Chapter 2
20th century physics: 1900–1933

The evening allocution of the Royal Institution of Great Britain on Friday April 27th, 1900, was entrusted to one of the most outstanding international scientific figures: Lord Kelvin. His speech was entitled "19th-century clouds over the dynamical theory of heat and light."[1] According to the celebrated physicist, there were two "clouds": the first was the failure of all attempts to detect the "ether wind," while the second was the energy equipartition theorem, whose applications to some physical phenomena were at variance with the empirical data. Kelvin's worries were fully justified, and stressed the importance and urgency of two basic theoretical issues. But these were also the consequences of the amazing scientific achievements of the century that was about to close: Maxwell's electromagnetic theory and Boltzmann's interpretation of thermodynamics in terms of the statistical analysis of molecular motion.

Kelvin's attempts to disperse the clouds originated the two different, albeit amply intersecting, main areas of physical research in the 20th century: relativity theory and quantum mechanics. These two fields of investigation basically correspond to two different scales of phenomena: large speeds and/or hugely massive stellar systems for the first, and the basic constituents of matter for the second. In this chapter we shall reconstruct the historical roots of our present-day scientific knowledge. As we already anticipated, we shall be concerned with global maps, made of shared knowledge and structured by some links that are different from the unstable local dynamics of the various specific investigations. We shall mainly pay attention to the researches on the structure of matter, whose main tool was quantum mechanics, just because this is the area of investigation where Fermi was most active.

[1]W. Thomson (Lord Kelvin), *Nineteenth Century Clouds over the Dynamical Theory of Heat and Light,* Philosophical Magazine 2 (1901), p. 1.

G. Bruzzaniti, *Enrico Fermi*, Springer Biographies,
DOI 10.1007/978-1-4939-3533-8_2

2.1 The roots of the relativistic program

The theory of relativity is a consequence of the systematization of electromagnetism done by James Clerk Maxwell at the end of the 19th century. Maxwell's celebrated equations raised deep questions whose solution required a radical revision of the (often rather intuitive) concepts underlying the 19th century physics.

The term "Maxwell equations" refers to a set of four equations that resume the properties of the electric and magnetic field. They express the following facts:

a) the way the electric field depends on its sources (the electric charges), namely, that electric charges interact among them with a force which is inversely proportional to the square of their distance;
b) magnetic monopoles (magnetic charges) do not exist;
c) a varying magnetic field produces an electric field;
d) the way the magnetic field depends on its sources (the electric currents), and the fact that a varying electric field produces a magnetic field.

One of the main consequences of Maxwell equations is that they prove the existence of electromagnetic waves, i.e., the fact that electric and magnetic field can propagate in space. One can intuitively understand this considering an oscillating electric charge; it produces a varying electric field that, according to the fourth equation, generates a varying magnetic field, which, as postulated by the third equation, produces a varying electric field, and so on. The surprising fact is that Maxwell's equations allow one to compute the speed c with which electromagnetic waves propagate in space. It is a value close to 185,790 miles per second, and coincides with the experimentally measured value.

It is hard to overestimate the historical importance of Maxwell's work, which eventually clarified the physical nature of light. However one major issue was still quite unclear. Light is a wave propagating with a certain speed; since this statement makes sense only after specifying a reference frame, in which reference frame does light propagate with speed c? At the end of the 19th century this question had a natural answer: ether. To understand the meaning of this mysterious term, we can start from the entry "Ether" written by Maxwell for the 9th edition of the *Encyclopaedia Britannica.* There ether was defined as "a material substance of a more subtle kind than visible bodies, supposed to exist in those parts of space which are apparently empty."[2] It is a rather generic definition, but the ambiguity was unavoidable. Ether played different roles and designated substances whose ontological status would have changed very rapidly. According to Maxwell himself, "æthers were invented for the planets to swim in, to constitute electric atmospheres and magnetic effluvia, to convey sensations from one part of our bodies to another,

[2] J. C. Maxwell, "Ether," *Enciclopedia Britannica*, 9th ed., VIII (1878). Reprinted in *The Scientific Papers of James Clerk Maxwell,* 2 vols., Dover, New York 1965, II, p. 763.

and so on, till all space had been filled three or four times over with æthers."[3] Some of these supposed features were gradually forgotten, but still the problem of the ether has been very important in the physics of the 19th century, as witnessed by the production of a large number of papers, especially in the years 1871–75 and 1891–95.[4]

Maxwell's impressive construction of the theory of the electromagnetic fields takes place within this scenario. Nowadays we know, as remarked by Feynman, that

> [. . .] what counts are the equations themselves and not the model used to get them. We may only question whether the equations are true or false. This is answered by experiments, and untold numbers of experiments have confirmed Maxwell's equations. If we take away the scaffolding he used to build it, we find that Maxwell's beautiful edifice stands on its own.[5]

Maxwell's scaffolding made reference to a mechanical model of the ether. While this was used more as an illustrative model than a factual explanation, yet one cannot underplay its basic role in Maxwell's research. As written in the above cited article in the *Encyclopaedia Britannica,*

> Whatever difficulties we may have in forming a consistent idea of the constitution of the aether, there can be no doubt that the interplanetary and interstellar spaces are not empty, but are occupied by a material substance or body, which is certainly the largest, and probably the most uniform body of which we have any knowledge.[6]

So the ether, this universal *plenum* which filled the empty space, air and everything else existing in the universe, was regarded as the absolute reference frame, a reference frame distinguished from all others, in which the electromagnetic waves, light, and the radiating heat propagate with the speed of 185,790 miles per second.

Maxwell's deduction of a finite speed of propagation of the electromagnetic waves, which seemed to imply the existence of an absolute reference frame, had a shattering effect on the theoretical framework of the 19th century's physics. It is a common experience that no mechanical experiment can detect a rectilinear uniform motion: if we are on a car or train and move on a straight path with constant speed, everything we do happens exactly the same way as when the car or train stands still. This easy but very important consideration bears the name of "Galilean relativity principle," as it was indeed Galilei who first mentioned it in a famous page, some parts of which deserve to be quoted here:

> Shut yourself up with some friend in the main cabin below decks on some large ship and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a vessel

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[3] *Ibid.*

[4] Cf. T. Hiroshige, *The ether problem, the mechanistic worldview and the origin of the theory of relativity,* Historical Studies in the Physical Science 8 (1976), p. 3.

[5] R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics,* Addison Wesley, California Institute of Technology 1963. Ch. 18, p. 3.

[6] J. C. Maxwell, *op. cit.*

> [with a narrow opening][7] beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air.
> [. . .]
> The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it and to the air also.[8]

If Galilei had known Maxwell's surprising result and had agreed on the existence of ether as an absolute reference frame, his conclusions would have been very different. If we travel on a highway at 40 miles per hour and see a car that is overtaking us at 60 miles per hour, we see it traveling at 20 miles per hour; if the same principle applied to electromagnetic phenomena, and the two travelers on Galilei's ship had measured the speed of a light ray under different conditions of motion of the ship, they would have obtained different results.

In 1881 Albert Michelson made one of the most celebrated experiments in the history of physics, aiming at revealing the Earth's motion with respect to the ether. It was necessary to achieve a very high precision; the point was to measure the effect due to the different traveling times of two light rays, one propagating in an arm of the apparatus parallel to the speed of the Earth, and one in an arm perpendicular to the first. If we denote by v the speed of the Earth, and c the speed of light, both with respect to the ether, theoretical calculations show that the effects are of the second order, i.e., of the same order of magnitude as the ratio v^2/c^2. Since the value of c is about 185,790 miles per second, while v is about 18 miles per second, the precision of the measurement had to be of one part over a hundred millions. That is, the ratio

[7] Drake translates the Italian "vaso di angusta bocca" into "wide vessel" instead of "vessel with a narrow opening," probably confusing "angusta" with "augusta."

[8] G. Galilei, *Dialogo sopra i due massimi sistemi del mondo.* Translated by S. Drake, *Dialogue Concerning the Two Chief World Systems,* University of California Press, Berkeley, 1953. p. 187.

between the weights of a piece of confetti and a car. The optical apparatus built by Michelson was capable of that performance, but the experiment gave a negative result: the speed in the two directions was the same. After the encouragement of two of the most reputed physicists of the time, Lord Kelvin and Lord Rayleigh, the measurement was repeated in 1887 by Michelson and Morley, but the result was once more negative.

The experiment was redone over and over till 1930, at least 13 times, and the result was always the same. Even with Michelson and Morley's apparatus, the two travelers on Galilei's ship could not have decided if the ship was standing still or was uniformly moving.

2.2 Special relativity

In 1905 Einstein provided his version of the story: the experiments gave a negative result for the simple reason that the ether does not exist. If so, what is the reference frame where the electromagnetic waves have speed c, as predicted by Maxwell's equations? Easy, said Einstein — that's the speed as measured in any inertial reference frame.[9] In other terms, a new law of nature was discovered: the speed of the electromagnetic waves, and so the speed of light, does not depend on the motion of the source.

Einstein's special relativity theory is based on the following postulates.

1. The Galilean relativity principle must hold for all physical phenomena, and not only for the mechanical ones, that is, *all* physical laws have the same form in all reference frames in relative uniform motion.
2. The speed of light in empty space does not depend on the motion of the source or of the receiver, in the sense that it is the same in all reference frames that move uniformly with respect to the source.

The second postulate is highly counterintuitive. We are accustomed to extend the explanation of everyday life's phenomena also to situations that are far away from it, and it seems natural to us that the same operation we make when we evaluate the speed of a car should apply also when something is moving at the speed of light. But this does not seem to be true.

Let us go back to the example with the cars. Let V be the speed of our car along a straight section of a highway, and let v' be the speed of a car that is overtaking us, with respect to us. What is the speed v of the car that is overtaking us? According to classical mechanics, the answer is very easy: $v = V + v' = 40 + 20 = 60$ miles per hour. According to Einstein's relativity theory, the formula we have used is just an approximation of the correct formula, which reads

[9] An inertial frame is a reference frame where any small body which is far away from any other matter — and is thus free from interactions — moves in a uniform rectilinear motion. The relative motion of two inertial frames is a uniform rectilinear motion.

$$v = \frac{V + v'}{1 + \dfrac{Vv'}{c^2}},$$

where c is the speed of light. The approximation $v = V + v'$ is actually very good, since the correction term Vv'/c^2 is appreciable only for speeds close to the speed of light. In our case indeed its value is 0,000000000000003. If our speed is half the speed of light, and a car overtakes us with the same speed relative to us, then the classical formula gives for the speed of the second car $c/2 + c/2 = c$, while the (correct) relativistic formula yields 0,8 c.

One may wonder what is the rationale of this strange way of summing speeds. This funny formula actually follows from a radical revisitation of the concepts of space and time, which is necessary for the speed of light to be the same in all reference frames. Two other celebrated consequences of this revolutionary approach are the *contraction of lengths* and the *dilation of time.* From a classical viewpoint, the length of a segment and the duration of a time interval do not depend on the state of motion of the observer; according to the theory of relativity, on the contrary, they do depend. So, if ℓ_0 is the length of a segment as measured by an observer for which the segment does not move, and t_0 is the interval of time between two events that for that observer happen at the same position, for an observer in relative motion with speed v, the length will be $\ell = \ell_0/\gamma$, and the interval of time $t = \gamma t_0$, where

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}.$$

The correction factor γ is greater than or equal to 1, and increases for bigger values of v; thus the motion of the observer induces a contraction of the lengths and a dilation of the durations. Also these effects depend on the ratio v^2/c^2 and so we do not perceive them in everyday's experience, as the speeds are very small with respect to the speed of light.

Such a drastic change of the concepts of space and time that were at the basis of Newtonian physics required a radical reformulation of the basic laws of mechanics. According to the theory of relativity, also the mass cannot be regarded as an unchangeable characteristic of a body; it is rather a quantity that increases with speed, according to a formula analogous to the previous ones: if m_0 is the mass of a body at rest, when the body moves with speed v, we have $m = \gamma m_0$. This result is a particular case of a more general relation between mass and energy, which is most likely the most extraordinary feature of the theory of relativity: the two quantities, mass and energy, can transform one into the other. If a body absorbs an energy E, its mass increases by an amount m (and, *vice versa,* if the mass decreases by an amount m, an energy E is released) according to the celebrated formula $E = mc^2$. This effect is terribly evident in the atomic bombs. The 20 kiloton bomb dropped on Hiroshima released the same amount of energy as due to the explosion of 20,000 tons of TNT; to get that enormous amount of energy it has been enough to convert into energy just 1 gram of the mass of the fissile material.

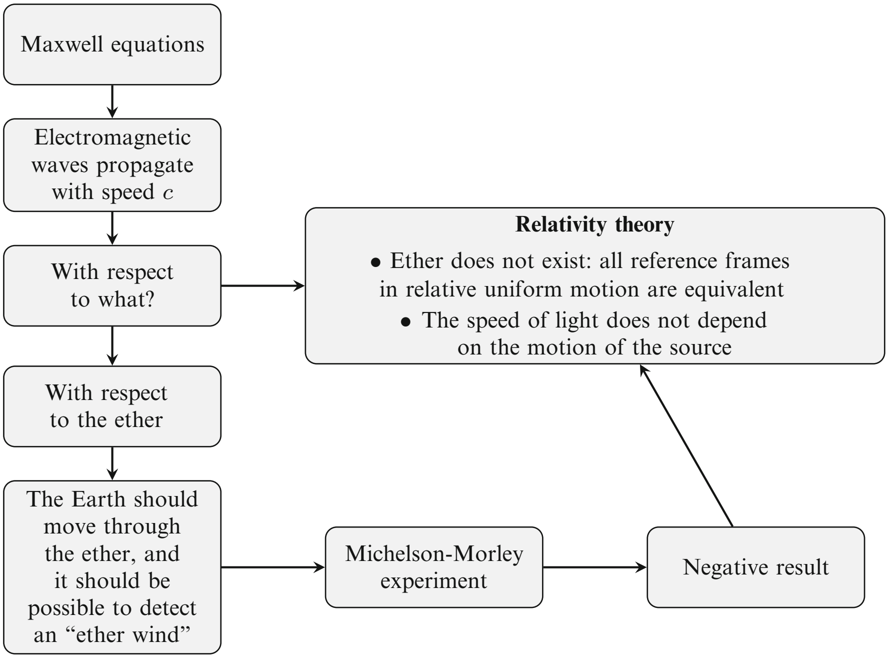

Fig. 2.1 The origins of the theory of relativity.

2.3 A note on global maps

As we have seen, the Michelson-Morley experiment played a central role in the birth of the theory of relativity, as a kind of logical premise to it. We can sketch the logical dependence among the various arguments as in Figure 2.1. One could ask if this is really the path followed by Einstein to formulate his theory; in particular, what was the role of Michelson-Morley's experiment? Specialists are still debating on this issue, and the arguments one can put forward are contradictory. In his original 1905 paper Einstein makes explicit reference — albeit in a few words, and without citing Michelson and Morley's papers — to 'the unsuccessful attempts to discover any motion of the earth relatively to the "light medium."'[10] In other circumstances, when he was interviewed and made some declarations, his attitude was more contradictory.[11] We do not mean here to enter into details about this issue; however, it is worth mentioning this question as it exemplifies very well the difference between the reconstruction of the global maps and of the local research

[10] H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, with notes by A. Sommerfeld, *The principle of relativity; a collection of original memoirs on the special and general theory of relativity*, Dover Publications, New York 1923, p. 37

[11] A. Pais, *Subtle is the Lord,* Clarendon Press — Oxford University Press, Oxford–New York 1982.

itineraries. It is only about the latter that the influence exerted by the Michelson and Morley experiment on Einstein's ideas is a meaningful problem. If we are interested to the reconstruction of the global maps, i.e., the network connecting the objective scientific problems and not the winding paths often followed by the scientists, then the question becomes utterly uninteresting.

2.4 General relativity

It is an extraordinary and fascinating aspect of the scientific enterprise that the most important mechanism of the scientific development is not the solution of existing problems, but rather the identification of new ones; this is the true engine of knowledge. Special relativity is no exception; if Einstein's 1905 theory solved the problems brought up by the Maxwellian synthesis of the electromagnetic theory, it also raised deep questions about one of the most successful theories of our time, Newtonian gravitation. This is expressed by the equation

$$F = G\frac{m_1 m_2}{d^2},$$

where F is the force between two bodies having masses m_1 and m_2 (mass, or to be more precise, gravitational mass, is an intrinsic property of bodies) at a distance d, and G is a constant. From the viewpoint of relativity theory, this equation has a problem: the gravitational force propagates instantaneously, that is, with infinite speed. Suppose that the bodies are located at a distance of several million miles, and that for some unknown reason, the mass m_1 doubles; according to Newton's law, the body with mass m_2 should immediately feel a force twice as big. How can this happen, if the special relativity theory singles out the speed of light c as an absolute upper bound for any speed, be it of a body, or of a signal?

This contradiction was solved by Albert Einstein in 1907. As he tells, while he was sitting at his desk at the Patent Office in Bern, he had the "happiest idea" of his life: he realized that "if a person falls freely he will not feel his own weight."[12] A seemingly simple idea, which however opens the way to the general theory of relativity. Einstein's "happiest idea" is fully formulated as the so-called "equivalence principle," which is the starting point of the general theory of relativity, and is empirically based on the proportionality between the inertial and the gravitational mass (see Appendix C.1). We can understand what that means from Einstein's words:

[12]There are two sources for this anecdote. The first is an unpublished paper written in 1921, known as the "Morgan Manuscript," which is at the Pierpont Morgan Library in New York. The second is a talk given by Einstein at the University of Kyoto in 1922. The two sentences as reported here are from A. Pais, *op. cit.,* p. 179.

> Imagine a great lift at the top of a skyscraper much higher than any real one. Suddenly the cable supporting the lift breaks, and the lift falls freely toward the ground. Observers in the lift are performing experiments during the fall. In describing them, we need not bother about air resistance or friction, for we may disregard their existence under our idealized conditions. One of the observers takes a handkerchief and a watch from his pocket and drops them. What happens to these two bodies? For the outside observer, who is looking through the window of the lift, both handkerchief and watch fall toward the ground in exactly the same way, with the same acceleration. We remember that the acceleration of a falling body is quite independent of its mass and that it was this fact which revealed the equality of gravitational and inertial mass.
> [. . .]
> We also remember that the equality of the two masses, gravitational and inertial, was quite accidental from the point of view of classical mechanics and played no role in its structure. Here, however, this equality reflected in the equal acceleration of all falling bodies is essential and forms the basis of our whole argument. Let us return to our falling handkerchief and watch; for the outside observer they are both falling with the same acceleration. But so is the lift, with its walls, ceiling, and floor. Therefore: the distance between the two bodies and the floor will not change. For the inside observer the two bodies remain exactly where they were when he let them go. [13]

This thought experiment explains Einstein's idea very well: there is no difference between the results of an experiment made by an observer inside a freely falling elevator, and those obtained by a hypothetical observer located in an identical elevator that is standing still in some region of the intergalactic space, so far form any other body that it feels no gravitational force. Moreover, let us imagine that the elevator that is isolated in space is hauled by a rope attached to its ceiling, so that it moves with an acceleration of 32.2 feet per squared second, the same of a falling object near the Earth's surface. If the observer in the elevator stands on a scale, she would read the same weight as if the elevator was standing still on the Earth's surface, and she would see the objects fall with the same acceleration as objects fall in the elevator at rest on the Earth's surface. In other words, the observer has no way to know if she is under the action of the terrestrial gravitational field, or she is accelerated by a force.

One should remark that this reasoning would not be valid if the inertial mass was not the same as the gravitational mass (better said, if they were not proportional). If that was not the case, indeed, the observer could detect the presence of a gravitational field by noting that different bodies fall with different accelerations. Let us also remark that the size of the elevator cannot be too big. As Einstein suggests, let us think of an imaginary elevator which extends from the North Pole to the Equator. In this situation the handkerchief and the watch, if they were located one at the North Pole and the other at the Equator, would fall with different accelerations — since the gravitational fields at the North Pole and the Equator are different — allowing the observer to detect the presence of gravity.

[13] A. Einstein, L. Infeld, *The Evolution of Physics,* Cambridge University Press, London 1938, pp. 226–227.

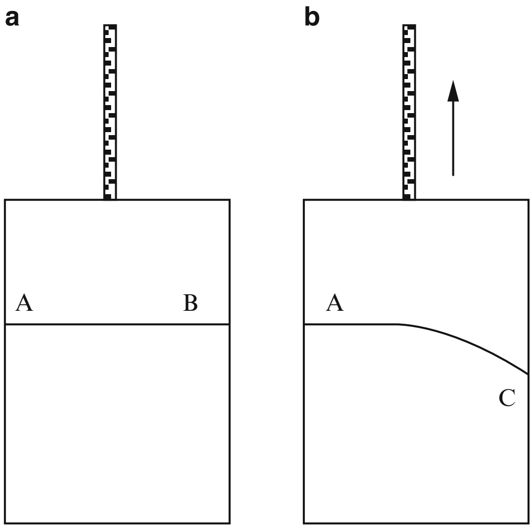

Fig. 2.2 The elevator in (**a**) is standing still in a gravitational field. The one in (**b**) is not subject to a gravitational field but is acted on by a force which accelerates it upward.

This "local equivalence principle" between gravitational fields and accelerated reference frames is the basis to extend the relativity principle: the physical law not only must have the same form in reference frames in relative uniform motion — they must be the same in *all* reference frames.

Let us analyze another thought experiment, and let us have a look at Figure 2.2. The elevator in (a) is at rest in a gravitational field, while in (b) the elevator is isolated in space and moves with an accelerate motion, dragged by a force applied to the ceiling by a rope. If the equivalence principle holds true, the observers inside the elevators cannot distinguish between the two situations. Think now that a light ray enters the elevator at the point A. How is this fact described in the two situations? At first it seems there is a violation of the equivalence principle. Indeed the observer in (a) sees a light ray moving along a straight line, entering the elevator in A and hitting the opposite wall in a point B exactly opposite to A, while (b) sees a curvilinear motion, since while the ray crosses the elevator, the latter has moved up.

But Einstein objected that this reasoning hides a serious mistake: also the observer (a) sees a light ray moving along a curvilinear path. Indeed light carries energy, hence has a mass, and is therefore subject to the action of the gravitational field, so that its trajectory curves toward the bottom of the elevator. There is no way to distinguish between the situations (a) and (b), and the equivalence principle holds.

This analysis is subject to experimental observation; it is enough to measure the position of the stars in the presence or the absence of the Sun. If the equivalence principle holds true, when the starlight passes near the Sun, it must be deflected; it should therefore be possible to detect a difference in the position of the stars according to whether their light grazes the Sun surface or not. In normal conditions this observation is impossible, as the strong solar light hides the stars, but it can be done during a total eclipse, when the Sun is present, but is obscured by

Moon, and the starlight which grazes the Sun surface is visible. On 29 May 1929 two British expeditions, one to Brazil led by Andrew Crommelin and one to St. Thomas and Prince, led by Arthur Eddington, made the measurement, during a total eclipse which was particularly suitable for the observations. The analysis of the photographic plates took a long time, but eventually on 6 November, an announcement was made at a common meeting of the Royal Society and the Royal Astronomical Society: "A very definite result has been obtained, that light is deflected in accordance with Einstein's law of gravitation."[14] The title at page 11 of the November 7 issue of *The Times* read "Revolution in science/New theory of the universe/Newtonian ideas overthrown."[15] Einstein's fame spread all over the world, and he became a myth and a legend.

Why all this fuss? Why Newton's conceptions had been demolished? If light carries energy and therefore, in accordance with the special theory of relativity, it also carries mass, why the classical theory is not enough to explain that light is deflected by a gravitational field? Actually this is what happens, but the problem is that the Newtonian theory only accounts for exactly half the deflection predicted by Einstein's theory. The plates made by the expeditions in Brazil and Guinea left no room for doubt: the measured deflection agrees with Einstein's prevision. But what is the "revolution" announced by *The Times*? Special relativity had already radically changed the standard conceptions of space and time. General relativity gave another blow to common sense, showing that the space and time intervals not only depend on the motion of the observer, but also on the location where the measurement is made. General relativity shows that the physical space is not "flat," i.e., it is not Euclidean, and that *the gravitational force is due to space-time curvature.*

The notion of space-time curvature is the true link between the equivalence principle and general relativity. To understand this point let us once more follow Einstein. Imagine a great disc (Fig. 2.3) on which two concentric circles have been drawn, one very small and the other very large. In our reference frame the disc is rapidly rotating. We use a rule to measure the lengths of the radii and the circumferences of the two circles, and find the well-known value 2π. What would be the result obtained by an observer on the disc? We assume she would be using the same rule. If classical mechanics — according to which the lengths and the time interval do not depend on the motion of the observer — was right, she would find exactly the same result. But as prescribed by the theory of relativity, there is a phenomenon of contraction of lengths. This effect depends on the speed, and to be more precise, only on the projection of the velocity along the direction of the length we measure. So the observer on the disc finds the same values of the radii, as they are orthogonal to the velocity of the rotating frame, while for the circumferences she finds different values, since the length of the rule has undergone a contraction. Actually for the small circumference she finds a value practically identical to ours, since the speed in this case is small (as the small circumference is not far for the

[14] A. Pais, *op. cit.,* p. 305.

[15] A. Pais, *op. cit.,* p. 307.

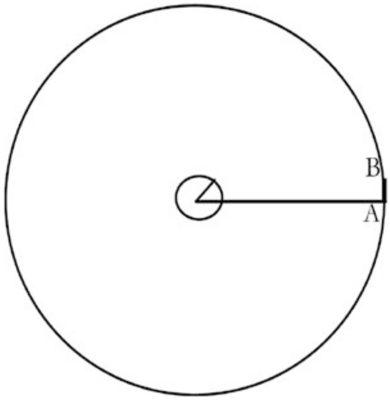

Fig. 2.3 The observer on the rotating disc uses the rule AB to measure the radii and the circumferences of the two circles.

center of rotation), while for the bigger circumference she finds, if the radius is big enough, an appreciably greater value. As a consequence, the ratio between the circumference and its radius, as measured by the observer on the disc, is no longer 2π, but rather a greater number, which increases with the value of the radius. This is a stunning result: all points in the bigger circle have the same distance from the center, as in any circle, yet the ratio between the length of the circumference and the distance of its points from the center is greater than 2π. One of the most celebrated and indisputable results of the Euclidean geometry seems to be no longer valid.

The remark that an accelerated observer has a different perception of the geometry of the physical space and of time, together with the equivalence principle, directly leads to general relativity. Indeed, if the equivalence principle holds, what happens in an accelerated frame of reference should also happen to an observer who is at rest but is acted upon by a gravitational field. Thus the presence of matter modifies the geometry of the space-time continuum; the distortions of the space and time intervals between two events depend not only on the state of motion of the latter, but also on their position with respect to matter. After three centuries the mechanism of gravity was understood: it is the curvature of space-time.

The mathematics of general relativity is very complicated, and it took many years for Einstein to reach a satisfactory formulation of the theory: from 1907's "happiest idea" to 1916, when the fundamental paper, containing the equations that relate the curvature of space-time to the matter distribution, was published.[16] It is not easy to talk about Einstein's equations without resorting to the mathematical formalism, whose conceptual content can be hardly visualized in an intuitive way. We are in a way like the inhabitants of Flatland (see Appendix C.2), who cannot conceive and describe an event involving a three-dimensional object (the arrival in

[16] A. Einstein, *Die Grundlagen der allgemeine Relativitätstheorie,* Annalen der Physik 49 (1916), pp. 769–822. English translation: *The Foundation of the General Theory of Relativity,* in H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, *The Principle of Relativity; a Collection of Original Memoirs on the Special and General Theory of Relativity,* Dover, New York 1952, p. 109.

their world of a sphere); in the same way, our intuition is unable to represent four-dimensional objects. We can however resort to some simplifications; for instance, we can neglect the time dimension, and consider a two-dimensional universe. In the absence of matter, there is no difference between the Newtonian and the Einsteinian pictures; in both cases, space is flat and its geometry is Euclidean. However, what happens if space contains a massive object? Using a very common imagery, it is like placing an iron ball on a stretched fishing net, which then undergoes a deformation; a heavier ball will produce a larger deformation. Thus, the relativistic picture of the phenomenon of gravitation is radically different from the Newtonian one. In Einstein's conception there is no room for the "gravitational force," which is replaced by the space-time curvature. Material bodies do not interact by means of a mysterious force, which is transmitted at infinite speed in a space which is just the infinite and inert container of all things existing, but rather by way of the reciprocal deformation of the space-time weave that each body generates in the point where the other is located.

Also the description of the way that matter is set in motion by gravitation is changed. For instance, what determines the trajectories of the planets in the solar system? According to both Newtonian mechanics and general relativity, in the absence of interactions the bodies move along straight lines. But in the presence of matter, and therefore in a space-time with a non-flat geometry, the straight lines are no longer the usual ones, but rather curves, called *geodesics.* The trajectories of the planets in the solar system are exactly the projections of these space-time curves onto the three-dimensional physical space.

The mass-induced space-time curvature and the motion of bodies along geodesics are two basic and tightly interrelated aspects of general relativity. As stated by the famous physicist John Wheeler, "mass grips space by telling it how to curve, space grips mass by telling it how to move."[17]

The imagery of the fishing net can also help us to deal with the contradiction between the instantaneous propagation of the gravitational force and the upper bound to the speed given by c. According to general relativity, the variation of a mass produces a perturbation which propagates in space exactly as the perturbation produced by throwing an iron ball on a fishing net propagates through it. And the speed of propagation of the perturbation, as computed by Einstein, is exactly c.

2.5 The quantum program

The history of quantum mechanics is long and tortuous. The "quantum of action" was introduced in 1900 by Max Planck, but it was not before 1925–27 that a rather heterogeneous complex of ideas and conjectures coalesced into a coherent formalism called "quantum mechanics." In contrast to the theory of relativity,

[17] In B. Greene, *The Elegant Universe,* W. W. Norton & Company, New York–London 1999, p. 85.

quantum mechanics was not the result of the efforts of a single scientist, but rather the implementation of a program to which many researchers contributed, coming from different schools. It was actually the birth of a new research style.

In the early 20s Niels Bohr reunited in his famous Institute in Copenhagen the best talents of theoretical physics, coming from many countries: Wolfgang Pauli, Werner Heisenberg, Lev Davidovič Landau, Paul Ehrenfest, Oskar Klein, Hendrick Kramers, George Gamow, and many more. It was the first example of a new approach to theoretical research, which, from a solitary pursuit, became a collective enterprise. Quantum mechanics was the product of the common efforts of these scientists. Its birth did not take place in a precise moment, in a rigorous and definitive form, and was not the resolutive answer to a specific, well-defined problem. The formulation of the quantum theory was rather a complicated and complex process, and its interpretation has given rise to heated disputes. One should also remark that quantum mechanics was born and developed together with atomic and nuclear physics, providing the language and the concepts to be used to study the structure of matter. According to Victor Weisskopf, a most authoritative physicist of that period:

> [Quantum mechanics] has been a leap into the unknown. With it we enter a world of phenomena which cannot be described in terms of the physics of the previous century. To construct and develop it, it has been necessary to look for new formulations and ways of thinking. It has opened the way to the understanding of the world of atoms and molecules, with its discrete quanta of energy and its spectra and chemical bounds. One can say that the beginning of this century has seen a radical change in the nature of the physical theory, and this change took place with quantum mechanics.[18]

In a very schematic way, one can divide the birth of the quantum theory into three stages, that also correspond to three levels of the investigation on the structure of matter: the first stage *(Old Quantum Theory)* is the birth of atomic physics; the second (quantum mechanics) is the development of atomic physics and the birth of nuclear physics; and the third (quantum field theory) is the beginning of elementary particle physics. In this chapter we shall reconstruct the global maps corresponding to the first two stages of development of the quantum program.

2.6 From radiation physics to atomic physics

"The foundation stones of the material universe [. . .] continue this day as they were created — perfect in number and measure and weight."[19] James Clerk Maxwell wrote these words at the end of the 19th century, referring to the atoms, the elementary constituents of matter. However, this was just a rough image, still rooted

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[18]Translated from the French original, V. Weisskopf, *La révolution des quanta,* Hachette, Paris 1989. p. 20.

[19]J. C. Maxwell, *A Discourse on Molecules,* Philosophical Magazine, 46 (1873), p. 468.

in a naive interpretation of the physical experience. The term "atom" indeed referred to indivisible entities that displayed at the microscopical level the observed macroscopic properties of matter. This is proved, for instance, by the criterion used to order the elements in the periodic table; as in common experience weight is one of the main properties of the material bodies, in Mendeleev's table the chemical elements were ordered by increasing atomic weight.

However, starting in the first few years after 1895, the 19th century conception of matter began crumbling. The ontological status of the atom changed radically: from being an unchangeable entity, it became a complex structure that can be split into its elementary constituents. The starting point of this process can be traced to what was at the time a very active field of the research, namely, radiation physics, and in particular the discovery of the cathodic rays, the X-rays and radioactivity.

Figure 2.4 summarizes the historical process that led from radiation physics to the investigations on the structure of matter based on the conception of a nuclear atom. The discovery of cathodic rays in the mid 19th century set off a lively controversy about their nature. The strange rays emitted by the cathode of a discharge tube when a deep vacuum was created in it were waves or particles? This question was just one element in a complicated conceptual scheme, which also touched even deeper issues about the relations between electricity and matter and the concept of ether. However, before the controversy about the cathodic rays could be solved, the scientific community was struck by two surprising discoveries. In 1895 Wilhelm Röntgen announced that the glass wall of a discharge tube opposite to the cathode, when hit by the cathodic rays, emits other rays, of an unknown nature (and indeed called "X-rays").[20]

These rays were able to cross bodies that are opaque to ordinary light, and to impress a photographic plate. According to the apt definition by Louis Olivier,[21] the photography of the invisible was born.

In 1896 Henri Becquerel[22] was investigating the relations between the phosphoresce of the natural substances and the X-rays. This was motivated by the fact that the emission of the X-ray was localized exactly in the area of the discharge tube that became phosphorescent. He discovered that minerals containing uranium emit

[20] W. Röntgen, *Über eine neue Art von Strahlen,* Sitzungberichte der physikalische-medikalische Gesellschaft Würzburg, December 1895, p. 132. This was followed by a second communication, published in 1896 in the same journal. The two papers were partially translated in English in *Nature* 53 (1896). The original text with a translation into English was also published in E. C. Watson, *The discovery of X-rays,* American Journal of Sciences 13 (1945), No. 5, pp. 281–291.

[21] L. Olivier, *La photographie de l'invisible,* Revue général de sciences pures et appliquées 7 (1896), No. 49, p. 2.

[22] H. Becquerel, *Sur quelques propriétés nouvelles des radiations invisibles émises par divers corps phosphorescents,* Comptes Rendus de l'Académie des Sciences 11 (1896), p. 559; *Sur les radiations invisibles émises par les corps phosphorescents,* ibid., p. 501; *Sur les propriétés différentes des radiations invisibles émises par les sels d'uranium, et du rayonnement de la paroi anticathodique d'un tube de Crookes,* ibid., p. 762.

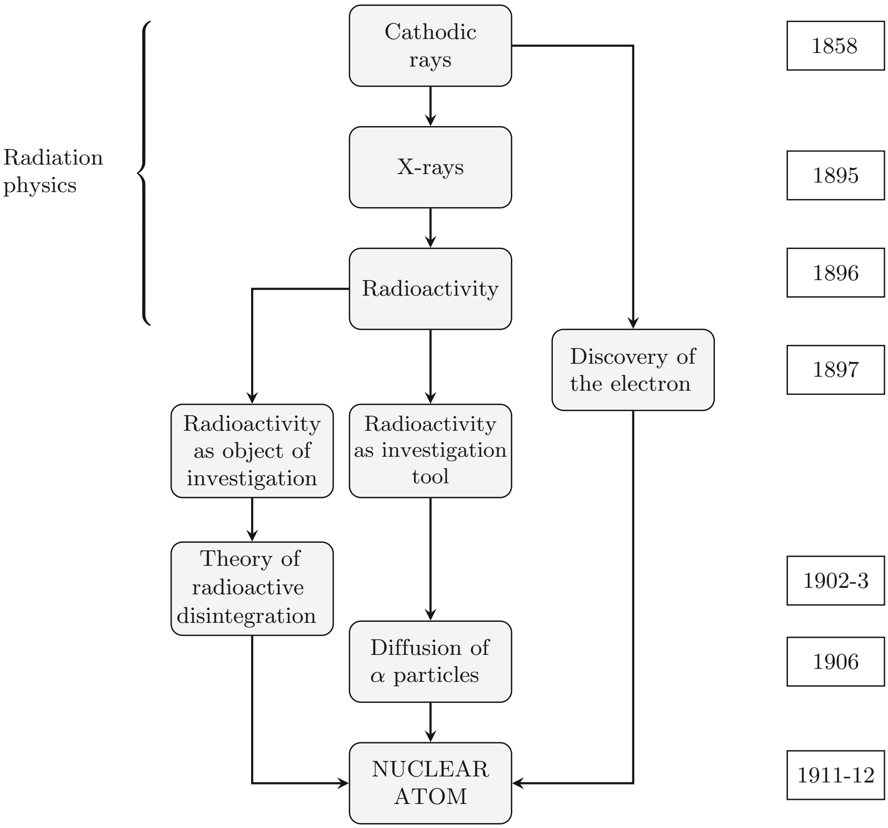

Fig. 2.4 A sketch of the line of research leading from radiation physics to the concept of nuclear atom.

penetrating radiations, having similar effects to the X-rays. Thus there were three different radiations of unknown nature: cathodic rays, X-ray, and uranium rays.[23]

The years between 1897 and 1903 were crucial. During that time span the mystery of the new radiations was deciphered; the nature of the cathodic and X-rays was understood, and to some extent, also that of the uranium rays. But more than that, during that period a process started, that would lead to a radical redefinition of the concept of atom. The main features of that process can be summarized in four stages:

1. In 1897 J. J. Thomson solved the problem of the cathodic rays by showing, beyond any doubt, that they are deflected by the electric and magnetic fields,

[23]This historical period has been analyzed in detail in G. Bruzzaniti, *Dal segno al nucleo* [From sign to nucleus], Bollati Boringhieri, Torino 1993.

as only the electrically charged particles do. The most significant success of Thomson however was the measurement of the mass/charge ratio of the particles that form the cathodic rays.[24] The result was surprising. In Thomson's words: "its value 10^{-7} is very small compared with the value 10^{-4}, which is the smallest value of this quantity previously known, and which is the value for the hydrogen ion in electrolysis." This fact, supported by some results by Eduard Anton von Lenard (an important German experimental physicist), and most of all, by the measurements of John Sealy Edward Townsend (a bright student of Thomson's at the Cavendish Laboratory), which showed the equivalence between the charge carried by the cathodic rays and the hydrogen ions in electrolysis, allowed Thomson to assign to the particles forming the cathodic rays a mass of about 1/1700 of the mass of the hydrogen atom. The electron had been discovered.

2. In 1899 Marie Slodowska Curie discovered that thorium displays an activity similar to that of uranium.[25] In the same year, together with her husband, she isolated two new elements which were much more active of uranium and thorium: polonium and radium. This discovery[26] confirmed a very bold hypothesis they had made, namely, that radioactivity is the manifestation of a property of the atom.
3. Between 1898 and 1902 a series of investigations made by Ernest Rutherford,[27] and also by Becquerel and Villard, clarified the nature of the radiation emitted by the radioactive substances; they were of three kinds: α, β, and γ rays. The first were particles, with twice the electric charge of the electron, and four times the mass of the hydrogen atom. β rays were electrons, and the γ rays were electromagnetic radiation of very high frequency.
4. Between 1902 and 1903, Rutherford and Soddy advanced the first complete theory on the causes of the radioactivity, which was supposed to be a manifestation

[24] J. J. Thomson, *Cathode Rays,* Philosophical Magazine, 44 (1897), p. 310.

[25] M. Slodowska Curie, *Rayons émis par les composés de l'uranium et du thorium,* Comptes Rendus de l'Académie des Sciences 126 (1896), p. 1101.

[26] P. Curie and M. Curie, *Sur un substance nouvelle radio-active contenue dans la pechblende,* Comptes Rendus de l'Académie des Sciences 128 (1898), p. 175; E. Demarçay, *Sur le spectre d'une substance radio-active, ibid.,* p. 128.

[27] E. Rutherford, *Uranium radiation and the electrical conduction produced by it,* Philosophical Magazine 47 (1899), p. 109. Reprinted in J. Chadwick (ed.), *The Collected Papers of Lord Rutherford of Nelson,* 3 vols., Allen & Unwin, London 1962–1965, vol. 1, p. 1169 (henceforth we shall refer to this work as *CPR*). Rutherford had the merit of detecting the α and β components of the uranium radiation, and discovering in 1903 the particle nature of the α rays (*The magnetic and electric deviation of the easily absorbed rays from radium,* Philosophical Magazine 5 (1903), p. 177). Henri Becquerel on the other hand was the first to identify the β rays with electrons (*Influence d'un champ magnétique sur le rayonnement des corps radio-actifs,* Comptes Rendus de l'Académie des Sciences 129 (1899), p. 996; *Sur le rayonnement des corps radio-actifs,* ibid., p. 1205). Paul Villard, finally, detected in the radiation emitted by radioactive bodies a component that is not deflected by the electromagnetic fields: the γ rays (*Sur le rayonnement du radium,* ibid., 130 (1900), p. 1178).

of the disintegration of the atom.[28] In this connection the discovery of the corpuscular nature of the α radiation was of paramount importance. According to this theory, the atom of a radioactive substance that emits α and β rays undergoes a disintegration which transforms it into another chemical element. The latter can in turn disintegrate, emitting more α or β particles. One of the strongest tenets of the 19th century physics, the immutability of the chemical elements, was thus disproved.

Transformation theory — as the Rutherford-Soddy theory was called — and the discovery of the electron played a fundamental role in the investigations about the structure of matter, radically changing the image of the atom, which became a complex structure, formed by many components. This point deserves to be stressed. For the first time, albeit in an embryonal and quite implicit form, a regulating principle was introduced, that was to govern the first steps of nuclear physics: if a particle is emitted by some (nuclear or atomic) structure, then it is a constituent of that structure. This principle guided the investigations on nuclear physics and stood against the strongest anomalies, till it was demolished by Fermi's theory of the β decay.

Starting with 1903 research on radioactivity went along two different lines, which were correlated, but also independent as far as their methods and aims were concerned. On the one hand, there was radioactivity as an object of investigation *per se*, in relation to the changes of the concepts of atom and chemical elements that it induces. On the other hand, radioactivity played the role of a research tool; a radioactive substance was regarded just a source of rays. In this case the emission was not considered as a problem, and was studied only in relation to the interaction between radiation and matter. Within this second project, in 1906 the *scattering* of α particles was discovered: a very important phenomenon for the understanding of the structure of matter.

The first evidence of that phenomenon was observed by Becquerel, who noticed that a beam of α particles leaves different tracks on a photographic plate according to whether it is propagating in the vacuum or in the air. Rutherford[29] guessed that the α particles hit the atoms in the air and deviated from their rectilinear path. Rutherford's hypothesis was carefully checked by Geiger in 1908[30] by letting the α particles go through solid bodies, such as metals. The most surprising and unexpected effect however was obtained by Geiger and Marsden in 1909;[31] on average, one out of 8000 α particles hitting a gold plate was deflected by an angle bigger than 90°. This result could not be explained within the most popular

[28] E. Rutherford and F. Soddy, *Radioactive change,* Philosophical Magazine 5 (1903), p. 576.

[29] E. Rutherford, *Some properties of the α rays from radium,* ibid., 11 (1906), p. 166; *Retardation of the α particle from radium in passing through matter,* ibid., 12 (1906), p. 134.

[30] H. Geiger, *On the scattering of the α particles by matter,* Proceedings of the Royal Society A 81 (1908), p. 174.

[31] H. Geiger and E. Marsden, *On a diffuse reflection of the α particles,* ibid., 82 (1909), p. 495.

atomic model of those years, proposed by Thomson in 1904, according to which the atom is formed by a uniformly charged sphere, with the electrons in its interior in some equilibrium configuration, like raisins in a cake.[32] The explanation of Geiger and Marsden's 1909 results is the content of two important papers published by Rutherford in 1911.[33]

As Rutherford wrote, "In order to explain these and other results, it is necessary to assume that the electrified particle passes through an intense electric field within the atom." Thus inside the atom there must be an electric charge, distributed over a very small volume, so that a very intense electric field is created. The atom became a structure formed by a central charge $\pm Ne$, where e is the charge of the electron, surrounded by an electric charge of the same amount but opposite sign, so that the whole atom is electrically neutral.

Rutherford's aim was to account for the electric — as opposed to the mechanical — structure of the atom. For this reason, it would not be correct to see his 1911 model as the birth of the nuclear atom, and in particular, of the concept of nucleus. Indeed, in his 1911 papers Rutherford never used the term "nucleus," but rather wrote "central charge"; moreover, in his model the central charge was surrounded by a spherical distribution of charge, uniformly spread over the entire volume of the atom.[34] While the importance of Rutherford's idea in this context is undeniable, however, the notion of nucleus originated from the confluence of various lines of research, as illustrated in Figure 2.4. Starting from very different considerations, around 1921 André Louis Debierne tackled the problem of the atomic structure from the viewpoint of the radioactive phenomena.[35] Every radioactive substance is characterized by a constant, usually denoted λ, which expresses the time that the substance takes to transform; it is in a way the substance's ID. The constant is unchangeable, even under extreme physical and chemical treatments, such as heating at very high temperatures; its value is independent of any external factor. As Debierne writes:

> One can however conclude that the infinitesimally small particle that we call atom is a very complex system. It is not only formed by electric charges moving in a more or less regular way. It must be formed by two quite distinct parts. The first region is the external part and manifests itself in many ways (electromagnetic radiation, molecular bounds, etc.); it is sensitive to the actions from the exterior (magnetic fields, electric discharges, etc).

[32] J. J. Thomson, *On the structure of the atom: an investigation of the stability and periods of oscillation of a number of corpuscles arranged at equal intervals around the circumference of a circle; with application of the results to the theory of the atomic structure,* Philosophical Magazine 7 (1904), p. 237.

[33] E. Rutherford, *The scattering of the α and β rays and the structure of the atom,* Memoirs of the Literary and Philosophical Society of Manchester, 55 (1911) (*CPF II,* p. 212); *The scattering of α and β particles and the structure of the atom,* Philosophical Magazine, 21 (1911) p. 669 (*CPR II,* p. 238).

[34] For a detailed analysis of this issue see G. Bruzzaniti, *op. cit.*

[35] A. L. Debierne, *Sur les transformations radioactives,* in *Les idées modernes sur la constitution de la matière,* Gauthier-Villars, Paris 1913.

> The regular movements of the electric charges take place in this region. The second region is so to say inaccessible; thanks to an unknown process, it is shielded from the external physical agents; it must contain some elements in a steady state of disordered agitation, and should be responsible for the gravitational phenomena. The volume of this inner part is most likely very small with respect to the total volume of the atom, so that atoms can be hit by external bodies and also be crossed throughout by a projectile without any influence on the nucleus. The presence of the latter is revealed when, due to the disordered internal agitation, a violent explosion takes place. This picture of the atom is similar to that of a planet whose atmosphere occupies a much bigger volume than the solid or liquid mass. The atmosphere is sensitive to the outer agents and is where the phenomena that are perceptible from the outside take place. The internal mass manifests itself in a tangible way only in the occasion of a cataclysm or volcanic eruption.[36]

At the beginning of 1913 the image of the atom had been substantially reshaped. Now it referred to a region of space of the dimension of about 10^{-8} cm, having a central nucleus of a radius of about 10^{-10} cm. The latter is responsible for the radioactive phenomena and accounts for the whole mass of the atom. The electrons, necessary to make the whole structure electrically neutral, orbit around the nucleus. There is however a big, unresolved problem: the stability of this structure. Indeed, an oscillating charge emits electromagnetic waves; this is the mechanism at the basis of radio and TV broadcasts. The circuits and the antennas of the transmitter stations contain charges oscillating with a certain frequency. The charges radiate electromagnetic waves that carry the signal. From the point of view of classical electromagnetism, the electrons around the nucleus look like a small transmitter that radiates electromagnetic waves of very high frequency — the frequency of revolution of the electrons around the nucleus. How is it possible that the electrons orbit around the electrically charged neutron without collapsing on it, since during their motion they emit energy? According to classical physics, an atom of this kind would live for one hundred-millionth of a second.

2.7 Atomic models and "Old Quantum Theory"

The problem of the stability of the atom was solved in 1913 by Bohr, who applied the hypothesis of the quantum of action to the nuclear model. This is a typical "confluence process," in which different lines of research merge and give rise to a new theory.[37] Using Bohr's words:

> In an attempt to explain some of the properties of matter on the basis of this atom-model [the nuclear model] we meet, however, with difficulties of a serious nature arising from the apparent instability of the system of electrons [. . .] Whatever the alteration in the laws of motion of the electrons may be, it seems necessary to introduce in the laws in question a quantity foreign to the classical electrodynamics, i.e., Planck's constant, or as it often is called the elementary quantum of action [. . .] This paper is an attempt to show that the

[36] *Ibid.,* p. 331.

[37] Cf. G. Bruzzaniti, *op. cit.*

> application of the above ideas to Rutherford's atom-model affords a basis for a theory of the constitution of atoms.[38]

The introduction of Planck's constant (see Appendix C.4) played here a fundamental role, analogous to that of the speed of light *c* in the theory of relativity.[39] The two constants set strict bounds on the validity of the 19th century theories, and delimited what nowadays is called "classical physics." Planck's original idea was that the process of emission and absorption of electromagnetic radiation takes place in a discrete way, by means of *quanta* of energy. However the electromagnetic radiation was still regarded to be continuous: only the emission and absorption processes were considered to have a new nature. George Gamow, a distinguished physicist and skilled popularizer of the last century, gave a nice picture of this state of affairs:

> Radiation is like butter, which can be bought or returned to the grocery store only in quarter-pound packages, although the butter as such can exist in any desired amount.[40]

It is Einstein's merit to have first used the quantum hypothesis to deduce the corpuscular nature of light. This first manifestation of the wave-particle duality appeared in 1905 in a paper devoted to the interpretation of the photoelectric effect (cf. Appendix C.4). Wave-particle duality for the electromagnetic radiation, i.e., the coexistence of antithetical properties in the same physical system, played an important role in the physics of that time. The research that Arthur Compton made in 1923[41] (see Appendix C.4) confirmed the idea that radiation has particle-like properties; that happened just before Louis de Broglie's contribution, which, by extending the wave-particle dualism to matter, opened the way to quantum mechanics.

Planck's hypothesis had a huge influence on the investigations on the structure of matter. If electromagnetic energy is quantized, why the same should not be true also for mechanical energy? With this ingenious idea Bohr solved the problem of stability. There are in the atom orbits where an electron can stay without irradiating energy; an electron radiates only when it "jumps" between two orbits, and then it emits a quantum of energy. Here is how Fermi described Bohr's idea in a talk, most likely addressed to high school teachers, which was published in 1925.

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[38]N. Bohr, *The constitution of atom and molecules,* Philosophical Magazine 26 (1913), p. 1.

[39]Bohr stressed that, by dimensional reasons, to give account of the existence of electronic orbits whose radii have the same order of magnitude as the atom, it was necessary to introduce a new constant: "By the introduction of this quantity the question of the stable configuration of the electrons in the atoms is essentially changed, as this constant is of such dimensions and magnitude that it, together with the mass and charge of the particles, can determine a length of the order of magnitude required." *Ibid.*

[40]G. Gamow, *Thirty Years that Shook Physics,* Dover, New York 1966. pp. 22–23.

[41]A. H. Compton, *The spectrum of scattered X-rays,* Physical Review 22 (1923), p. 409.

> The two basic principles of quantum theory are the following:
>
> a) the motion of an atomic system is a mechanically possible motion (i.e., it can be computed by using only the three laws of mechanics and the Coulomb law); however, not all mechanically possible motions actually take place, but only a discrete sequence of them, called quantum or static motions; so that in particular the energy w of the system can only take a discrete sequence of values $w_1, \ldots, w_n$.
> b) As long as the system moves according to a static motion there is no irradiation of energy (contrary to the results of classical electrodynamics). The irradiation of energy is always due to the non-mechanical jump or an electron between two quantum motions. If w and w' are the energies of the two motions, the radiated energy will be $w - w'$, and one assumes that it will be radiated in only one quantum. So the energy will be radiated with the frequency $\nu = (w - w')/h$.
>
> These two principles can be by now considered as being experimentally proved.[42]

In the perspective of the reconstruction of a global map, the contributions by Planck, Einstein, and Bohr can be considered as the basic elements of what nowadays we call *Old Quantum Theory* (henceforth shortened into *OQT*). It can be dated between 1913 and 1924 and formed the ground over which quantum mechanics would have been built. The succession of events that led to *OQT* and quantum mechanics is sketched in Figure 2.5.[43]

As it happens with all theories that mark radical changes from the received view, also *OQT* at the beginning was tightly linked with the existing theory, that is, classical mechanics. The quantum aspects of the theory were the result of suitable quantization rules superimposed to a conceptual structure still based on classical physics. In an attempt to provide a sounder basis to the theory, several researchers, including Bohr and Einstein, looked for more general principles. In the talk cited above, Fermi expressed the following point of view.

> Obviously, [the two postulates] are not enough to solve all the problems of atomic physics; therefore they need to be integrated with the solutions of other problems, which however are for the moment incomplete and uncertain. In this talk I will make an attempt to explain how one can try to find a systematic solution to some of these problems. In particular we shall deal with the following questions:
>
> a) What are the rules for choosing the quantum motions among all mechanically possible motions. Evidently a complete solution to this problem would be of paramount importance, since it would allow one to use [a formula previously written] to compute all frequencies that the atom under consideration can emit, namely, to completely solve the problem of determining theoretically all spectral lines.
> b) What is the probability that, at a given temperature, an atom will be in the state corresponding to a specified mechanically possible motion.
> c) What is the probability that an atom in a given quantum state during a time interval dt will pass to another given state.

[42]E. Fermi [22], *CPF I,* p. 138.

[43]From an idea of A. Pais, *op. cit.,* p. 385.

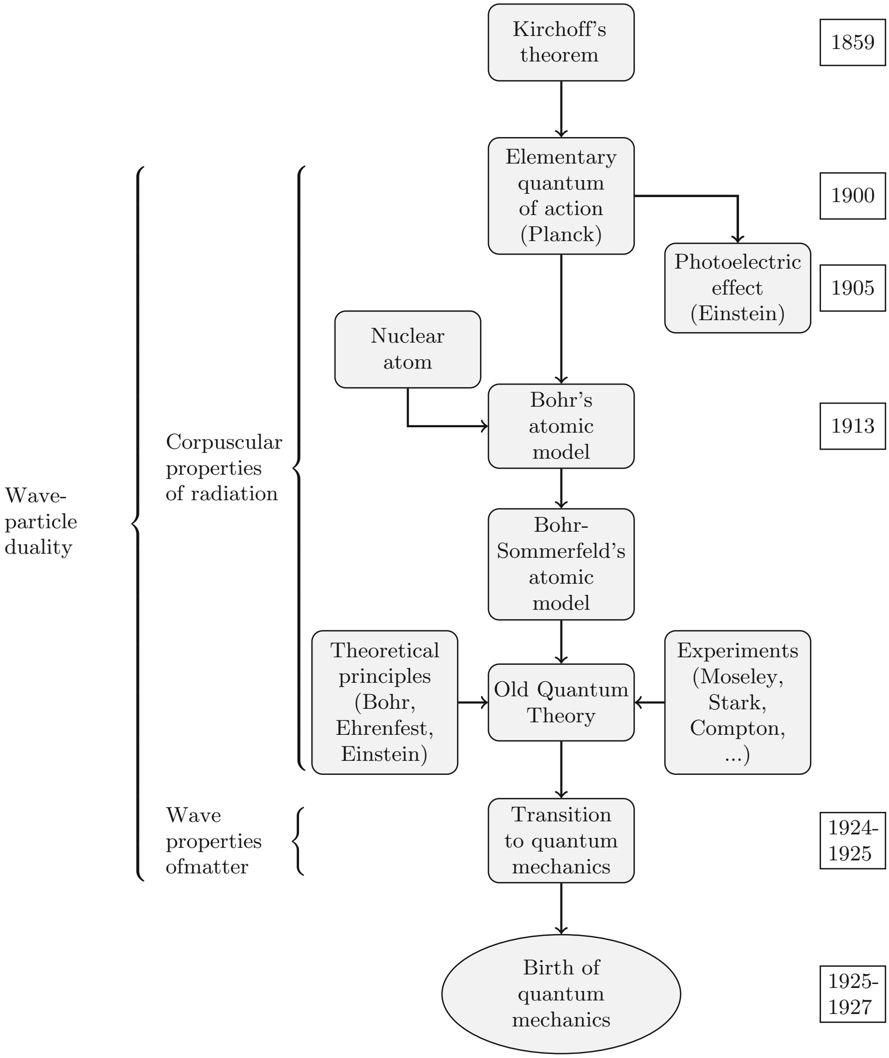

Fig. 2.5 Development of the *OQT* and main lines of influence in the birth of quantum mechanics.

> These three problems can be at least partially solved by resorting to some general principles, in particular to Ehrenfest’s adiabatic principle and the correspondence principle.[44]

Bohr’s correspondence principle, Ehrenfest adiabatic principle (see Appendix C.10), and Einstein’s introduction of statistical elements were the main attempts to give *OQT* a sounder theoretical foundation, and this was accomplished, especially

[44] E. Fermi [22], *CPF I,* p. 139.

in the first two cases, by anchoring it to the conceptual structure of classical physics. The correspondence principle, which was given this name only in 1920,[45] was introduced by Bohr in its primitive form in 1913, at the end of the third paper of his trilogy devoted to his celebrated atomic model.[46] It is a strong constraint on the new theory, which, in the limit of large orbit and large masses, must reproduce the results of classical mechanics. Ehrenfest's adiabatic principle was published in its definitive form in 1917.[47] It allowed the determination of the quantities of a physical system that are quantized, i.e., those that can only assume a discrete series of values (more details can be found in Appendix C.10). Einstein's attempt to strengthen the theoretical foundations of the *OQT* started in 1916, as shown by a letter written to his dear friend Michele Besso on November 18: "A splendid light has dawned on me about the absorption and emission of radiation."[48] Einstein hypothesis, published in 1917 after a preliminary version of the previous year,[49] establishes a deeper connection between Bohr's and Planck's hypotheses.[50]

OQT's experimental foundations were provided by a wealth of data about atomic spectra. Every atom, if heated at a certain temperature, emits a characteristic radiation. A standard example is provided by the light emitted by table salt (which contains sodium) or by a calcium salt: yellow for sodium, brick red for calcium. Every element has an ID provided by the wavelength of the emitted radiation. The atomic spectra, which can be measured by means of optical instruments called spectrometers, are formed by a set of lines, each corresponding to a certain frequency or wavelength. Figure 2.6 shows the hydrogen spectrum, with wavelengths expressed in Ångströms (one Ångström equals 10^{-10} m).

[45]Cf. A. Pais, *Niels Bohr's Times, in Physics, Philosophy, and Polity,* Clarendon Press, Oxford 1991.

[46]N. Bohr, *The constitution of atom and molecules, op. cit.*

[47]P. Ehrenfest, *Adiabatic invariants and the theory of quanta,* Philosophical Magazine 33 (1917), p. 500.

[48]A. Pais, *Subtle is the Lord, op. cit.,* p. 405.

[49]A. Einstein, *Zur Quantentheorie der Strahlung,* Physikalische Zeitschrift 18 (1917), pp. 121–128. English translation *On the quantum theory of radiation,* in B. L. van der Waerden, *Sources of Quantum Mechanics,* North-Holland Publ. Co., Amsterdam 1967. p. 63.

[50]To understand Einstein's idea, let us consider a gas in thermal equilibrium in an electromagnetic radiation field. Einstein conjectured that the probability that a molecule of the gas absorbs energy to pass between two energy levels is proportional to the energy of the electromagnetic field, while the probability to release energy to move between two energy levels is the sum of two terms, one independent of the radiation density (spontaneous emission) and one proportional to it. From this hypothesis Einstein obtained an important result, namely, that a necessary condition for Planck's law to hold is that during the transitions between the energy levels a single quantum of energy is absorbed or emitted, with energy given by (and frequency proportional to) the difference between the two energy levels — exactly Bohr's hypothesis.

Fig. 2.6 Wavelengths of the light emitted by the hydrogen atom. The first line on the right is very strong; moving to the left the lines are more densely distributed and less intense.

The investigation of atomic spectra was one of the main experimental research areas in the 19th century physics. In an interview cited by Pais,[51] Bohr, referring to the situation of spectroscopy during those years, expressed the following viewpoint.

> One thought [spectra are] marvelous, but it is not possible to make progress there. Just as if you have the wing of a butterfly then certainly it is very regular with the colors and so on, but nobody thought that one could get the basis of biology from the coloring of the wing of a butterfly.[52]

But one of the first successes of Bohr's model was indeed about the atomic spectra: the theoretical calculation of the constant called R in Balmer's formula (the empirical rule that relates the spectral lines with their frequencies). More than that, the basic force behind the development of *OQT* was the comparison with the atomic spectra. In this connection, Arnold Sommerfeld's words in his celebrated treatise *Atombau und Spektrallinien* are revealing.

> What we are nowadays hearing of the language of spectra is a true "music of the spheres" in order and harmony that becomes ever more perfect in spite of the manifold variety. The theory of spectral lines will bear the name of Bohr for all time. But yet another name will be permanently associated with it, that of Planck. All integral laws of spectral lines and of atomic theory spring originally from the quantum theory. It is the mysterious organon on which Nature plays her music of the spectra, and according to the rhythm of which she regulates the structure of the atoms and nuclei.[53]

Bohr's model involves another assumption: the order of the chemical elements in the periodic table is given by the value of the charge of the atomic nucleus.[54] This was at variance with the principle that had underlain the structure

[51] A. Pais, *Niels Bohr's Times, op. cit.,* p. 146.

[52] *Ibid.,* p. 142.

[53] A. Sommerfeld, *Atombau und Spektrallinien,* English translation *Atomic structure and spectral lines,* Methuen & Co., London 1934.

[54] This idea is due Antonius van den Broek, a Dutch lawyer who studied the periodic system as a hobby. Between 1907 and 1914 he published, in some authoritative English and German journals, a number of papers about new interpretations of the periodic system. His most important papers are *Das Mendelejeffsche "kubische" periodische System der Elemente und die Ernondnung der Radioelemente in diesem System,* Physikalische Zeitschrift 12 (1911), p. 490; *The number of possible elements and Mendeléeff's "cubic" periodic system,* Nature 92 (1911), p. 78; *Die*

of Mendeleev's table until 1913, namely, that elements are ordered by their atomic weight, as the chemical properties of the elements were expressed by their atomic weight. It was known however that this principle gave rise to three anomalies: in the three pairs argon-potassium, cobalt-nickel, and tellurium-iodine, the first elements had a bigger atomic weight than the second, however potassium, nickel, and iodine, in view of their chemical properties, were accommodated in the periodic table after argon, cobalt, and tellurium, thus violating the underlying principle of the periodic table. The idea of ordering the periodic table according to the electric charge revealed the need to base Mendeleev's systematics on a deeper and less intuitive notion than the atomic weight: the electronic structure. An idea of this significance required an experimental verification. Indeed in 1914 Moseley's results[55] confirmed that hypothesis: the atomic number, i.e., the number corresponding to the position of a chemical element in the periodic table, has a precise physical meaning: it is the value of the electrical charge of the nucleus.

Initially, Bohr, according to the idea we have just described, quantized the orbits of the hydrogen atom with a number n, which determines the energy of the state and therefore the radius of the corresponding circular orbit. But very soon it became apparent that a single number was not enough to describe the complex phenomenology of the atom; in particular, it could not explain the "fine structure" of the spectral lines. Indeed, each line, if analyzed with suitably sensitive instruments, appears to be formed by many, very close lines, corresponding to very small variations of the energy. It became necessary to improve the model, by adding new quantum numbers and suitable selection rules, able to discriminate the admissible transitions between atomic states among all those that could be a priori possible. A fundamental contribution to this program was given by Sommerfeld from 1914 onwards.[56]

The "Bohr-Sommerfeld model" (see Fig. 2.7) is a sophisticated evolution of Bohr's rudimentary atomic model. In accordance with classical mechanics, the orbits of the electrons are no longer circular but elliptic, so that the speed of the electrons is no more constant. This modification allowed Sommerfeld to take advantage of the results of theory of relativity: the variation of the speed induced, as a relativistic effect, a variation of the mass of the electron, and this fact justified the fine structure of the spectrum of the hydrogen atom. The quantum numbers were now three: n, l, and m. The first (principal) quantum number quantized the

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Radioelemente, das periodische System und die Konstitution der Atome, Physikalische Zeitschrift 14 (1913), p. 32; *Intra-atomic charge,* Nature 92 (1913), p. 372. For a more detailed study of van den Broek's contribution see G. Bruzzaniti, *op. cit.*

[55] H. G. J. Moseley, *The high-frequency spectra of the elements. Part I,* Philosophical Magazine 26 (1913), p. 1024; *The high-frequency spectra of the elements. Part II,* Philosophical Magazine 27 (1913), p. 403.

[56] A. Sommerfeld, *Die Feinstruktur der Wasserstoff und der Wasserstoff-ähnlichen Linien,* Sitzungsberichte der mathematisch-physikalischen Klasse der K. B. Akademie der Wissenschaften zu München (1915), p. 459; *Zur Quantentheorie der Spektrallinien,* Annalen der Physik 51 (1916), pp. 1–94, 125–67.

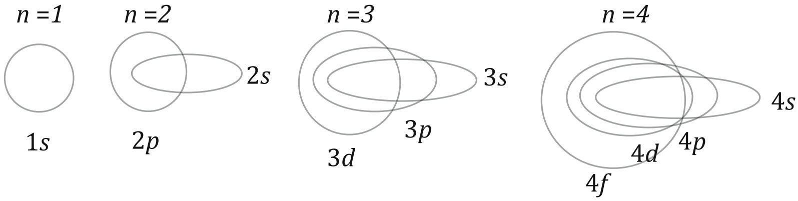

Fig. 2.7 Elliptical orbits in the hydrogen atom according to the Bohr-Sommerfeld model. They are labeled by a number and a letter; the number n is the main quantum number, and the letter l is the azimuthal quantum number, with the correspondence $s = 1, p = 2, d = 3, f = 4$, etc. For example, $4d$ denotes the orbit with $n = 4$ and $l = 3$.

energy of the orbit, as we saw above, and was geometrically related to the major axis of the ellipses; the second number (azimuthal quantum number) quantized the angular momentum and fixed the ratio between the axes of the ellipses; and the third (magnetic quantum number) quantized the projection of the magnetic moment of the electron along the direction of the external magnetic field (if any). In the absence of an external magnetic field, the quantum number m has no meaning, but an external magnetic field singles out a direction in space. Thus, the Bohr-Sommerfeld model provided a complete explanation of the Zeeman effect; the different spatial orientations of the orbits, each characterized by a different value of m, correspond to different values of the magnetic energy, and this induces a separation of the spectral lines, in full agreement with the experimental results.

Around 1920 the *OQT* was a very successful theory.[57] Due to its strong connection with classical mechanics, it was called "semiclassical theory." The name given to the separation of the spectral lines in a magnetic field, the "normal Zeeman effect," is very suggestive in this connection; the effect was "normal" because classical physics was able to predict that in the presence of an external magnetic field every spectral line splits into three. However, very few lines split into just three parts; most of them split into many more parts. This is the "anomalous" Zeeman effect (see Fig. 2.8). This showed how the theory was based on unsound principles. In his autobiography Einstein writes about this period:

> It was as if the ground had been pulled out from under one's feet, with no firm foundation to be seen anywhere, upon which one could have built. That this insecure and contradictory foundation was sufficient to enable a man of Bohr's unique instinct and tact to discover the major laws of the spectral lines and of the electron-shells of the atoms together with their

[57] In 1914 Franck and Hertz made an experiment which proved the existence of quantized atomic energy levels. Franck and Hertz bombarded the vapors of different elements with electrons having a known kinetic energy, and observed that the atoms in the vapor were excited only for specific values of the energy of the incident electrons. J. Franck and G. Hertz, *Über Zusammenstöße zwischen Elektronen und den Molekülen des Quecksilberdampfes und die Ionisierungsspannung desselben,* Verhandlungen der Deutsches Phykalische Gesellschaft 16 (1914), p. 457.

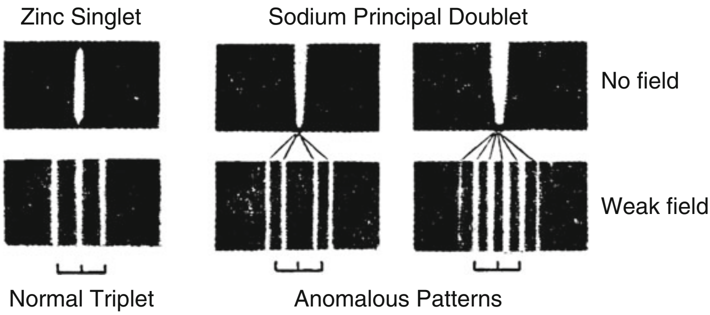

Fig. 2.8 On the left, the normal Zeeman effect in zinc; in the presence of a week magnetic field, the line splits into three. On the right, anomalous Zeeman effect in sodium; in the presence of the magnetic field, the two lines of the doublet split into more than three lines.

> significance for chemistry appeared to me like a miracle — and appears to me as a miracle even today. This is the highest form of musicality in the sphere of thought.[58]

In spite of the contradictions and the uncertainty, it seemed quite evident that the *OQT* hinted at the existence of an underground river, a deeper theory which, if discovered, would explain the microscopic properties of matter. The failure to explain the spectrum of the helium atom, the anomalous Zeeman effect, and the intensity of the spectral lines, the difficulty in relating the electronic structure with the position of the elements in the periodic table, the need to introduce new quantum numbers to cope with particular, problematic situations, all testified to a crisis of the *OQT* which was not the manifestation of a single problem, but was rather due to the accumulation of several different anomalies.

The formulation of Pauli's exclusion principle[59] and the discovery of spin by Uhlenbeck and Goudsmit in 1925[60] (see Appendix C.5) were the forerunners of an imminent, radical turn in the theory. Pauli, in particular, spoke very keenly about a quantum property of the electron that he called "a double valence that cannot be

[58] P. A. Schilpp, *Albert Einstein, Philosopher-Scientist,* MJF Books, New York, 1949, pp. 46–47.

[59] W. Pauli, *Über den Einfluß der Geschwindigkeitsabhängigkeit der Elektronenmasse auf den Zeemaneffekt,* Zeitschrift für Physik 31 (1925), p. 373; *Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren, ibid.,* p. 765. The first paper claims the existence of a fourth quantum number than cannot be described classically; the second contains the formulation of Pauli's principle: "There can never be two or more equivalent electrons in an atom for which in strong fields the values of all quantum numbers [. . .] are the same. If an electron is present in the atom for which these quantum numbers (in an external field) have definite values, this state is 'occupied'." (From the English translation *On the connexion between the completion of electron groups in an atom with the complex structure of spectra,* in D. ter Haar, *The Old Quantum Theory,* Pergamon Press, Oxford-London-Edinburgh 1924. pp184–203.)

[60] G. E. Uhlenbeck and S. Goudsmit, *Ersetzung der Hypothese vom unmechanischen Zwang durch eine Forderung bezüglich des inneren Verhaltens jedes einzelnen Elektrons,* Naturwissenschaften 13 (1925), p. 953.

described classically.” In other words, physicists were more and more aware that adding *ad hoc* hypotheses to a theory whose structure was still basically classical was not sufficient to provide a viable description of nature at the atomic scale.

1924 and 1925 were decisive years for the *OQT.* In addition to the difficulties in the comparison with the empirical data, as in the above mentioned case of the helium atom, there was the audacious theoretical proposal by Bohr, Kramers, and Slater,[61] known as BKS theory, which basically represented a refusal of the corpuscular nature of the electromagnetic radiation. A few years later this would be remembered by Heisenberg as the apex of the crisis of the *OQT.*[62] To give a historical interpretation of the BKS theory, which was the most evident manifestation of the crisis, one should remark that the survival of a theory, in spite of its uncertain and shaky principles, is usually guaranteed by the continuous agreement with the experimental data. If the latter fails, there is no reason why the basic principles should not be critically rediscussed, to settle the contradictions of the theory.

The BKS proposal originated from an idea of young Slater, which interpreted the wave-particle duality for the electromagnetic radiation by assuming that waves and particles coexist:

> I have both the waves and the particles, and the particles are sort of carried along by the waves, so that the particles go where the waves take them, instead of just shooting in straight lines, as other people assume.[63]

This idea aimed to dispose of the autonomous existence of the light quantum, in an attempt to recover the classical, wave-theoretic foundations of the theory. This is quite evident from the abstract of the paper:

> In this paper we endeavor to provide a reasonable description of the optical phenomena, closely related to the meaning of spectra according to the quantum theory, without deviating from the classical law of propagation of radiation in empty space. The continuous phenomena that characterize radiation are related with the discrete atomic processes by means of probabilistic laws, according to Einstein’s procedure. The introduction of virtual oscillators, which can be related to the continuous processes thanks to the correspondence principle, allows these laws to be interpreted in a rather different way than it is usually done.[64]

The price paid by the BKS proposal to save the preeminence of the undulatory concept was, however, very heavy; the principles of conservation of energy and of impulse were no longer valid in a universal sense, but only statistically. In a single process at the atomic level energy and impulse needed not be conserved. In 1925 Compton and Simon made an experiment to verify the principles of conservation

[61] N. Bohr, H. A. Kramers and J. C. Slater, *Über die Quantentheorie der Strahlung,* Zeitschrift für Physik 24 (1924), p. 69.

[62] W. Heisenberg, *Die Entwicklung der Quantentheorie 1918–1928,* Naturwissenschaften 17 (1929), p. 490.

[63] From a letter by Slater to his family, reported by A. Pais, *Niels Bohr’s Times, op. cit.*, p. 235.

[64] N. Bohr, H. A. Kramers and J. C. Slater, *op. cit.*

of energy and impulse at the level of a single atomic process.[65] The results left no room for doubt: : "They are, on the other hand, in direct support of the view that *energy and momentum are conserved during the interaction between radiation and individual electrons.*"[66] As a consequence, in a letter to Darwin, Bohr wrote "It seems ... that there is nothing else to do than to give our revolutionary efforts as honourable a funeral as possible."[67]

The *OQT* was declining, leaving behind many shadows and unresolved problems. As we already mentioned, the community of physicists started realizing the impossibility of building a theory describing the microscopic world that was founded on the principles of classical physics. Again in 1925, Bohr concludes a paper by writing:

> In this state of affairs one must be prepared to find that the generalization of the classical electrodynamic theory that we are striving for will require a fundamental revolution in the concepts upon which the description of nature has been based until now.[68]

This resounded the words of Pierre Duhem, about the perseverance in maintaining

> at all costs, at the price of continuous reparations and of a forest of tangled posts, the worm-eaten pillars of a shaking building, while by throwing these pillars away, one could build a simple, elegant and robust system based on new hypotheses.[69]

2.8 Nuclear protophysics and the proton-electron nuclear model

The introduction of the notion of nuclear atom marked the beginning of the *OQT,* but also set off, around 1920, a new area of research that we have called "nuclear protophysics."

Figure 2.9 shows the two main areas of research which originated nuclear protophysics: isotopy and the scattering of α particles. These investigations allowed the collection of a huge amount of experimental data, and highlighted a complex phenomenology, whose interpretation led to the construction of many nuclear models, each suited to the particular set of phenomena considered. The theoretical environment in which the theory developed is the *OQT.* The problems that this project met reflect the general issues which still affected the semiclassical approaches which were at the basis of the physical explanation of the atomic and nuclear phenomena.

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[65] A. H. Compton and A. W. Simon, *Directed quanta of scattered X-rays,* Physical Review 26 (1925), p. 289.

[66] *Ibid.,* p. 299. Italics in original.

[67] Letter by N. Bohr to Ch. G. Darwin, cited in A. Pais, *Niels Bohr's Times, op. cit.*, p. 238.

[68] *Ibid.,* p. 239.

[69] P. Duhem, *op. cit.,* p. 357.

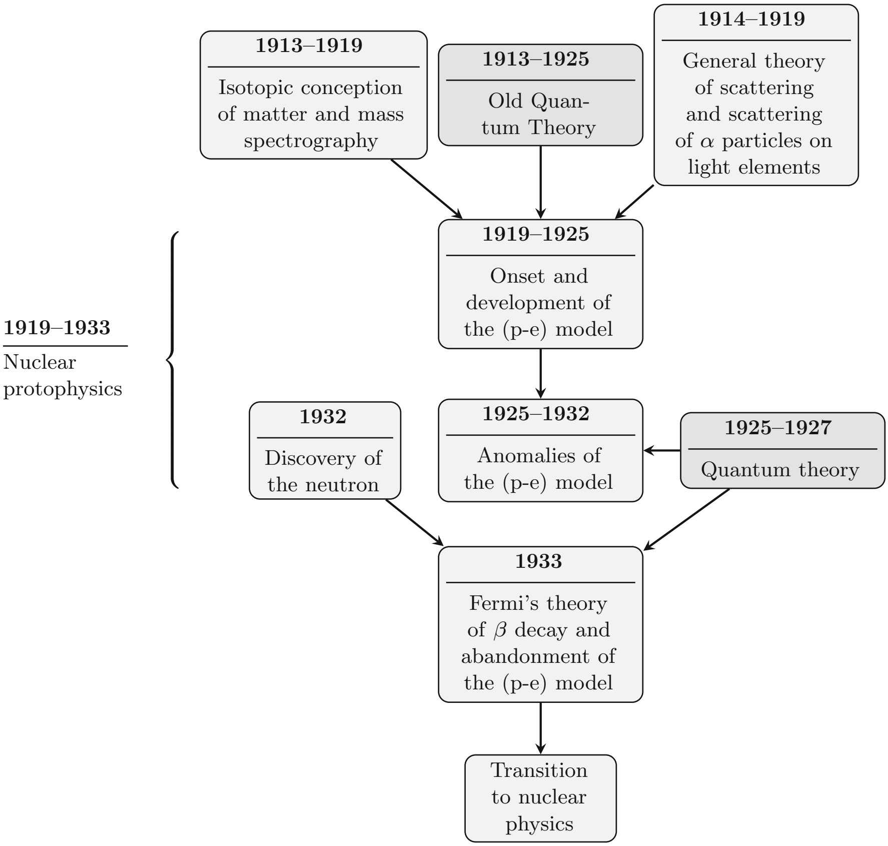

Fig. 2.9 A historical scheme of the "nuclear protophysics," characterized by the onset and decline of the proton-electron nuclear model.

2.8.1 The isotopic conception of matter

The term "isotope" was used for the first time by Frederick Soddy in a 1913 paper:

> The successive expulsion of one α and two β particles in three radio-active changes in any order brings the intra-atomic charge of the element back to its initial value, and the element back to its original place in the table, though its atomic mass is reduced by four units [. . .] The same algebraic sum of the positive and negative charges in the nucleus, when the arithmetical sum is different, gives what I call "isotopes" "isotopic elements," because they occupy the same place in the periodic table. [70]

The notion of isotopy was a synthesis between van den Broek's hypothesis (the order of the elements in the periodic table is not given by their weight but

[70]F. Soddy, *Intra-atomic charge,* Nature 92 (913), p. 399.

rather by their nuclear charge) and the law of radioactive displacement. To illustrate the meaning of this law, let us recall that α particles have charge $+2$ (assuming the electron charge as unit) and mass 4 (with unit given by the hydrogen nucleus mass), while β particles have charge -1. If a nucleus with atomic number Z and atomic weight A emits an α particle, its charge decreases by 2 and its weight by 4, if it emits a β particle, its charge increases by 1 and its mass remains unchanged, as the electron mass is negligible. The law of radioactive displacement affirmed that if an element with atomic number Z emits an α particle, it transforms into the element with atomic weight $Z - 2$, that is, the element preceding the original one by two places in the period table; if it emits a β particle, it transforms into the element with atomic number $Z + 1$, i.e., the element which immediately follows the original one in the table.

The investigations made by Francis W. Aston in 1919[71] extended the notion of isotopy from the radioactive elements to all elements in the periodic table. To settle a long-standing controversy about the nature of the atmospheric neon, Aston set up an experimental apparatus, the mass spectrograph, which allowed him to obtain remarkable results. The data showed, with an accuracy of one part over a thousand, that atmospheric neon (whose atomic weight is 20,20) is actually a mixture of two isotopes, whose atomic weights are 20 and 22, in the proportion of 90 and 10 percent. In the same way Aston was able to show that many elements are a mixture of different isotopes. After Aston's work, the notion of isotopy was unanimously accepted by the scientific community, as shown, for instance, by the famous 1921 *Discussion on isotopes*.[72] A few years later Marie Curie published a treatise devoted to the isotopic elements. In the introduction one can read:

> [They] extended and completed the notion of isotopy in all its generality, and definitely deprived the atomic weight of the role it had been assigned in the periodic system. The place that atomic weight occupied in that system has been given to a physical quantity which has a more fundamental importance: the positive charge of the *nucleus,* the central part of the atom.
> The periodic classification, thus generalized with a new interpretation, takes us to the idea of unity of matter; a grandiose idea, as old as the atomic theory, which however was superficially refuted by the very precise determination of the atomic weights. These were assigned by chemistry a simple meaning which, however, they do not possess.[73]

Curie's reference to the unity of matter expressed a concept which was quite common within the scientific community of that time. It evoked Prout's old "law of the integers," according to which all atoms were made from a primordial entity, identified with the hydrogen atom. Ever more, the possibility appeared to formulate

[71] F. W. Aston, *A positive ray spectrograph,* Philosophical Magazine 38 (1919), p. 707; *The constitution of atmospheric neon,* Philosophical Magazine 39 (1920), p. 449; *The mass-spectra of chemical elements,* Philosophical Magazine 40 (1920), p. 628. A more detailed historical reconstruction can be found in G. Bruzzaniti, *op. cit.*

[72] *Discussion on isotopes opened by Sir J. J. Thomson,* Proceedings of the Royal Society A 99 (1921), p. 87.

[73] M. Curie, *L'isotopie et les éléments isotopes,* Presses Universitaires de France, Paris 1924, p. 12.

hypotheses about the constituents of a structure that seemed to be more and more complex, and was most likely formed by hydrogen nuclei, electrons, and, perhaps, α particles. (For further information about the notion of isotope see Appendix C.6.)

2.8.2 The scattering of α particles

In 1911 Rutherford developed a theory for the scattering of α particles by atoms with a large atomic weight. In particular, in Rutherford's experiment, the target was formed by gold atoms. But what was one to expect in case of other atoms? In an important paper published in 1914,[74] Charles G. Darwin tried to answer this question, with a theoretical study of the scattering of α particles by light elements, including the cases when the target had the same mass as the α particles (scattering of α particles by helium) or even smaller (scattering by hydrogen). It was a theory based on the classical conservation principles, where nuclei were assumed to be point-like and to interact by means of the Coulomb force. The first experiments to verify Darwin's theory were made by Marsden in 1914,[75] who analyzed the scattering of α particles by hydrogen. The results were in good agreement with the predictions. In any case these first experiments were just the preliminary stages of a more comprehensive project, which aimed at a detailed investigation of all the consequences of the theory.

However, the onset of the First World War determined an almost complete stop of the scientific activity. Rutherford, at the end of the war, resumed Marsden's experiments, and in 1919 published a series of four very important papers. The most striking result, published in the fourth paper of the series, was the first evidence of the disintegration of the nitrogen atom:

> From the results so far obtained it is difficult to avoid the conclusion that the long-range atoms arising from collision of alpha particles with nitrogen are not nitrogen atoms but probably atoms of hydrogen, or atoms of mass 2. If this be the case, we must conclude that the nitrogen atom is disintegrated under the intense forces developed in a close collision with a swift alpha particle, and that the hydrogen atom which is liberated formed a constituent part of the nitrogen nucleus. [...] It is of interest to note, that while the majority of the light atoms, as is well known, have atomic weights represented by $4n$ or $4n+3$ where n is a whole number, nitrogen is the only atom which is expressed by $4n + 2$. We should anticipate from radioactive data that the nitrogen nucleus consists of three helium nuclei each of atomic mass 4 and either two hydrogen nuclei or one of mass 2.[76]

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[74] Ch. G. Darwin, *On collision of α particles with light atoms,* Philosophical Magazine 27 (1914), p. 499.

[75] E. Marsden, *The passage of α particles through hydrogen, ibid.,* p. 824.

[76] E. Rutherford, *Collision of α particles with light atoms,* Philosophical Magazine 37 (1919), pp. 537–61. The paper is made of four parts: *I. Hydrogen,* pp- 537–561; *II. Velocity of the hydrogen atom,* pp. 562–571; *III. Nitrogen and oxygen atoms,* pp. 571–580; *IV. An anomalous effect in nitrogen,* pp. 581–587.

The naturality with which Rutherford mentions the constituents of the nitrogen nucleus makes it clear that the idea of the atomic nucleus as a complex entity, formed by more elementary constituents, was by then well established. It was supported by several experimental evidences, such as isotopy, the radioactive phenomena, and the disintegration of nitrogen.

Around 1920 the investigation of the nuclear structure has become an autonomous field of study, characterized by a specialized language and specific programs. Rutherford's introduction in 1921 of the term "proton" is symptomatic; it attests not only the understanding of the role played by the hydrogen nucleus as a constitutive element of the atomic nuclei, but also the interest of the scientific community for the new research area.[77]

2.8.3 The (p-e) model

As it commonly happens, also nuclear protophysics showed uncertainties and interpretative problems, due to the lack of a theoretical language powerful enough to describe the huge amount of available experimental data. By inspecting the literature around the year 1920, these uncertainties are in particular shown by the great number of nuclear models, interpretative schemes, and bold conjectures.[78]

We use here the generic term "(p-e) model" to denote a number of different models, all built on the same assumption and a common methodological principle, i.e., that all atomic nuclei are made of elementary particles (protons and electrons);

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[77] It is interesting to recall what Rutherford wrote in 1921 as a remark about a paper of D. O. Masson, where the term *baron* was suggested to designate the hydrogen nucleus: 'At the time of writing this paper in Australia, Professor Orme Masson was not aware that the name "proton" had already been suggested as a suitable name for the unit of mass nearly 1, in terms of oxygen 16, that appears to enter into the nuclear structure of atoms. The question of a suitable name for this unit was discussed at an informal meeting of a number of members of Section A of the British Association at Cardiff this year. The name "baron" suggested by Professor Masson was mentioned, but was considered unsuitable on account of the existing variety of meanings. Finally the name "proton" met with general approval, particularly as it suggests the original term "protyle" given by Prout in his well-known hypothesis that all atoms are built up of hydrogen. The need of a special name for the nuclear unit of mass 1 was drawn attention to by Sir Oliver Lodge at the Sectional meeting, and the writer then suggested the name "proton." Professor Orme Masson sent the present paper for publication through the writer, and in order to avoid the long delay involved in correspondence, his paper is printed in its original form. If the name "proton" is generally approved, it is merely necessary to change the symbol "b" into "p" in the chemical equations given in the paper. It should be pointed out that a somewhat similar type of nomenclature for the constituents of atoms has been suggested in the interesting paper of Professor W. D. Harkins, entitled *The Nuclei of Atoms and the New Periodic System* (Physical Review, 15 (1920) p. 73), in D. O. Masson, *The constitution of atoms,* Philosophical Magazine 41 (1921), p. 281'.

[78] Almost 25 different nuclear models were proposed during those years. The situation has been analyzed in detail in G. Bruzzaniti, *op. cit.,* and R. H. Stuewer, *The nuclear electron hypothesis,* in W. R. Shea (ed.), *Otto Hahn and the Rise of Nuclear Physics,* Reidel, Dordrecht 1983.

the principle assumed that if a nucleus emits a particle, then the latter already existed inside the nucleus. The principle was so deeply rooted in the scientific community that it was never explicitly stated. However, Aston wrote in 1922:

> It has been stated that the presence of helium nuclei inside the nuclei of radioactive atoms is definitely proved by the ejection of α particles by the latter. In the writer's opinion this is much the same as saying that a pistol contains smoke, for it is quite possible that the α particle, like the smoke of the pistol, is only formed at the moment of its ejection.[79]

Nevertheless, Aston's voice remained unheard, and had no consequence on the development of nuclear protophysics. The principle was held true until 1933, when it was eventually demolished by Fermi's theory of the β decay.

2.8.4 Quantum statistics

Ludwig Boltzmann's profound intuition that "the problems of the mechanical theory of heat are [...] problems in the theory of probability" gave rise to statistical mechanics.[80] This merit is acknowledged by the inscription on the great Austrian physicist's grave in Vienna's central cemetery: $S = k \log W$. This equation relates the entropy S of the state of a thermodynamical system with a quantity W, which is proportional of the probability of that state. Actually the equation was not written by Boltzmann, but rather by Planck, who, in paper which followed Boltzmann's after a few weeks, wrote the equation if a more complete form, stressing that the entropy is defined up to an additive constant: $S = k \log W + \text{const}$. However, it remains true that the equation was the result of Boltzmann's research and of his understanding "the problems of the mechanical theory of heat as [...] problems of the theory of probability," of his perception — as written by Abraham Pais — that "the second law of thermodynamics can be understood only in terms of a connection between entropy and probability."[81] For a better understanding of the relations between thermodynamics and statistical mechanics let us once more resort to Enrico Fermi (he called π the quantity we denoted by W).

> The fact that the entropy of an isolated system can never decrease during any transformation has a very clear interpretation from the statistical point of view. Boltzmann has proved that the entropy of a given state of a thermodynamical system is connected by a simple relationship to the probability of the state. We have already emphasized the difference between the dynamical and thermodynamical concepts of the state of a system. To define the dynamical state, it is necessary to have the detailed knowledge of the position and motion of all the molecules that compose the system. The thermodynamical state, on the other hand, is defined by giving only a small number of parameters, such as the temperature, pressure, and so forth. It follows, therefore, that to the same thermodynamical state there corresponds a

[79] F. W. Aston, *Isotopes,* Arnold, London 1922, p. 102.

[80] L. Boltzmann, *Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen,* Wiener Berichte 66 (1872), pp. 275–370.

[81] A. Pais, *Subtle is the Lord, op. cit.,* p. 60.

> large number of dynamical states. In statistical mechanics, criteria are given for assigning to a given thermodynamical state the number π of corresponding dynamical states. [...] This number π is usually called the probability of the given thermodynamical state, although, strictly speaking, it is only proportional to the probability in the usual sense. The latter can be obtained by dividing π by the total number of possible dynamical states.
> We shall now assume, in accordance with statistical considerations, that in an isolated system only those spontaneous transformations occur which take the system to states of higher probability, so that the most stable state of such a system will be the state of highest probability consistent with the given total energy of the system.
> We see that this assumption establishes a parallelism between the properties of the probability π and the entropy S of our system, and thus suggests the existence of a functional relationship between them. Such a relationship was actually established by Boltzmann, who proved that $S = k \log \pi$.[82]

The main objective of statistical mechanics is to determine W, the quantity that expresses the number of microscopic ways in which the same macroscopic state can be realized. As Erwin Schrödinger wrote

> There is, essentially, only one problem in statistical thermodynamics: the distribution of a given amount E of energy over N identical systems. Or perhaps better: to determine the distribution of an assembly N of identical systems over the possible states in which the assembly can find itself, given that the energy of the assembly is a constant E.[83]

The merit for finding the general solution to this problem goes to Boltzmann, who introduced the so-called "Boltzmann statistics," or "classical statistics," to distinguish it from the "quantum statistics" of Bose-Einstein and Fermi-Dirac. We shall use a simple example to understand, at least in principle, what is going on.[84] Let us suppose we have two identical small balls, one white and one black, and two glasses, A and B. How can we place the balls under the glasses? Clearly, as in Figure 2.10. Would the situation be anyhow different if the balls were of the same color? According to classical physics, the answer is no. The balls are identical, but nevertheless they can always be distinguished, since after observing one of them at some instant of time, we can follow its trajectory, and after some time we know for sure that we are still watching the same ball. So we still have the four cases depicted in the figure: 1) two balls under A; 2) two balls under B; 3) one ball under A, and the other under B; and 4) the same as in 3), but with the two balls swapped. The probabilities to have one of the configurations are easily computed: the probability that the two balls are under A is 1/4; that they are under B is 1/4; and that they are one under A and one under B is 1/2.

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[82] E. Fermi, *Thermodynamics,* Dover, New York 1936, p. 56–57.

[83] E. Schrödinger, *Statistical Thermodynamics,* Cambridge University Press, Cambridge, U.K. 1946.

[84] This example is taken, with some small changes, from G. Parisi, *La statistica di Fermi,* in C. Bernardini and L. Bonolis (eds.), *Conoscere Fermi,* Compositori, Bologna 2001. That is in turn basically taken from an example by Fermi, published in the entry "Meccanica Statistica" of G. Treccani's *Enciclopedia Italiana* (all in Italian).

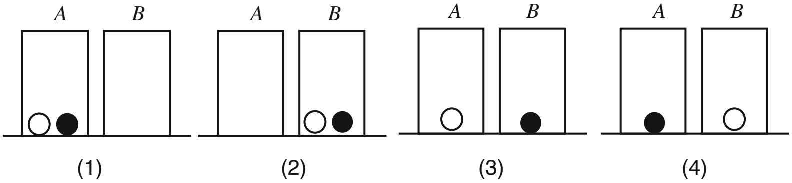


Fig. 2.10 There are four ways to place a white and a black ball under two glasses.

Fig. 2.11 There are three ways to place two indistinguishable balls under two glasses.

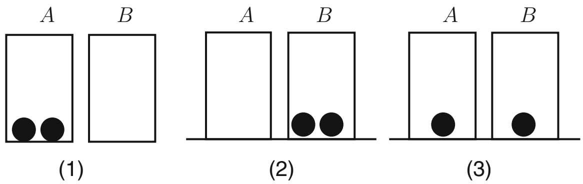
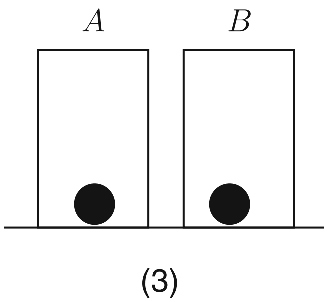


Fig. 2.12 The only possible configuration for two indistinguishable balls that obey Pauli's exclusion principle.



From the viewpoint of quantum mechanics the situation is radically different. Indeed, as we shall discuss in the next Section, we cannot follow the trajectory of a ball, at least not without strongly affecting the physical system. The two balls become indistinguishable, and this has deep effects on the probability pattern. Taking the indistinguishability into account, the cases 3) and 4) of Figure 2.10 become the same, as in Figure 2.11. Now the probabilities are 1/3 for all cases: two balls under A, two balls under B, and one ball under A and one under B. Let us eventually assume that the two indistinguishable balls, like electrons, satisfy Pauli's exclusion principle, so that they cannot be put under the same glass. The statistics changes again: the cases (1) and (2) in the figure no longer take place, and the only possibility is case (3), as in Figure 2.12. So, the probability to have one ball under A, and one ball under B, is 1.

The three cases (identical distinguishable particles, identical indistinguishable particles, identical indistinguishable particles following Pauli's exclusion principle) correspond, respectively, to the three different statistics we have previously mentioned: Boltzmann's, Bose-Einstein's, and Fermi-Dirac's. It seems that to formulate the two non-classical statistics one needs to know one of the most important features of quantum mechanics, namely, the relation between identity and distinguishability, which is a direct consequence of the loss of the notion of trajectory. However, the historical documents show that the two statistics were discovered somehow before the formulation of quantum mechanics. They were, so to say, *OQT*'s swan song.

Very briefly, the first documents attesting the birth of the non-classical statistics are two. The first is a letter the young and unknown Bengali student Satyendra Nath Bose wrote to Einstein in 1924. The letter included the text of a paper that had been rejected by the authoritative journal *Philosophical Magazine,* and contained a new deduction of the spectral distribution of Planck's black body radiation. The deduction used a very original method; the electromagnetic radiation was treated as a gas of particles (a photon gas we would say nowadays) and Planck's formula was deduced by means of statistical arguments, using however a counting of the distributions different from Boltzmann's. Einstein was impressed by the paper, translated it into German, and had it published in the journal *Zeitschrift für Physik,* adding the note

> In my opinion, Bose's derivation of the Planck formula constitutes an important advance. The method used here also yields the quantum theory of the ideal gas, as I shall discuss elsewhere in more detail.[85]

However, it was not clear why Bose was getting the correct result, and indeed Einstein wrote "[the] derivation is elegant but the essence remains obscure."[86] Not even Bose had it clear:

> I had no idea that what I had done was really novel. . . . I was not a statistician to the extent of really knowing that I was doing something which was really different from what Boltzmann would have done, from Boltzmann statistics.[87]

Most likely, Pais is right in affirming "I believe there had been no such successful shot in the dark since Planck introduced the quantum in 1900."[88] In any case, the Bose-Einstein statistics was born.

The second document is a 1926 paper by Fermi entitled *Sulla quantizzazione del gas perfetto monoatomico.*[89] The paper concluded a research project that we shall analyze more in detail in Chapter 3. As in Bose's paper, the topic is the statistical properties of a gas, but while Bose treated a photon gas, here Fermi considered a gas made of particles. By extending Pauli's exclusion principle, introduced since less than a year to interpret the behavior of the atomic electrons, Fermi got a new statistics, different from Boltzmann's classical statistics. Fermi's work was carried out in the *OQT* framework and offered a solution to some problems of the *OQT* treatment of the monoatomic ideal gas, such as the independence of the specific heat from temperature. With Fermi's statistics, the specific heat of the gas is no longer constant with respect to temperature, but goes to zero linearly with the temperature.

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[85] A. Pais, *op. cit.,* p. 423.

[86] *Ibid.,* p. 424.

[87] *Ibid.*

[88] *Ibid.,* p. 428.

[89] On the quantization of the ideal monoatomic gas. Fermi [30].

Since the relation between spin and statistics was not yet clear at the time, Fermi's proposal had some inaccuracies, and nobody in those years attempted to relate it to Bose's statistics. In any case, a new statistics was born, which would have been called "Fermi-Dirac statistics," since about six months later, Paul Adrien Maurice Dirac[90] obtained the same results, albeit in a completely different way and in the framework of the arising quantum mechanics.

2.8.5 Quantum mechanics

Between 1924 and 1925 the *OQT* lost most of its impetus, due to some incurable difficulties whose solution required the construction of a new theoretical perspective. This state of affairs was very well described by Enrico Fermi in 1930, in a popular talk in which he reviewed the development of the "new physics":

> In the first attempts, one tried to understand the atom by means of those physical laws we know to hold for macroscopic phenomena. However, it became very soon clear that those laws cannot be applied to bodies of such small dimensions. Then Bohr's theory came, which attempted to modify the usual laws to adapt them to the new problems, obtaining a great number of results, mostly of qualitative nature. Almost invariably, however, when one tried to get precise quantitative predictions, Bohr's theory was insufficient. Thus it became clear to the physicists that it was not enough to modify the old laws, but it was rather necessary to replace them with new ones.[91]

The "new laws" were formulated following two different paths. The first line of research, initiated by Louis de Broglie,[92] resulted in Schrödinger's "wave mechanics";[93] and the second, which started with Werner Heisenberg's "matrix mechanics,"[94] led, via researches done by Born, Jordan, and Heisenberg himself,[95] to what was initially called "quantum mechanics."

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[90] P. A. M. Dirac, *On the theory of quantum mechanics,* Proceedings of the Royal Society A 92 (1926), p. 661.

[91] Fermi [62], *CPF I*, p. 375.

[92] L. de Broglie, *Recherches sur la théorie des quanta,* Annales de Physique 3 (1925), p. 22.

[93] E. Schrödinger, *Quantisierung als Eigenwertproblem,* Annalen der Physik 49 (1926), pp. 361–76.

[94] W. Heisenberg, *Über die quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen,* Zeitschrift für Physik 33 (1925), p. 879.

[95] M. Born and P. Jordan, *Zur Quantenmechanik,* Zeitschrift für Physik 34 (1925), p. 858 (English translation B. L. van der Waerden, *Sources of Quantum Mechanics, op. cit.*); M. Born, W. Heisenberg and P. Jordan, *Zur Quantenmechanik II, ibid.* (1926), p. 557 (English translation in B. L. van der Waerden, *op. cit.*).

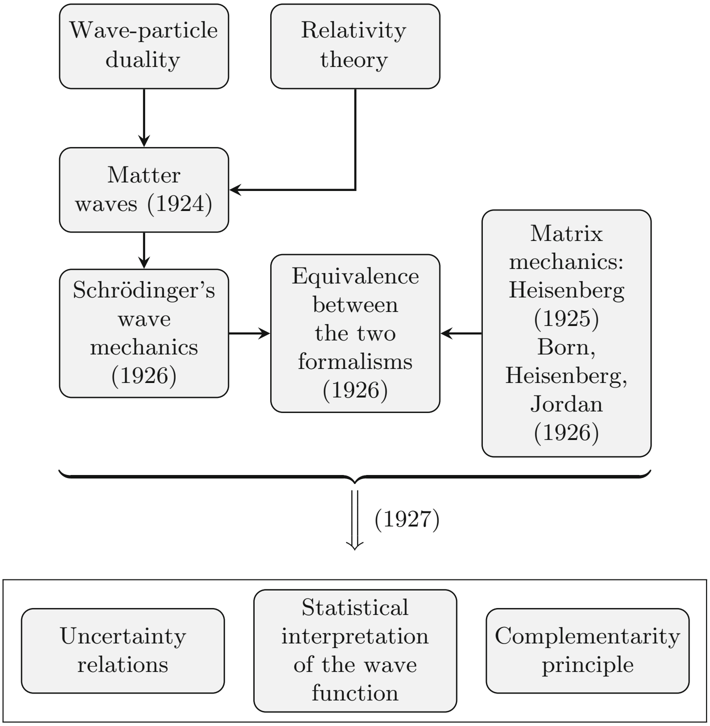

Fig. 2.13 Lines of research from the beginnings of quantum mechanics to the Copenhagen interpretation.

Schrödinger[96] and Carl Eckart[97] are to be credited for proving the equivalence between the two formalisms. Starting from the end of the 1920s, the term "quantum mechanics" referred both to wave and matrix mechanics, and became synonymous to "quantum theories," denoting the complex of theories and interpretative apparatuses that replaced the *OQT*.

Figure 2.13 sketches the most important moments of the formulation of quantum mechanics. In this theory, perhaps more than in any other, the interpretative and more essentially philosophical level played a fundamental role in its birth, and in its acceptance by the scientific community. The physical interpretation of the new theory turned out to be a much more challenging problem than just writing the equations.

[96]E. Schrödinger, *Über das Verhältnis des Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen,* Annalen der Physik 79 (1926), p. 734.

[97]C. Eckart, *Operator calculus and the solution of the equations of quantum dynamics,* Physical Review 27 (1926), p. 711.

De Broglie's proposal, as he told in his 1929 Nobel prize speech,[98] was rooted in a deep feeling of inadequacy of the wave-particle duality, together with the impossibility to understand why, among the infinite number of classical motions that are possible according to classical physics, only a few actually take place. On the other hand, De Broglie remarked again, the equation defining the energy of a light quantum ($E = h\nu$) is not, from the particle viewpoint, a satisfactory definition, due to the presence of the quantity ν that expresses a frequency. De Broglie stressed another aspect: the stationary motions of the electrons inside the atomic structure. These were characterized by integer numbers, and it was known that "the physical phenomena that involve only integer numbers are those related to interference." It was this consideration that suggested De Broglie to represent electrons not just as simple corpuscles, but rather to assign them a "certain periodicity":

> On the other hand the determination of the stable motions of the electrons in the atom involves whole numbers, and so far the only phenomena in which whole numbers were involved in physics were those of interference and of eigenvibrations. That suggested the idea to me that electrons themselves could not be represented as simple corpuscles either, but that a periodicity had also to be assigned to them too. I thus arrived at the following overall concept which guided my studies: for both matter and radiations, light in particular, it is necessary to introduce the corpuscle concept and the wave concept at the same time. In other words the existence of corpuscles accompanied by waves has to be assumed in all cases.[99]

Relativity theory was the formal instrument which allowed the French scientist to associate a wave with the motion of a particle. In relativity, energy and momentum are treated as quantities of the same kind.[100] If a particle of energy E, according to the relation $E = h\nu$, is associated with a wave of frequency ν, then a wave of wavelength λ should be associated with a particle with momentum $p = mv$, according to the relation $\lambda = h/p = h/mv$.

Schrödinger's "wave mechanics" was based indeed on this relation between momentum and the associated wavelength. By assigning a wave nature also to matter, one could establish a strict analogy between optical and mechanical phenomena; as in optics the behavior of a wave packet is determined by the refraction index of the medium in which it propagates, in the same way in mechanics the motion of a point particle is determined by the force field in which it moves. Moreover, as a ray of light traveling between two points in a certain medium chooses the path that is traversed in the least time (Fermat's principle), also in mechanics the trajectory of a point particle is determined by a minimum principle; it should indeed minimize the *action,* a quantity depending on the kinetic and potential energy. But there is more: this analogy allows one to understand why classical physics fails to describe

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[98] L. de Broglie, *La nature ondulatoire de l'électron,* Nobel Prize acceptance speech given in Stockholm on 12 December 1929. In *Nobel Lectures, Physics 1922–1941*, Elsevier Publishing Company, Amsterdam, 1965, p. 247.

[99] *Ibid.*

[100] In the theory of relativity, momentum is the spatial part of a four-vector, whose time component is energy.

the atomic structure. In optics indeed one can assume that light rays propagate along straight lines (the approximation on which geometric optics is based) until the dimensions of the physical system under study are big as compared with the wavelength of the light ray. In the same way, the laws of classical mechanics cease to be valid when the wavelength of the particle, as given by De Broglie's relation, is comparable with the dimensions of the system with which it interacts.

De Broglie's hypothesis was confirmed by an experiment performed by Davisson and Germer in 1927,[101] exactly as the photoelectric and Compton effects[102] had provided an empirical foundation to support the particle-like nature of the electromagnetic radiation. Davisson and Germer's experiment showed beyond any doubt that electrons are subject to diffraction phenomena, the trademark of a wave-like behavior.

This analogy between optics and mechanics is very important from the formal viewpoint. In the same way as electric and magnetic fields satisfy a wave equation, one can write an equation that the wave associated with the particles, the *wave function* $\Psi(x, y, z, t)$, a function of the coordinates and time, must obey. This is Schrödinger's equation. To study the dynamics of a system, thus, one needs to determine the time evolution of the wave function. Schrödinger's equation, moreover, allows one to compute the permitted values of the energy for a given atomic system; this is one of the most important and innovative features of wave mechanics. The discontinuity of the stationary states, which was one of the most unsatisfactory aspects of quantum theory, is a natural consequence of the properties of the solutions of Schrödinger's equation. The latter only admits regular solutions when the energy E takes some well-determined values (the *eigenvalues* of the system), which, according to the specific situation under study, can be distributed continuously in an interval, or form a discrete series.

Also for Heisenberg the starting point for the construction of the new mechanics was a matter of interpretation. In this case it was the belief that the theory had to be founded only on relations among observable quantities. In the new formalism there was to be no room for quantities such as the diameter or the eccentricity of the orbit of an electron, as opposed, for instance, to the frequency of the emitted radiation; this is an observable quantity, contrary to the diameter of the orbit. To understand the implications of Heisenberg's perspective for the formalism of the theory, let us consider the radiation emitted by an atom. Its frequency depends on the transition of an electron between two energy levels; we can therefore denote it by ν_{nm}, where the indexes n and m denote the two levels. If we want to represent all emitted frequencies, we need a matrix with an infinite number of rows and columns:

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[101] C. J. Davisson and L. H. Germer, *Diffraction of electrons by a crystal of nickel,* Physical Review 30 (1927), p. 705.

[102] The Compton effect is due to the particle-like interaction between electrons and electromagnetic radiation, and may be thought of as the scattering between an electron at rest and a photon of energy $h\nu$ and momentum $h\nu/c$. During the scattering the photon is deviated from its original direction, and its frequency changes, due to the loss of energy. As in all scattering processes, momentum is conserved, and the momentum of the incident photon is the same as the sum of the momenta of the electron and the photon after the collision.

$$\begin{vmatrix} \nu_{11} & \nu_{12} & \dots & \nu_{1m} & \dots \\ \nu_{21} & \nu_{22} & \dots & \nu_{1m} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \nu_{n1} & \nu_{n2} & \dots & \nu_{nm} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix}$$

Matrices are not just plain tables of numbers, but rather are mathematical objects, which can be summed and multiplied. The algebra of matrices is substantially different from that of the real numbers; for instance, it is not commutative, that is, the product AB, where A and B are matrices, is in general different from BA.

Heisenberg's 1925 proposal took place within this formalism. It was developed over the next few months by Born and Jordan, and completed, still in 1925, by Born, Heisenberg, and Jordan. The idea was to associate a matrix to any physical quantity, and transpose the equations of classical physics into the new mechanics, considering them as relations between matrices. In this perspective, also the position q and the momentum p of a particle are non-commuting matrices, i.e., $qp \neq pq$. This relation expresses the deviation between the new "matrix mechanics" and classical mechanics; while position and momentum commute in classical mechanics, they do not in quantum mechanics. The deviation between qp and pq is given by the Planck constant, according to the formula

$$pq - qp = \frac{h}{2\pi i} I,$$

where i is the imaginary unit and I is the identity matrix, with entries 1 on the main diagonal, and 0 elsewhere (it is the equivalent of the number 1 for the product of real numbers).

A few months after the publication of the long paper by Born, Heisenberg, and Jordan, whose final part was devoted to the physical application of the theory, Pauli published an important paper,[103] where matrix mechanics was applied, with brilliant results, to the computation of the spectrum of the hydrogen atom.

After 13 years, Bohr's queer quantization rules, which had raised so many perplexities, found a formal interpretation within two theories that led to the same results, but were otherwise very different in what concerns their starting point, methods, concepts, and formalism. The matrix mechanics of Born, Heisenberg, and Jordan, by replacing the continuous variables of classical physics with discrete sets of numerical quantities, signed, as it was indeed in the intentions of its authors, a radical break from classical physics; on the contrary, Schrödinger's wave mechanics moved in the opposite direction, toward a theory of the continuum.

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[103] W. Pauli, *Über das Wasserstoffspektrum vom Standpunkt der neuen Quantenmechanik,* Zeitschrift für Physik 36 (1926), p. 336 (English translation in L. van der Waerden, *Sources of Quantum Mechanics, op. cit.*).

Schrödinger's proof, and the independent one by Eckart, of the substantial equivalence between the two theories, was no little surprise for the scientific community. As George Gamow famously said,

> It was just as surprising as the statement that whales and dolphins are not fish like sharks or herring but animals like elephants or horses.[104]

But what is really a wave function? What really are the matter waves? To answer these questions is not easy now, as it was not at that time. Schrödinger, however, was firmly convinced that the true nature of particles was wave-like. Particles move like the impulsion given to a stretched string causes a perturbation that propagates along the string (a *wave packet*). Already in 1926, Max Born's statistical interpretation of the wave function[105] showed that the truth was different; the function $\Psi(x, y, z, t)$, or better said, the quantity $|\Psi(x, y, z, t)|^2\, dx\, dy\, dz$, represents the probability that the particle at the instant t is in the volume $dx\, dy\, dz$ located at the position (x, y, z).

Heisenberg in 1927 introduced the celebrated "uncertainty relations,"[106] and Bohr formulated the "complementarity principle."[107] In this way, the last conceptual cruxes of the so-called "Copenhagen interpretation" were fixed, and that extraordinary process of construction of the new physics, that had started only a few years before, came to an end (see Appendix C.7 for a more detailed discussion). As we have already mentioned several times, the cognitive importance of the scientific theory does not reduce to having solved the problems that originated it, but rather, is its capacity to be a starting point for the formulation of new problems and the creation of new interpretation paradigms. The creation of quantum mechanics was not only a goal, but also opened new and promising research paths. The most important among these was the construction of a *quantum electrodynamics* (see Appendix C.8), namely, a theory capable of quantizing the electromagnetic field, also when interacting with electrons, and compatible with relativity (Schrödinger's equation, indeed, is not relativistic). The first hints of this project can be found in the two already cited 1925 papers, one by Born and Jordan, the other by Born, Heisenberg, and Jordan, where the electromagnetic field *in vacuo* (in the absence of matter) was quantized. However, a viable and successful theory of quantum electrodynamics was eventually formulated by Dirac in his 1927 papers.[108] That theory would have been studied and improved over the following 20 years.

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[104] G. Gamow, *op. cit.*, p. 105.

[105] M. Born, *Zur Quantenmechanik der Stoßvorgange,* Zeitschrift für Physik 37 (1926), p. 863.

[106] W. Heisenberg, *Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik,* Zeitschrift für Physik 43 (1927), p. 172. English translation *The actual content of quantum theoretical kinematics and mechanics,* in *NASA Technical Memorandum,* TM-77379, https://archive.org/details/nasa_techdoc_19840008978.

[107] N. Bohr, *The quantum postulate and the recent development of atomic theory,* Nature 121 (1928), p. 580.

[108] P. A. M. Dirac, *The quantum theory of the emission and absorption of radiation,* Proceedings of the Royal Society A 114 (1927), p. 243.

2.8.6 *Quantum mechanics and nuclear protophysics: the anomalies of the (p-e) model*

Quantum mechanics played a decisive role in the development of nuclear protophysics. The several variants of the (p-e) model, which were created to give a coherent organization to the rapidly increasing amount of experimental data, needed to be confronted with a theory which was becoming the official language of atomic physics. This confrontation had two effects. On the one hand, the success of the application of the theory to some nuclear phenomena was regarded as a token of the effectiveness of the theory, which therefore could claim to be the most natural framework to develop a theory of the atomic nucleus. On the other hand, some features of the (p-e) model, considered within the new theory, led to apparent anomalies.

The most striking result of quantum mechanics in the interpretation of nuclear phenomena is the explanation of the α emission given in 1928 by George Gamow, and, independently, by Roland W. Gurney and Edward Condon.[109] In the latter paper one can read:

> It seems, however, that the new quantum mechanics has had sufficient success to justify the hope that it is competent to carry out an effective attack on the problem. The quantum mechanics has in it just those statistical elements which would seem appropriate to an explanation of the phenomenon of radioactive decay.

Both papers aimed to explain the radioactive decay as a consequence of the quantum mechanical laws, in particular, of the wave-like properties of matter. Let us have a look at Figure 2.14. It represents a "potential well" containing a particle. We may think of a person in an enclosure surrounded by a fence; the gray areas of the graph represent the fence, and its height denotes the energy spent by the person to jump over the fence. From the classical viewpoint, if the energy of an α particle is smaller than the confining potential, the particle will never leave the nucleus that contains it. The viewpoint of quantum mechanics is completely different. The solution of the Schrödinger equation for a particle in that potential is an oscillating

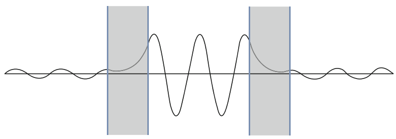

Fig. 2.14 Particle in a potential well. The curves in the three regions (inner, outer, and inside the walls) represent the wave function of the particle, computed by solving Schrödinger's equation.

[109] G. Gamow, *Zur Quantentheorie des Atomkernes,* Zeitschrift für Physik 51 (1928), p. 204; R. W. Gurney and E. Condon, *Wave mechanics and radioactive disintegration,* Nature 122 (1928), p. 439; *Quantum mechanics and radioactive disintegration,* Physical Review 33 (1928), p. 127.

function inside and outside the well, and is exponentially decreasing (toward the exterior) inside the walls (the regions a, b, and c in the figure). According to the probabilistic interpretation of the wave function, these solutions show that there is a nonzero probability that an α particle can be found outside of the potential well (i.e., the nucleus). In other terms, there is no need to figure out cataclysmic scenarios to account for the expulsion of an α particle; it is enough to solve a differential equation to understand that a particle has a certain probability to escape from the nucleus, although the confining potential is larger than its energy. As Gurney and Condon observe,

> Much has been written of the explosive violence with which the α-particle is hurled from its place in the nucleus. But from the process pictured above, one would rather say that the α-particle slips away almost unnoticed.[110]

The anomalies of the (p-e) model, i.e., its contradictions with quantum mechanics, are essentially four, all about the presence of electrons inside the nucleus. Let us examine them one by one.

Confinement anomaly. The uncertainty principle tells us that the indeterminacies Δp and Δq in the measurements of the position and impulse of a particle are related by the inequality $\Delta p \cdot \Delta q \geq \hbar/2$. If one assumes that an electron is confined inside the nucleus of an atom, the uncertainty Δq is of the order of the linear dimensions of the nucleus. Plugging the data into the equation, one obtains a completely unreasonable value for the momentum of the electron. This problem was first mentioned in a 1929 paper by Rutherford,[111] and was afterwards examined in detail by Klein[112] and Gamow,[113] who stressed the importance of the anomaly.

Spin-statistics anomaly. From the quantum mechanical perspective, the notion of spin, and the relations between Pauli's principle and the statistical properties of a collection of identical particles, establish new features of the electron and of the photon. As a consequence, also the atomic nucleus, regarded as a system formed by electrons and protons, can be studied in more detail. This poses three problems.

a) Given a system formed by a collection of particles whose spin is known, what is the spin of the system?
b) Given a system formed by a collection of particles, and knowing what statistic every particle obeys, what is the statistics of the system?
c) What are the spin and statistics of the proton?

[110] R. W. Gurney and E. Condon, *Wave mechanics and radioactive disintegration, op. cit.*

[111] E. Rutherford, *Discussion on the structure of atomic nuclei,* Proceedings of the Royal Society A 123 (1929), p. 373.

[112] O. Klein, *Die Reflexion von Elektronen an einem Potentialsprung nach der relativistischen Dynamik von Dirac,* Zeitschrift für Physik 53 (1929), p. 157.

[113] G. Gamow, *Constitution of Atomic Nuclei and Radioactivity,* Oxford University Press, Oxford 1931.

We shall consider the first two questions in the most significant case, when all particles have half-integer spin, or have Fermi statistics, respectively. Quantum mechanics gives an immediate answer to a):

a) A system formed by N particles, each having half-integer spin, has integer spin if N is even, half-integer spin if N is odd.

The answer to b) is

b) A system formed by N particles, each having the Fermi statistics, obeys the Bose-Einstein statistics if N is even, the Fermi-Dirac statistics if N is odd.

The story of this question is more complex.[114] Its first rigorous deduction was given in a 1929 paper by Wigner.[115]

Question c) was studied by David M. Dennison[116] on the basis of a preliminary study by Friedrich Hund.[117] Dennison analyzed the anomalous behavior of the hydrogen at low temperatures and deduced that the proton has spin 1/2 and obeys the Fermi-Dirac statistics.

The first signs of the anomalous behavior of the nuclear electrons with respect to spin appeared before the formulation of quantum mechanics.[118]

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[114] A detailed analysis was made in G. Bruzzaniti, *op. cit.*

[115] E. P. Wigner, *Ősszetett rendszerek statisztikája az új quantum-mechanika szerint,* Matematikai és Természettudománti Értesítő 46 (1929), pp. 576 ff. (German abstract at p. 584). Being written in Hungarian, the paper was unnoticed by most physicists, and indeed in 1931 P. Ehrenfest and J. R. Oppenheimer (*Note on the statistics of nuclei,* Physical Review 37 (1931) p. 333), starting from different considerations, gave a new and definitive proof.

[116] D. M. Dennison, *A note on the specific heat of the hydrogen molecule,* Proceedings of the Royal Society A 115 (1927), p. 483.

[117] F. Hund, *Zur Deutung der Molekelspektren. II,* Zeitschrift für Physik 42 (1927), p. 93.

[118] Kronig was the first to notice an immediate consequence of Uhlenbeck and Goudsmit's hypothesis about the electron spin (R. Kronig, *Spinning electrons and the structure of spectra,* Nature 117 (1926), p. 550). Kronig had already thought of relating the electron spin to Pauli's exclusion principle, but, after a suggestion of Pauli's, did not publish the idea. According to Kronig, due to its spin, the electron must have an intrinsic magnetic moment, of the order of magnitude of Bohr's magneton. One should expect the same behavior when an electron is part of the nuclear structure. Then the nucleus should have an intrinsic magnetic moment of the same order of magnitude, unless a very unlikely mechanism takes place, namely, that the magnetic moments of all the electrons cancel each other. If this were true, by the Zeeman effect there would be a splitting of the energy levels, which is not observed. Fermi and Rasetti also entered the discussion, remarking that Kronig's objection had another consequence; a nuclear magnetic moment should induce a paramagnetic behavior of the atom, which, however, is not observed. Fermi and Rasetti, on the other hand, thought that the magnetic moments of the nuclear electrons could well cancel each other. However, they put forward another important consideration. The electron magnetic moment corresponds to some energy, which, according to the electromagnetic theory of mass, yields an increase of the mass, and therefore of the electron radius. They wrote "This value is about 20 times larger of what the electron radius is usually supposed to be. Actually, there are no direct measures of the electron radius; however, this is a serious drawback, as we know that the nucleus contains a large number of electrons. On the other hand, the linear dimensions of the nuclear structure are known with a fairly good precision [...] and they are of about 10^{-12} cm. The two facts cannot be

However, the most striking evidence was provided by a 1928 paper by Kronig,[119] who, interpreting Ornstein and van Wijk's[120] experiments on the spectrum of the nitrogen molecule, concluded that the spin of that molecule is 1. This allowed Kronig to detect the following anomaly: the nitrogen nucleus, having charge 7 and mass 14, is formed by 14 protons and 7 electrons, i.e., by an odd number (21) of particles of spin 1/2, so one should expect a half-integer value of the total spin, contrary to the experimental data. As Kronig wrote,

> We know that both electrons and protons have a rotational impulse $s = 1/2$, and one expects, therefore, that, as for the electrons outside the nucleus, the rotational impulse of an odd number of particles can only take half-integer values. [...] One is therefore compelled to believe that inside the nucleus protons and electrons do not maintain their identity, as they do out of the nucleus.[121]

Kronig's conclusion was typical of the standing of the entire scientific community; the anomalies concerning the presence of electrons inside the nucleus did not affect the conviction of their existence, but rather incepted a critical process that will end in a redefinition of the properties of the electrons.

The anomaly of the statistics followed a similar pattern. In 1929 Heitler and Herzberg,[122] while analyzing the results of a series of experiments on the Raman effect for hydrogen and nitrogen made by Rasetti,[123] concluded that the nitrogen nucleus obeys the Bose-Einstein statistics. Since according to the (p-e) model, as we saw above, the nucleus was formed by an odd number of particles obeying the Fermi-Dirac statistics, it should have obeyed that statistics as well. From Heitler-Herzberg's paper:

> This is quite an unexpected fact. The nitrogen nucleus contains altogether 14 protons and 7 electrons, or, if we figure out the maximum possible number of α-particles, it contains three α particles, two protons, and one electron. From quantum mechanics one obtains that systems made up of an even/odd number of protons or electrons obey the Bose/Fermi statistics, since protons and electron themselves obey the Fermi statistics. This rule, if Rasetti's measurements are correct, does not apply anymore in the nucleus [...] It seems

reconciled, unless one assumes that the nature of the electron changes substantially when it is part of the nuclear structure." (Fermi [35], p. 227.)

[119]R. Kronig, *Der Drehimpuls des Stickstoffkerns,* Naturwissenschaften 16(1928), p. 335.

[120]L. S. Ornstein and W. R. van Wijk, *Untersuchungen über das negative Stickstoffbandenspektrum,* Zeitschrift für Physik 49 (1928), p. 315.

[121]R. Kronig, *Der Drehimpuls des Stickstoffkerns, op. cit.*

[122]W. Heitler and G. Herzberg, *Gehorchen die Stickstoffkerne der Boseschen Statistik?*, Naturwissenschaften 17 (1929), p. 673.

[123]F. Rasetti, *Raman effect in gases,* Nature 123 (1929), p. 205; *On the Raman effect in diatomic gases,* Proceedings of the National Academy of Sciences 15 (1929), p. 234; *Selection rules in the Raman effect,* Nature 122 (1929), p. 757; *On the Raman effect in diatomic gases II,* Proceedings of the National Academy of Sciences 15 (1929), p. 515; *Incoherent scattered radiation in diatomic molecules,* Physical Review 34 (1929), p. 367; *Alternating intensities in the spectrum of nitrogen,* Nature 124 (1929), p. 792; *Sopra l'effetto Raman nelle molecole biatomiche,* Nuovo Cimento 6 (1929), p. 356.

> that the electron, inside the nucleus, loses, together with its spin, also the right to take part in the statistics of the nucleus (in the sense of the previously cited rule). [. . .] Quite evidently, the mechanical properties of nuclear electrons are modified much more deeply than those of protons and α particles. Only the conservation of charge is certain.[124]

As with spin, also in this case the scientific community, faced by the failure of all attempts to apply quantum mechanics to the (p-e) model, did not doubt of the existence of the electrons inside the nucleus, but rather hypothesized a possible dependence between the properties of the electron and its role inside the atom.

Anomaly of the continuous spectrum of the β decay. The history of the discovery of the β decay is very long. It started in the first years of the 20th century and ended in 1933 with Fermi.[125] The most important dates are 1914 and 1927; let us see what happened in those years.

In 1914 Chadwick discovered that the electrons emitted by the radioactive substances have a continuous spectrum. This gave rise to a serious problem. Indeed this result radically deviated from the general picture of radioactivity as provided by the *OQT,* which on the other hand was well confirmed by the α and γ emissions:[126]

$$\text{parent nucleus} \rightarrow \text{daughter nucleus} + \text{emitted particle}.$$

If one admitted the existence of stationary states for the nuclei, i.e., states with well-defined constant energy, so that the energies of the parent and of the daughter nucleus had some fixed value, then the β particles had to have a well-defined energy, while the experiments showed a range of values for the energy, continuously distributed between a minimum and a maximum (see Figure 2.15). The principle of conservation of energy was at stake, unless one speculated, like Lisa Meitner,[127] that the measured energy distribution originated from a secondary phenomenon. According to Meitner, the electrons were emitted by the nuclei with a fixed speed, corresponding to the upper limit of the energy spectrum, but then, due to collision processes with the atomic electronic structure, lost part of their energy, giving

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[124] W. Heitler and G. Herzberg, *Gehorchen die Stickstoffkerne der Boseschen Statistik?, op. cit.*

[125] There is an extensive literature about the history of the β decay. We may cite, among the works of general nature that contain further references: A. Pais, *Inward Bound,* Oxford University Press, New York 1986; *Niels Bohr's Times, op. cit.;* C. S. Wu and S. A. Moszkowski, *Beta Decay,* Academic Press, New York 1966; G. Bruzzaniti, *op. cit.*.

[126] Very accurate measurements of the γ-rays frequencies allowed Ellis (Ch. D. Ellis, *The magnetic spectrum of the β-rays excited by γ-rays,* Proceedings of the Royal Society 99 (1921), p. 261; *β-rays spectra and their meaning, ibid.* 101 (1922), p. 1) to introduce also for nuclei the notion of "stationary state." In the second paper, Ellis remarkably wrote, about the measurement of the frequency of the γ-rays emitted by radium B: "The information [. . .] about the energies of the stationary states of the radium B nucleus is extra-ordinarily detailed, but, on the other hand, this information is very limited. There is no evidence which indicates whether these levels are occupied by positively charged particles or by electrons."

[127] L. Meitner, *Über die Entstehung der β-Strahl-Spektren radioaktiver Substanzen,* Zeitschrift für Physik 9 (1922), p. 131; *Über den Zusammenhang zwischen β und γ Strahlen, ibid.*, p. 145; *Über die β-Strahl-Spektra und ihren Zusammenhang mit der γ-Strahlung, ibid.* 11 (1922), p. 35.

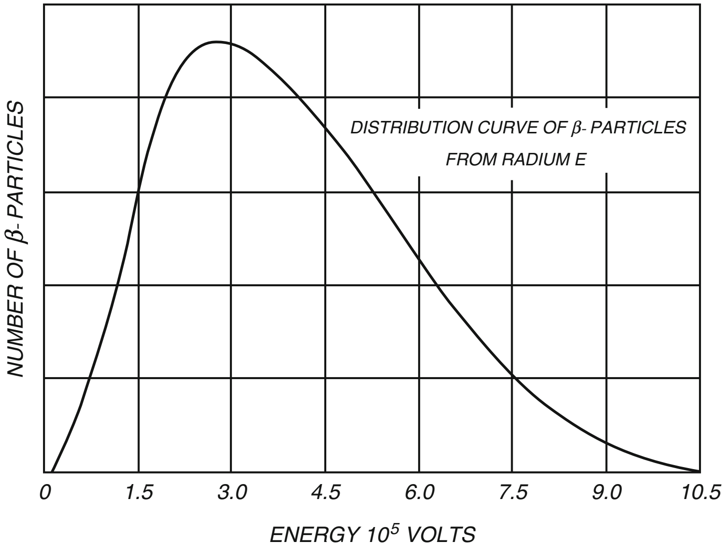

Fig. 2.15 The average energy of electrons emitted by radium E as measured by Ellis and Wooster.

rise to a continuous spectrum. This proposal immediately sparked an animated controversy; the continuous spectrum was the result of a primary phenomenon, the emission of electrons with the measured energy, or rather of a secondary phenomenon?

The answer was given in 1927 by a famous experiment made by Ellis and Wooster,[128] based on a simple procedure that made it "completely free from any hypothesis." They placed the substance emitting the electrons inside a calorimeter, whose walls could completely absorb the β radiation, together with any other radiation. Now, if quantum mechanics was correct, each emitted electron carried an energy e, and if N electrons were emitted, a total energy $E = e \times N$ was discharged during the process. Even if there was some secondary process (the emitted electrons interacted with the orbital electrons, for instance), and if the conservation of energy held, all the energy E would have been sooner or later revealed by the calorimeter. The average energy would have amounted to 1,05 MeV. However, the measurements gave the value of 350 keV, with an experimental uncertainty of 40 keV. This perfectly fitted the value provided by the curve in Figure 2.15 (390 keV), thus confirming that the energy spectrum was continuous.

Three hypotheses were put forward to solve this serious anomaly (the existence of a continuous spectrum): a) abandon the principles of quantum mechanics; b) abandon the principle of conservation of energy; and c) assume that during the β

[128] C. D. Ellis and W. A. Wooster, *The average energy of disintegration of radium E,* Proceedings of the Royal Society A 117 (1927), p. 109.

emission, an unknown particle is emitted, so that the sum of its energy with that of the emitted electron equals the difference between the energies of the parent and the daughter nucleus. The proposal a) was mainly supported by Ellis,[129] b) by Bohr,[130] while c), which would turn out to be correct explanation, was backed by Wolfgang Pauli. Appendix B.2 includes the letter in which Pauli introduced, in a somehow unusual way, the hypothesis of the existence of this new particle, called "neutron" by Pauli, and then identified by Fermi with the neutrino. There Pauli used the term "exchange theorem" to refer to the spin-statistics connection.[131]

2.8.7 New discoveries and first nuclear theories

The years from 1928 to 1930 were decisive for the nuclear protophysics. The application of quantum mechanics to the theory of the nucleus sparked the crisis of the (p-e) model, and triggered a growing interest in nuclear physics, as witnessed by the first Conference of Nuclear Physics that took place in Rome in 1931, in the eve of a series of discoveries that laid the foundations for Fermi's theory of the β decay. This in turn induced the abandonment of the (p-e) model.

In 1932, the "annus mirabilis" according to Emilio Segrè's definition, three extraordinary discoveries were made.[132]

Deuterium. On 5 December 1931 Harold C. Urey, Ferdinand C. Brickwedde, and George M. Murphy announced the discovery of deuterium, an isotope of hydrogen of mass 2, whose relative abundance with respect to usual hydrogen is about 1/4,000.[133] The hydrogen atom, which is the simplest atomic structure, was the best candidate for investigations on atomic physics, with the hope of getting a better understanding of more complex atomic structures; analogously, the deuterium nucleus, the simplest among the complex nuclei, gave hopes to reach a better understanding of the laws that regulate, in general, the structure of all nuclei.

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[129] *Ibid.*

[130] N. Bohr, *Faraday Lectures: chemistry and the quantum theory of the atoms constitution,* Journal of the Chemical Society (1932), p. 349.

[131] For more details see Appendix C.5.

[132] Among the many historical reconstructions of these discoveries, we mention the following: E. Amaldi, *From the discovery of the neutron to the discovery of nuclear fission,* Physics Reports 11 (1984), p. 1; J. Six, *La découverte du neutron,* Editions du Centre National de la Recherche Scientifique, Paris 1987; M. De Maria and A. Russo, *The discovery of positron,* Rivista di Storia della Scienza 2 (1985), p. 237.

[133] H. C. Urey, F. C. Brickwedde and G. M. Murphy, *A hydrogen isotope of mass 2,* Physical Review 39 (1932), p. 164. The paper by R. Stuewer, *The naming of the deuteron,* American Journal of Physics 54 (1986), p. 206, is an interesting rendering of the debate that between 1993 and 1935 took place about the naming of the mass 2 isotope of hydrogen.

Positron.

On August 2, 1932, during the course of photographing cosmic-ray tracks produced in a vertical Wilson chamber (magnetic field of 15,000 gauss) designed in the summer of 1930 by Professor R. A. Millikan and the writer, the tracks shown in Fig. 1 were obtained, which seemed to be interpretable only on the basis of the existence in this case of a particle carrying a positive charge but having a mass of the same order of magnitude as that normally possessed by a free negative electron.[134]

With these words, Carl D. Anderson communicated in 1933 the discovery of a new particle, that he called "positron." The previous year he had published a short paper in the journal *Science.* Anderson's discovery was confirmed by Blackett and Occhialini,[135] who related the existence of the positron with the hypotheses formulated by Dirac in 1930. Dirac had shown that the validity of a relativistic wave equation for the electron implied the existence of electron states of both positive and negative energy. To overcome this difficulty, Dirac made the hypothesis that all negative energy states, called "gaps," are fully occupied. Since electrons obey the Pauli exclusion principle, no transition to a negative energy state is possible. If, for some reason, a gap should not be occupied, it would behave as a positively charged electron.

Neutron. The neutron was discovered in 1932, but the story actually started in 1930, when Bothe and Becker,[136] bombarding light atoms with α particles emitted by polonium, obtained very penetrating secondary rays. The effect is very strong with beryllium, but is also present with lithium, boron, fluorine, aluminum, magnesium, and sodium. Bothe and Becker's interpretation was that the radiation is of an electromagnetic nature. By measuring the absorption by lead of the radiation emitted by beryllium and boron, they managed to estimate the energy of the radiation. The next step toward the discovery of the neutron was taken by Irène Curie and Frédéric Joliot, who repeated Bothe and Becker's experiment with the aim of testing the existence of other effects produced by the penetrating radiation while traveling in matter.[137] The most striking result was obtained when the secondary radiation produced by beryllium hit some hydrogenated substance; in this case the ionizing effect became almost twice as strong with respect to the Po + Be case (i.e., when radiation emitted by beryllium was absorbed by lead). Substances not containing hydrogen produced on the contrary no increase in the ionization. The Joliot-Curies, after several experimental checks, concluded that the hydrogenated

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[134] C. D. Anderson, *The positive electron,* Physical Review 43 (1933), p. 491.

[135] P. M. S. Blackett and G. P. S. Occhialini, *Some photographs of the tracks of penetrating radiation,* Proceedings of the Royal Society A 139 (1933), p. 699.

[136] W. Bothe and H. Becker, *Kunstliche Erregung von Kern γ–Strahlen,* Zeitschrift für Physik 64 (1930), p. 289.

[137] F. Joliot and I. Curie, *Emission de protons de grande vitesse par les substances hydrogénées sous l'influence de rayons-γ très pénétrants,* Comptes Rendus de l'Académie des Sciences 194 (1932), p. 273; *Effet d'absorption des rayons-γ de très haute frequence par projection de noyaux légers, ibid.,* p. 708.

substances hit by the radiation emitted protons, due to a Compton effect between the photons in the Po + Be radiation and the hydrogen nuclei contained, e.g., in paraffin. James Chadwick, after a careful analysis, stressed some serious problems in this interpretation of the emission of protons as due to the Compton effect.[138] His observations left no room to doubt: Joliot-Curies' interpretation necessarily implied the violation of the principles of conservation of energy and momentum. The difficulties disappeared if one interpreted the radiation exited by α particles in beryllium as formed by particles of zero charge and mass one, i.e., "neutrons." In Chadwick's words:

> It has been shown by Bothe and others that beryllium when bombarded by α-particles of polonium emits a radiation of great penetrating power, which has an absorption coefficient in lead of about 0.3 cm^{-1}. Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionisation produced by this beryllium radiation in a vessel with a thin window, that the ionisation increased when matter containing hydrogen was placed in front of the window. The effect appeared to be due to the ejection of protons [...] They suggested that the transference of energy to the proton was by a process similar to the Compton effect, and estimated that the beryllium radiation had a quantum energy of 50×10^6 electron volts. I have made some experiments [...] These results, and others I have obtained in the course of the work, are very difficult to explain on the assumption that the radiation from beryllium is a quantum radiation, if energy and momentum are to be conserved in the collisions. The difficulties disappear, however, if it be assumed that the radiation consists of particles of mass 1 and charge 0, or neutrons. [...] It is to be expected that many of the effects of a neutron in passing through matter should resemble those of a quantum of high energy, and it is not easy to reach the final decision between the two hypotheses. Up to the present, all the evidence is in favour of the neutron, while the quantum hypothesis can only be upheld if the conservation of energy and momentum be relinquished at some point.[139]

The discovery of the neutron had an enormous impact on nuclear protophysics, but not because it was an expected event, that the scientific community was awaiting to solve the problem of the nuclear structure. In a sense, the discovery of the neutron did not directly trigger the end of the (p-e) model; it rather made the problem more complicated, but in an intelligent way.

Dmitri Ivanenko was among the first who regarded the neutron as new constituent of the nucleus. The Russian physicist, in a short but important paper in *Nature,* wrote:

> Is it not possible to admit that neutrons play also an important rôle in the building of nuclei, the nuclei electrons being all packed in α-particles or neutrons?[140]

It is not hard to interpret Ivanenko's question as the germ of a strategy that would characterize the first "post-neutron" nuclear theories: try to solve the anomalies of the (p-e) model by means of suitable hypotheses on the structure of the neutron.

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[138] J. Chadwick, *Possible existence of a neutron,* Nature 129 (1932), p. 312.

[139] *Ibid.,* pp. 312 ff. A more extended and detailed discussion of the experiment was communicated by Chadwick to the Royal Society on 10 May 1932; cf. J. Chadwick, *The existence of a neutron,* Proceedings of the Royal Society A 136 (1932), p. 692.

[140] D. Ivanenko, *The neutron hypothesis,* Nature 129 (1932), p. 798.

Another sentence by Ivanenko shows how the underlying principles of the (p-e) model were still strongly upheld by the scientific community:

> The lack of a theory of nuclei makes, of course, this assumption rather uncertain, but perhaps it sounds not so improbable if we remember that the nuclei electrons profoundly change their properties when entering into the nuclei, and lose, so to say, their individuality, for example, their spin and magnetic moment.[141]

The proceedings of the 7th Solvay Conference (1933)[142] provide a detailed testimonial of the discussions about the structure of the neutron that were taking place at that time. The main question was, is the neutron an elementary particle, or it is formed by a proton and an electron? The second hypothesis was corroborated by the measurement of the neutron mass; if it was smaller than the mass of the hydrogen atom, made up by a proton and an electron, one could think that the mass difference was due to the energy necessary to bind the electron and the proton. However, Chadwick[143] made it clear that that was not a correct reasoning. According to quantum mechanics, the hydrogen atom is the only possible combination of a proton and an electron. Moreover, if the neutron had that structure, the hydrogen atom should transform into a neutron, releasing energy. There is also an argument about spin. Experimental data attribute the neutron spin 1/2 and Fermi statistics, and since also the proton has spin 1/2, if the neutron was an electron-proton combination, the electron should have spin zero and obey the Bose statistics, contrary to the experimental evidence.

So, the discovery of neutron in 1932 was not enough to dispose of the nuclear electrons. This clearly emerges from Gamow's report at the Solvay Conference, where he stressed the possibility of building up models where the nucleus was formed by electrons and protons, together their compounds, such as neutrons and α particles. And also accepting that the neutron was elementary, the nuclear electrons still existed. Dirac clearly stated this:

> I think there is no final evidence against the hypothesis that electrons, having a nonzero spin and obeying the Fermi statistics, can be inside the nucleus as elementary particles. If we consider protons and neutrons as elementary particles, we have three kinds of elementary particles forming the nuclei. This number may seem to be big, but from this viewpoint, two is already a big number.[144]

The neutron thus was regarded as a new nuclear constituent, which joined the previously known constituents, but did not replace them.

This incertitude about the role of the nuclear electrons can be seen in what was the first attempt to a general schematization of the atomic nucleus, the Heisenberg-

[141] *Ibid.*

[142] *Structure et propriétés des noyaux atomiques. Rapports et discussions du Septième Conseil de Physique,* Gauthier-Villars, Paris 1934.

[143] J. Chadwick, *Diffusion anomale des particules α, ibid.,* p. 102.

[144] P. A. M. Dirac, *Théorie du positron, ibid.,* p. 203; W. Heisenberg, *Considérations théoriques générales sur la structure du noyau, ibid.,* p. 328.

Majorana theory. This term refers to a paper by Heisenberg, published in three parts in 1932–1933,[145] followed by a 1933 paper by Ettore Majorana, which included important changes. Heisenberg's proposal was based on five assumptions:

a) Nuclei are made of protons and neutrons. The neutron has spin 1/2 (in units $h/2\pi$) and obeys the Fermi-Dirac statistics.
b) The neutron may be formed by an electron and a proton. In this case the electron inside the neutron loses its spin and obeys the Bose-Einstein statistics. Moreover the disintegration of a neutron onto an electron plus a proton does not obey the conservation of energy.
c) The interaction between neutron and proton is not representable as an ordinary force, but rather as an "exchange force." The model which suggests this hypothesis to Heisenberg is the ionized hydrogen molecule H_2^+, whose chemical bond results from the two protons sharing one electron. In the same way, according to Heisenberg, a proton and a neutron interact inside a nucleus by exchanging a common electron. As D. G. Cassidy nicely put it,

   > In a sense, the neutron and proton play a wild game of catch with the one available electron, the proton turning into a neutron when it catches the electron, the neutron turning into a proton when it releases the electron — then back again.[146]

d) Again by analogy with the hydrogen molecule, this time not ionized, the neutron-neutron interaction is attractive, and is negligible with respect to the neutron-proton interaction. Both must vanish at distances greater than approximately 10^{-12} cm.
e) The proton-proton interaction is only due to the Coulomb force.

These assumptions were part of a framework that was very innovative as regards the way the proton electric charge was treated, in analogy with the electron spin; for this reason, Heisenberg's hypothesis was called "isotopic spin formalism." Its basic feature was the attribution to the heavy particles of a new coordinate r', that could only take the values 1 and -1: when $r' = 1$ the particle was a neutron, when $r' = -1$ a proton. The theory had many consequences, some of which, as it has been noticed in several historical reconstructions,[147] were evidently contradictory. As written by Cassidy,

> In other words, Heisenberg's neutron was simultaneously indivisible (fundamental), compound, and a contradiction of both quantum mechanics and conservation laws![148]

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[145] W. Heisenberg, *Über den Bau der Atomkerne, I,* Zeitschrift für Physik 77 (1932), p. 1; II, *ibid.* 78 (1932), p. 156; III, *ibid.* 80 (1933), p. 587. On this aspect of Heisenberg's work, see the excellent biography by D. Cassidy, *Uncertainty. The Life and Science of Werner Heisenberg,* Freeman, New York 1992, and J. Bromberg, *The impact of the neutron: Bohr and Heisenberg,* Historical Studies in the Physical and Biological Sciences 3 (1971), p. 307; D. M. Brink, *Nuclear Forces,* Pergamon Press, London 1965. The latter also contains the English translation of some papers of Heisenberg and Majorana's.

[146] David G. Cassidy, *Beyond Uncertainty: Heisenberg, Quantum Physics, and the Bomb.* Bellevue Literary Press, New York 2009. p. 203.

[147] See references in note 145.

[148] David G. Cassidy, *op. cit.,* p. 202.

Indeed, on the one hand, the isotopic spin formalism introduced a perfect symmetry between proton and neutron, and therefore gave the two particles the same ontological status; but on the other hand, the theory hypothesized an attractive interaction between neutrons, and a Coulomb interaction between protons. This, at least implicitly, denoted that the proton had a truly elementary nature, while the neutron was still composite: the attractive force between two neutrons was a manifestation of the fact that actually they are two protons which exchange two electrons.

In any case, Heisenberg's theory was prone to a very serious criticism: the proton-neutron interaction depended on the directions of the spin of the two particles. This implied that the proton-neutron bond leads to saturation: it is not possible that a second neutron binds to the proton. The theory also implied that the deuteron (the deuterium nucleus) was the most stable nucleus, contrary to the evidence that, as an analysis of the binding energies shows, it is the α particle which is most stable. Majorana had the merit of disposing of this difficulty by modifying the exchange interaction mechanism.[149] In his report at the 7th Solvay Conference, Heisenberg abandoned his original model in favor of the interaction mechanism proposed by Majorana. While the Heisenberg-Majorana theory was immediately hailed as "a general scheme of the nuclear structure," it was precarious and incomplete, as it was unable to solve the problem of the neutron structure. Moreover, the existence of the positron greatly amplified the problem of what really were the elementary particles; as observed by Anderson, why it is not possible that the neutron is elementary, and the proton is formed by a neutron and a positron?

As we see, the nuclear physics scenario had become much more complicated. The new discoveries had added new difficulties to the already existing anomalies, adding new pieces to a huge and intricate jigsaw. But at least, all pieces were there. The problem was to find the key idea to decipher the mystery, which at the time was still the methodological principle of the (p-e) model; if a particle is emitted by a nucleus, then it already existed inside the nucleus. This was clearly expressed by Heisenberg's above mentioned report at the 7th Solvay Conference: "To believe that electrons are part of the nuclei has no precise meaning, up to the fact that some nuclei emit β particles."

Enrico Fermi had the merit to understand that the way to reconstruct the jigsaw was to get rid of that principle. The decisive paper appeared in 1933,[150] and contained the basic idea of his theory; the electrons emitted in the β decay do not pre-exist inside the nucleus, but are created at the moment of their emission. This is analogous to what happens with the electromagnetic field; a photon is created at moment of its emission, as a consequence of the transition of a charged particle between two different quantum states. Moreover, to preserve the conservation of

[149] As already noted, according to Heisenberg, the proton-neutron interaction was due to the exchange of an electron, while spin was left unchanged. Majorana, on the contrary, postulated that the interaction involved the exchange of both charge and spin.

[150] Fermi [76, 80a].

energy, Fermi endorsed Heisenberg's proposal of a new particle, the neutrino, which is created together with the electron during a β decay process.

In addition to the genial assumption that nuclei are only formed by protons and neutrons, Fermi's theory was based on several ideas, that we give here in the author's words.

a) The total number of electrons and neutrinos is not necessarily constant. Electrons (or neutrinos) can be created or annihilated. This possibility however has no analogy with the creation of an electron-positron pair; indeed, if one interprets a positron as a Dirac "hole," the latter process can be simply regarded as a quantum jump of an electron from a negative energy state to one with positive energy, with conservation of the total (infinite) number of electrons.
b) According to Heisenberg, the heavy particles, neutron and proton, are two different inner quantum states of the same particle. This will be formalized by introducing an internal coordinate of the heavy particle, which takes only two values: $r = 1$ if the particle is a neutron, $r = -1$ if the particle is a proton.
c) The Hamiltonian function[151] of a system formed by heavy and light particles must be chosen so that every transition from neutron to proton is accompanied by the creation of an electron and a neutrino. Note that in this way there is conservation of the total electric charge.[152]

Fermi's formalism allowed him to encompass inside a unified theoretical framework the non-contradictory aspects of the Heisenberg-Majorana theory (nuclei formed only by protons and neutrons), the method of second quantization, the quantization of the electromagnetic field, and the description of the 1/2 spin particles provided by the Dirac equation. By disposing of the nuclear electrons, the theory solved the spin, statistics, and confinement anomalies. Moreover, the reaction hypothesized by Fermi

$$\text{neutron} \rightarrow \text{proton} + \text{electron} + \text{neutrino}$$

explains the continuous spectrum of the β decay without giving up the principle of conservation of energy. The last section of Fermi's paper was indeed devoted to the "comparison with the experimental data." He showed that the speed distribution of the β rays as predicted by his theory fitted the experimental data, up to a small disagreement at low energies.

These were already remarkable achievements, but even more important was the conceptual structure of the theory, which decreed the end of the (p-e) model, setting the grounds for the construction of a theoretical language capable of treating not only problems in nuclear physics, but also in a new theory, that was indeed incepted by Fermi's work: the theory of elementary particles. But this is another story, and we shall deal with it in the 4th Chapter.

[151]The Hamiltonian function is a function which is associated with a physical system and determines its evolution, both in classical and quantum mechanics.

[152]Fermi [80a], *CPF I*, p. 560.

2.9 A note of the dynamics of the global maps: the regulating principles and the birth of nuclear physics

Figure 2.16 describes the research lines that in the first 30 years of the 20th century led to the creation of a subject called "nuclear physics." The picture reproduces the plan of this second chapter: the reconstruction of a fragment of the global maps of our knowledge in which it is possible to identify Enrico Fermi's legacy. This reconstruction is possible only *a posteriori,* and does not include processes that can be attributed to the protagonist of the moment.[153] It is like describing Christopher Columbus's 1492 enterprise: from the viewpoint of global maps we are talking of the discovery of a new continent, that cannot be reduced to the project of the Genoese sailor of discovering a new way to the Indies; this is rather the local research itinerary.

We have given special attention to the development of nuclear protophysics, stressing how it can be identified with the birth, maturation, and final decline of the (p-e) model, that is, of the idea that there are electrons inside the nucleus. In 1931 Gamow published the first treatise on nuclear physics, entirely based on the (p-e) model. From the first page the author makes it clear that, in accordance with the tenets of modern physics, all nuclei are "built up elementary particles — protons and electrons."[154] The book pays much attention to the role of the α particles. Formed by four protons and two electrons, an α particle is so stable that it can be treated as a single entity. It is this remark that gives rise to the classification of the atomic nuclei into four classes: $4n$, $4n+1$, $4n+2$, and $4n+3$, with n an integer number. Since protons aggregate in the most stable configurations, the nuclei of the first type must be formed only by α particles, while in the nuclei of the other types there are one, two, or three free protons, respectively. A similar argument is applied to the electrons; there exist nuclei where not all nuclear electrons are bound to α particles. The question here becomes very delicate, and indeed Gamow decided to mark with a special typographical character ($\sim$) the sections concerning the difficulties raised by the properties of the nuclear electrons.

Gamow's treatment essentially concerned the difficulties that here we have called "anomalies," whose structure is represented in Figure 2.17. As shown by the documents, these difficulties did not affect the plausibility of the underlying principles of the (p-e) model, but rather changed the meaning of the term "electron."

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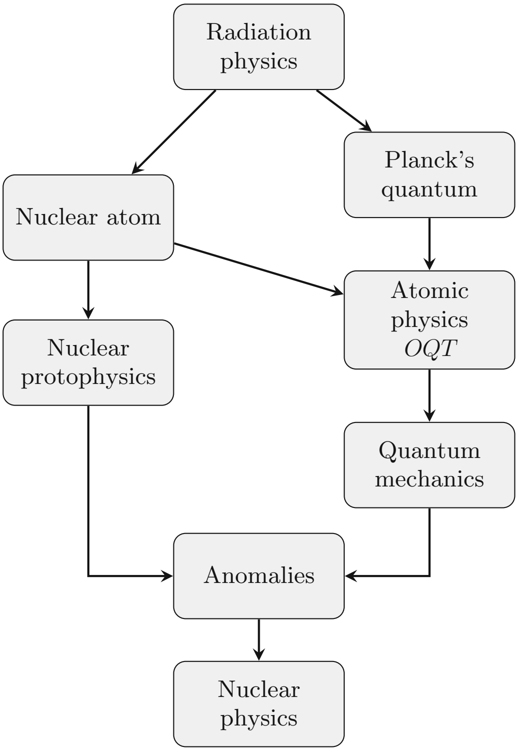
[153] I owe this remark to G. Bachelard, who characterized the history of science in these terms: "[...] it is a history that starts from the certainties of the present time and discovers in the past the progressive forms of the truth. Thus the history of science appears as the most irreversible of all histories. By discovering the truth, the man of science cancels irrationality. Irrationalism may certainly appear elsewhere, but by now some routes are impossible. The history of science is the history of the defeats of irrationalism." (G. Bachelard, L'activité rationaliste de la physique contemporaine, PUF, Paris 1965.)

[154] G. Gamow, *Constitution of Atomic Nuclei and Radioactivity,* Clarendon Press, Oxford 1931. p. 1.

Fig. 2.16 Main research lines leading to the creation of nuclear physics.



In the early 30s, so, the term "electron" denoted a rather complex object. When it was not inside an atomic nucleus, it had mass, charge, statistical properties, and spin, and participated in processes where the conservation of energy held. On the contrary, when it was inside a nucleus, it lost all its properties, with the only exception of charge and mass. Moreover, it took part in processes where the energy was not conserved. Even the extraordinary discoveries made in 1932 did not affect the (p-e) model. Neutrons entered the atomic nuclei, but did not solve the anomalies of the nuclear electrons, that indeed were regarded as constituents of the neutrons.

It is not difficult to find the roots of such an obstinate defense of the (p-e) model in the methodological principle that we have often mentioned; if a particle is emitted by a nucleus, then it pre-existed in the nucleus. It was a deeply rooted conviction, that had controlled from the outset the evolution of the nuclear protophysics, jeopardizing any attempt to develop a quantum theory of the atomic nucleus. It is interesting to compare the starting sentence of Gamow's book we have cited above, or Heisenberg's statements at the 7th Solvay Conference, with what Bethe and Bacher wrote in a 1936 paper that would have become the Bible of nuclear physics:

> Therefore we are forced to assume that the *electrons observed in β-disintegration did not pre-exist in the emitting nucleus. We suppose that they are formed in the same moment when*

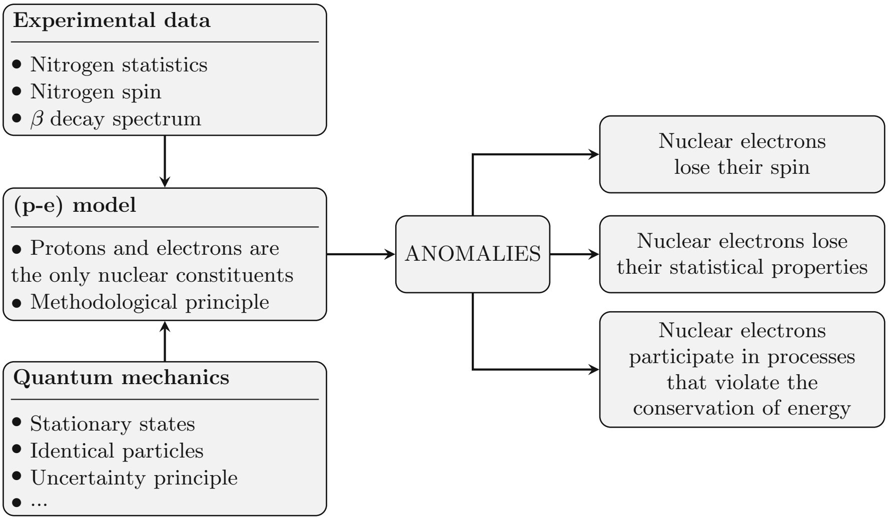


Fig. 2.17 Anomalies of the (p-e) model. These did not induce the abandonment of the notion of nuclear electrons, but changed the properties that were attributed to them.

> *they are actually emitted,* and that it is this process of formation which is so improbable that it accounts for the long lifetime of β-emitting nuclei.[155]

and again, with reference to the nature of the neutron:

> [...] the *neutron* should *not* be considered as *composed of a proton, an electron and a neutrino, but is only able of transforming into these three particles,* and similarly for the proton.[156]

It was the final demolition of the principle of pre-existence of the emitted electrons in the nuclei, and the birth certificate of the modern theories of the atomic nucleus. This certificate, as recorded in the global maps, bears Enrico Fermi's signature.

[155] H. A. Bethe, R. F. Bacher, *Nuclear Physics,* Review of Modern Physics vol. 8 (1936), p. 184. Italics in original.

[156] *Ibid.,* p. 189. Italics in original.

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