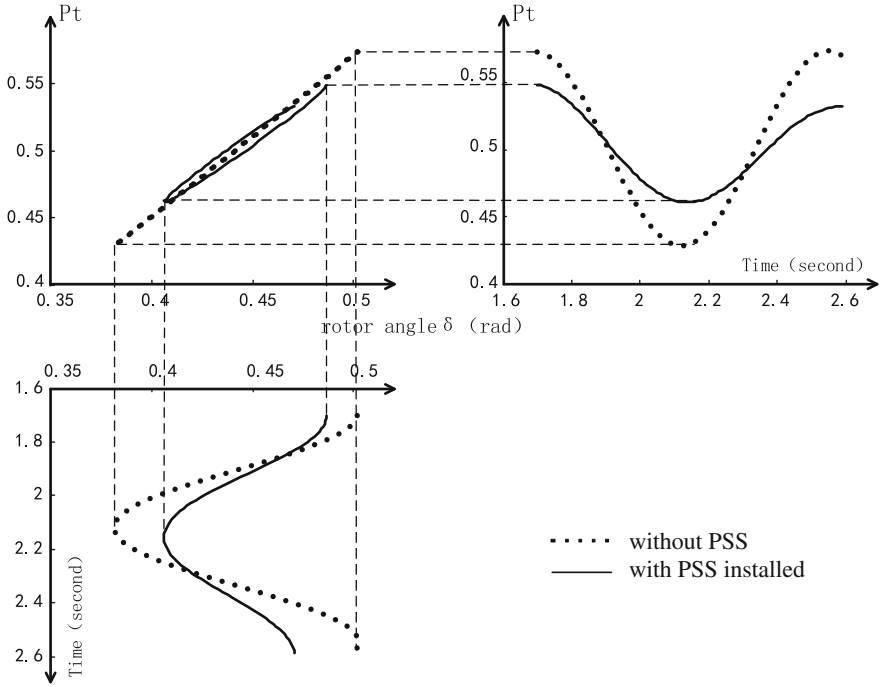


Fig. 2.21 $P_t - \delta$ curve, corresponding variation of rotor angle and power output of the generator of example power system

increased by 1 % and then returned to its original value in 10 ms. $P_t - \delta$ curve depicted in Fig. 2.20 is the first cycle of power oscillation starting from its first peak. Dashed curve is the case without the PSS installed.

With the PSS designed in Sect. 2.4.3.2 being installed,

$$\begin{aligned}
 \Delta P_t &= K_1 \Delta \delta + F_{\text{delta}}(\bar{\lambda}_c) \Delta \delta + \bar{F}_{\text{pss}}(\bar{\lambda}_c) \bar{T}_{\text{pss}}(\bar{\lambda}_c) \Delta \omega(\bar{\lambda}_c) \\
 &= K_1 \Delta \delta + T_{s1} \Delta \delta + T_{d1} \Delta \omega + D_{\text{pss}} \Delta \omega(\bar{\lambda}_c) \\
 &= 1.5207 \Delta \delta + 15.1553 \Delta \omega
 \end{aligned}$$

where $\bar{\lambda}_c = -1.2279 + j8.0264$ is the electromechanical oscillation mode of example power system with the PSS being installed. Solid $P_t - \delta$ curve in Fig. 2.21 is the case that the PSS is installed in the example power system. Figure 2.21 confirms the graphical explanation of the DTA illustrated in Fig. 2.16.

2.4.4 Equivalence Between the Damping Torque and Modal Analysis

It is concluded in Sect. 2.3.1.3 that at a complex frequency $\bar{\lambda}_i$, the residue is equal to the forward path of the PSS multiplied by the sensitivity of the mode to the damping torque contribution. In this section, this conclusion is to be demonstrated by example power system.

2.4.4.1 Demonstration by Use of Heffron–Phillips Model of Example Power System

In Sect. 2.4.2.3, state-space model of the example power system is obtained to be

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{E}'_q \\ \Delta \dot{E}'_{fd} \end{bmatrix} = \begin{bmatrix} 0 & 314.16 & 0 & 0 \\ -0.2178 & 0 & -0.1086 & 0 \\ -0.1345 & 0 & -0.5977 & 0.2 \\ 26.5821 & 0 & -3245 & -100 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10000 \end{bmatrix} \Delta u_{\text{pss}}$$

$$\Delta \omega = [0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix}$$

The electromechanical oscillation modes are as follows:

$$\bar{\lambda}_{2,3} = -0.0114 \pm j8.2610$$

In Sect. 2.4.2.1, matrix formed by right eigenvectors is as follows:

$$\bar{\mathbf{V}} = [\mathbf{v}_1 \quad \bar{\mathbf{v}}_2 \quad \bar{\mathbf{v}}_3 \quad \mathbf{v}_4]$$

Thus, the right eigenvectors related to the electromechanical oscillation modes are as follows:

$$\bar{\mathbf{v}}_2 = \begin{bmatrix} 0.9014 \\ j0.0237 \\ -0.0050 + j0.0050 \\ 0.3868 - j0.1930 \end{bmatrix} \quad \bar{\mathbf{v}}_3 = \begin{bmatrix} 0.9014 \\ -j0.0237 \\ -0.0050 - j0.0050 \\ 0.3868 + j0.1930 \end{bmatrix}$$

Matrix formed by left eigenvectors is as follows:

$$\bar{\mathbf{W}}^T = [\mathbf{w}_1 \quad \bar{\mathbf{w}}_2 \quad \bar{\mathbf{w}}_3 \quad \mathbf{w}_4]^T = \bar{\mathbf{V}}^{-1}$$

The left eigenvectors related to the electromechanical oscillation modes are as follows:

$$\begin{aligned} \bar{\mathbf{w}}_2^T &= [0.5554 - j0.0001 \quad -0.0312 - j21.1226 \quad 0.1626 + j0.1475 \quad 0.0003 + j0.0003] \\ \bar{\mathbf{w}}_3^T &= [0.5554 + j0.0001 \quad -0.0312 + j21.1226 \quad 0.1626 - j0.1475 \quad 0.0003 - j0.0003] \end{aligned}$$

Thus, according to Eq. (2.64), for the electromechanical oscillation modes, the residue is calculated to be

$$\begin{aligned} \bar{\mathbf{R}}_2 &= \bar{\mathbf{w}}_2^T \mathbf{b}_o \mathbf{c}_o^T \bar{\mathbf{v}}_2 \\ &= [0.5554 - j0.0001 \quad -0.0312 - j21.1226 \quad 0.1626 + j0.1476 \quad 0.0003 + j0.0003] \\ &\quad \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10000 \end{bmatrix} [0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} 0.9014 \\ j0.0237 \\ -0.0050 + j0.0050 \\ 0.3868 - j0.1930 \end{bmatrix} \\ &= -0.0633 + j0.0822 \end{aligned}$$

$$\begin{aligned} \bar{\mathbf{R}}_3 &= \bar{\mathbf{w}}_3^T \mathbf{b}_o \mathbf{c}_o^T \bar{\mathbf{v}}_3 \\ &= [0.5554 + j0.0001 \quad -0.0312 + j21.1226 \quad 0.1626 - j0.1475 \quad 0.0003 - j0.0003] \\ &\quad \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10000 \end{bmatrix} [0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} 0.9014 \\ -j0.0237 \\ -0.0050 - j0.0050 \\ 0.3868 + j0.1930 \end{bmatrix} \\ &= -0.0633 - j0.0822 \end{aligned}$$

From Eq. (2.104), sensitivity of the electromechanical oscillation modes to the coefficient of damping torque provided by the PSS can be computed to be

$$\begin{aligned} \frac{\partial \bar{\lambda}_2}{\partial D_{\text{pss}}} &= -\frac{\bar{\mathbf{w}}_{22} \bar{\mathbf{v}}_{22}}{\mathbf{M}} = -\frac{(-0.0312 - j21.1226)(j0.0237)}{7} = -0.0715 \\ \frac{\partial \bar{\lambda}_3}{\partial D_{\text{pss}}} &= -\frac{\bar{\mathbf{w}}_{32} \bar{\mathbf{v}}_{32}}{\mathbf{M}} = -\frac{(-0.0312 + j21.1226)(-j0.0237)}{7} = -0.0715 \end{aligned}$$

At the complex frequency $\bar{\lambda}_{2,3} = -0.0114 \pm j8.2610$, the forward path can be calculated from Eq. (2.96) as

$$\begin{aligned} F_{\text{pss}}(\bar{\lambda}_{2,3}) &= K_2 \frac{K_A}{(K_3 + \bar{\lambda}_{2,3} T'_{d0})(1 + \bar{\lambda}_{2,3} T_A) + K_6 K_A} \\ &= \frac{0.7602 \times 100}{[2.9885 + (-0.0114 \pm j8.2611) \times 5][1 + (-0.0114 \pm j8.2611) \times 0.01] + 0.3245 \times 100} \\ &= -0.8845 \pm j1.1493 \end{aligned}$$

Hence,

$$\begin{aligned} R_{2,3} &= -\frac{\partial \bar{\lambda}_{2,3}}{\partial D_{\text{pss}}} F_{\text{pss}}(\bar{\lambda}_{2,3}) = -(-0.0715) \times (-0.8845 \pm j1.1493) \\ &= -0.0633 \pm j0.0822 \end{aligned}$$

Thus, it is conformed that the residue is equal to the forward path of the PSS multiplied by the sensitivity of electromechanical oscillation modes to the coefficient of damping torque contribution from the PSS.

2.4.4.2 Demonstration by Use of General Linearized Model of Example Power System

In Sect. 2.4.1.1, state matrix and control vector of state-space model of example power system are obtained to be

$$\begin{aligned} \mathbf{A}_{\text{gc-dq}} &= \begin{bmatrix} 0 & 314.16 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.045 & 0 & 0 & 0.20 & 0.47 & -0.09 & -0.104 & -0.39 \\ 714.29 & 0 & -100 & 6116.3 & -1404.6 & -2646 & -3127 & 1154.5 \\ 295.81 & -89.35 & 0 & -6.65 & 478.4 & 2.88 & 3.40 & -135.03 \\ -105.79 & -318.31 & 0 & -513.64 & -5.48 & 86.29 & 101.98 & 4.50 \\ 0 & 0 & 314.16 & 0.43 & 0 & -1.22 & 0.71 & 0 \\ 0 & 0 & 0 & 1.36 & 0 & 1.88 & -3.49 & 0 \\ 0 & 0 & 0 & 0 & 36.0 & 0 & 0 & -46.8 \end{bmatrix} \\ \mathbf{b}_{\text{pss}} &= \begin{bmatrix} 0 \\ 0 \\ 10000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

Eigenvalues of state matrix are as follows:

$$\begin{aligned}\bar{\lambda}_{1,2} &= -24.12 \pm j971.56 \\ \bar{\lambda}_{3,4} &= -36.32 \pm j359.63 \\ \lambda_5 &= -35.57 \\ \bar{\lambda}_{6,7} &= -0.71 \pm j8.44 \\ \lambda_8 &= -5.71\end{aligned}$$

The pair of electromechanical oscillation modes are as follows:

$$\bar{\lambda}_{6,7} = -0.71 \pm j8.44$$

For each of eigenvalues, right eigenvector is calculated to be

$$\begin{aligned}\bar{\mathbf{v}}_1 &= \begin{bmatrix} 0.000005 - j0.000011 \\ 0.000032 + j0.000016 \\ 0.9507 \\ -0.0001 + j0.0182 \\ -0.0369 + j0.0014 \\ -0.0072 - j0.3072 \\ -0.000569 + j0.000026 \\ j0.0014 \end{bmatrix} & \bar{\mathbf{v}}_2 &= \begin{bmatrix} 0.000005 + j0.000011 \\ 0.000032 - j0.000016 \\ 0.9507 \\ -0.0001 - j0.0182 \\ -0.0369 - j0.0014 \\ -0.0072 + j0.3072 \\ -0.000569 - j0.000026 \\ -j0.0014 \end{bmatrix} \\ \bar{\mathbf{v}}_3 &= \begin{bmatrix} -0.000215 - j0.000026 \\ 0.000055 - j0.000244 \\ 0.7251 \\ 0.0163 - j0.2196 \\ 0.1654 + j0.025 \\ -0.0615 - j0.6274 \\ -0.0041 + j0.0006 \\ 0.003 - j0.0165 \end{bmatrix} & \bar{\mathbf{v}}_4 &= \begin{bmatrix} -0.000215 + j0.000026 \\ 0.000055 + j0.000244 \\ 0.7251 \\ 0.0163 + j0.2196 \\ 0.1654 - j0.025 \\ -0.0615 + j0.6274 \\ -0.0041 - j0.0006 \\ 0.003 + j0.0165 \end{bmatrix} & \mathbf{v}_5 &= \begin{bmatrix} -0.0527 \\ 0.006 \\ -0.0539 \\ 0.1055 \\ 0.2563 \\ 0.4925 \\ -0.0334 \\ 0.8209 \end{bmatrix} \\ \bar{\mathbf{v}}_6 &= \begin{bmatrix} 0.7004 \\ -0.0016 + j0.0188 \\ 0.000061 - j0.000914 \\ -0.1482 + j0.0043 \\ -0.5516 + j0.0283 \\ -0.0317 + j0.0041 \\ -0.0077 + j0.0284 \\ -0.413 + j0.0977 \end{bmatrix} & \bar{\mathbf{v}}_7 &= \begin{bmatrix} 0.7004 \\ -0.0016 - j0.0188 \\ 0.000061 + j0.000914 \\ -0.1482 - j0.0043 \\ -0.5516 - j0.0283 \\ -0.0317 - j0.0041 \\ -0.0077 - j0.0284 \\ -0.413 - j0.0977 \end{bmatrix} & \mathbf{v}_8 &= \begin{bmatrix} 0.000104 \\ -0.000002 \\ 0.0095 \\ -0.000116 \\ -0.000082 \\ -0.7636 \\ 0.6456 \\ -0.000072 \end{bmatrix}\end{aligned}$$

They form the following matrix

$$\bar{\mathbf{V}} = [\bar{\mathbf{v}}_1 \quad \bar{\mathbf{v}}_2 \quad \dots \quad \bar{\mathbf{v}}_8]$$

As $\bar{\mathbf{W}}^T = [\bar{\mathbf{w}}_1 \quad \bar{\mathbf{w}}_2 \quad \dots \quad \bar{\mathbf{w}}_8]^T = \bar{\mathbf{V}}^{-1}$, left eigenvectors corresponding to eigenvalues are calculated to be

$$\begin{aligned} \bar{\mathbf{w}}_1^T &= \begin{bmatrix} -1.2419 - j0.4483 \\ -0.0486 - j0.262 \\ 0.4494 + j0.0412 \\ -0.2153 - j3.8659 \\ -1.9686 + j0.7712 \\ -0.0188 + j1.3997 \\ -0.0288 + j1.6542 \\ 0.6025 - j0.5408 \end{bmatrix}^T & \bar{\mathbf{w}}_2^T &= \begin{bmatrix} -1.2419 + j0.4483 \\ -0.0486 + j0.262 \\ 0.4494 - j0.0412 \\ -0.2153 + j3.8659 \\ -1.9686 - j0.7712 \\ -0.0188 - j1.3997 \\ -0.0288 - j1.6542 \\ 0.6025 + j0.5408 \end{bmatrix}^T \\ \bar{\mathbf{w}}_3^T &= \begin{bmatrix} 1.6552 + j0.1633 \\ -0.1388 + j0.9214 \\ 0.1003 - j0.0038 \\ 0.2788 + j1.9563 \\ 2.5805 - j0.126 \\ 0.0246 + j0.1141 \\ 0.0277 + j0.1353 \\ -0.7412 - j0.2714 \end{bmatrix}^T & \bar{\mathbf{w}}_4^T &= \begin{bmatrix} 1.6552 - j0.1633 \\ -0.1388 - j0.9214 \\ 0.1003 + j0.0038 \\ 0.2788 - j1.9563 \\ 2.5805 + j0.126 \\ 0.0246 - j0.1141 \\ 0.0277 - j0.1353 \\ -0.7412 + j0.2714 \end{bmatrix}^T & \bar{\mathbf{w}}_5^T &= \begin{bmatrix} 0.7411 \\ -6.8756 \\ -0.000219 \\ -0.0934 \\ -0.0106 \\ -0.000045 \\ -0.00006 \\ 1.3311 \end{bmatrix}^T \\ \bar{\mathbf{w}}_6^T &= \begin{bmatrix} 0.7422 + j0.06 \\ -0.2586 - j27.632 \\ 0.000019 + j0.00094 \\ -0.0034 + j0.0146 \\ 0.0004 + j0.000131 \\ -0.000019 + j0.000298 \\ -0.000139 + j0.0002727 \\ 0.0499 + j0.2027 \end{bmatrix}^T & \bar{\mathbf{w}}_7^T &= \begin{bmatrix} 0.7422 - j0.06 \\ -0.2586 + j27.632 \\ 0.000019 - j0.00094 \\ -0.0034 - j0.0146 \\ 0.0004 - j0.000131 \\ -0.000019 - j0.000298 \\ -0.000139 - j0.0002727 \\ 0.0499 - j0.2027 \end{bmatrix}^T \\ \bar{\mathbf{w}}_8^T &= \begin{bmatrix} 0.0803 \\ -2.7921 \\ 0.0021 \\ 0.003 \\ 0.0283 \\ 0.000637 \\ 1.5496 \\ 0.079 \end{bmatrix}^T \end{aligned}$$

According to Eq. (2.64), for the pair of electromechanical oscillation modes, the residue is calculated to be

$$\begin{aligned}
 \bar{\mathbf{R}}_{6,7} &= \bar{\mathbf{w}}_{6,7}^T \mathbf{b}_o \mathbf{c}_o^T \bar{\mathbf{v}}_{6,7} \\
 &= \begin{bmatrix} 0.7422 \pm j0.06 \\ -0.2586 \mp j27.632 \\ 0.000019 \pm j0.00094 \\ -0.0034 \pm j0.0146 \\ 0.0004 \pm j0.000131 \\ -0.000019 \pm j0.000298 \\ -0.000139 \pm j0.0002727 \\ 0.0499 + j0.2027 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 10000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
 &\quad \times [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \begin{bmatrix} 0.7004 \\ -0.0016 \pm j0.0188 \\ 0.000061 \mp j0.000914 \\ -0.1482 \pm j0.0043 \\ -0.5516 \pm j0.0283 \\ -0.0317 \pm j0.0041 \\ -0.0077 \pm j0.0284 \\ -0.413 \pm j0.0977 \end{bmatrix} \\
 &= -0.1772 \mp j0.0114
 \end{aligned}$$

From Eq. (2.104), the sensitivity of electromechanical oscillation modes to the coefficient of damping torque provided by the PSS can be computed to be

$$\begin{aligned}
 \frac{\partial \bar{\lambda}_{6,7}}{\partial D_{\text{pss}}} &= -\frac{\bar{\mathbf{w}}_{62,72} \bar{\mathbf{v}}_{62,72}}{\mathbf{M}} = -\frac{(-0.2586 \mp j27.632)(-0.0016 \pm j0.0188)}{7} \\
 &= -0.0743 \mp j0.0056
 \end{aligned}$$

At the complex frequency $\bar{\lambda}_{6,7} = -0.71 \pm j8.44$, the forward path can be calculated to be

$$\begin{aligned}
F_{\text{pss}}(\bar{\lambda}_{6,7}) &= -\mathbf{a}_{23-8}^T (\mathbf{sI} - \mathbf{A}_{33})^{-1} \mathbf{b}_{\text{pss}-3} \\
&= [0 \quad 1.425 \quad 3.283 \quad -0.617 \quad -0.729 \quad -2.698] \\
&\quad \left(\begin{array}{c} \left[\begin{array}{cccccc} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{array} \right] \\ (-0.71 \pm j8.44) \end{array} \right)^{-1} \begin{bmatrix} 10000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&\quad - \begin{bmatrix} -100 & 6116.3 & -1404.6 & -2646 & -3127 & 1154.5 \\ 0 & -6.65 & 478.4 & 2.88 & 3.40 & -135.03 \\ 0 & 513.64 & -5.48 & 86.29 & 101.98 & 4.50 \\ 314.16 & 0.43 & 0 & -1.22 & 0.71 & 0 \\ 0 & 1.36 & 0 & 1.88 & -3.49 & 0 \\ 0 & 0 & 36.0 & 0 & 0 & -46.8 \end{bmatrix} \\
&= -2.3823 \pm j0.0253
\end{aligned}$$

Hence,

$$\begin{aligned}
R_{6,7} &= -\frac{\partial \bar{\lambda}_{6,7}}{\partial D_{\text{pss}}} F_{\text{pss}}(\bar{\lambda}_{6,7}) = -(-0.0743 \mp j0.0056)(-2.3823 \pm j0.0253) \\
&= -0.1772 \mp j0.0114
\end{aligned}$$

It is thus confirmed that the residue is equal to the forward path of the PSS multiplied by the sensitivity of oscillation modes to the coefficient of damping torque contribution from the PSS.

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