

Preface

The practical interest in contact problems stems from the fundamental role of contact in mechanics of solids and structures. Indeed, the contact of one body with another is a typical way how loads are delivered to a structure and the typical mechanism which supports structures to sustain the loads. Thus we do not exaggerate much if we say that the contact problems are in the heart of mechanical engineering.

The contact problems are also interesting from the computational point of view. The conditions of equilibrium of a system of bodies in mutual contact enhance a priori unknown boundary conditions, which make the contact problems strongly nonlinear, and when some of the bodies are “floating,” then the boundary conditions admit rigid body motions and a solution need not be unique. After the discretization, the contact problem can be reduced to a finite dimensional problem, such as the minimization of a possibly non-differentiable convex function in many variables (currently from tens of thousands to billions) subject to linear or nonlinear inequality constraints for surface variables, with a specific sparse structure. Due to the floating bodies, the cost function can have a positive semidefinite quadratic part. Thus the solution of large discretized multibody contact problems still remains a challenging task, which can hardly be solved by general algorithms.

The main purpose of this book is to present scalable algorithms for the solution of multibody contact problems of linear elasticity, including the problems with friction and dynamic contact problems. Most of these results were obtained during the last twenty years. Let us recall that an algorithm is said to be *numerically scalable* if the cost of the solution increases nearly proportionally to the number of unknown variables, and it enjoys *parallel scalability* if the computational time can be reduced nearly proportionally to the number of processors. The algorithms which enjoy numerical scalability are in a sense optimal as the cost of the solution by such algorithms increases as the cost of duplicating the solution. Taking into account the above characterization of contact problems, it is rather surprising that such algorithms exist.

Our development of scalable algorithms for contact problems is based on the following observations:

- There are algorithms which can solve relevant quadratic programming and QCQP problems with asymptotically linear complexity.
- Duality based methods like FETI let us define a sufficiently small linear subspace with the solution.
- The projector to the space of rigid body motions can be used to precondition both the linear and nonlinear steps of solution algorithms.
- The space decomposition used by the variants of FETI is an effective tool for solving multibody contact problems and opens a way to the massively parallel implementation of the algorithms.

The development of scalable algorithms represents a challenging task even when we consider much simpler linear problems. For example, the computational cost of solving a system of linear equations arising from the discretization of the conditions of equilibrium of an elastic body with prescribed displacements or traction on the boundary by direct sparse solvers increases typically with the square of the number of unknown nodal displacements. The first numerically scalable algorithms for linear problems of computational mechanics based on the concept of multigrid came in use only in the last quarter of the last century and fully scalable algorithms based on the FETI (Finite Element Tearing and Interconnecting) methods were introduced by Farhat and Roux by the end of the twentieth century.

The presentation of the algorithms in the book is complete in the sense that it starts from the formulation of contact problems, briefly describes their discretization and the properties of the discretized problems, provides the flowcharts of solution algorithms, and concludes with the analysis, numerical experiments, and implementation details. The book can thus serve as an introductory text for anybody interested in contact problems.

Synopsis of the book:

The book starts with a general introduction to contact problems of elasticity with the account of the main challenges posed by their numerical solution. The rest of the book is arranged into four parts, the first of which reviews some well-known facts on linear algebra, optimization, and analysis in the form that is useful in the following text.

The second part is concerned with the algorithms for minimizing a quadratic function subject to linear equality constraints and/or convex separable constraints. A unique feature of these algorithms is their rate of convergence and the error bounds in terms of bounds on the spectrum of the Hessian of the cost function. The description of the algorithms is organized in five chapters starting with a separate overview of two main ingredients, the conjugate gradient (CG) method (Chap. 5) for unconstrained optimization and the results on gradient projection (Chap. 6), in particular on the decrease of the cost function along the projected-gradient path.

Chapters 7 and 8 describe the MGP (Modified Proportioning with Gradient Projections) algorithm for minimizing strictly convex quadratic functions subject to separable constraints and its adaptation MPRGP (Modified Proportioning with Reduced Gradient Projections) for bound constrained problems. The result on the rate of convergence in terms of bounds on the spectrum of the Hessian matrix is given that guarantees a kind of optimality of the algorithms—it implies that the number of iterates that are necessary to get an approximate solution of any instance of the class of problems with the spectrum of the Hessian contained in a given positive interval is uniformly bounded regardless the dimension of the problem. A special attention is paid to solving the problems with elliptic constraints and coping with their potentially strong curvature, which can occur in the solution of contact problems with orthotropic friction.

Chapter 9 combines the algorithms for solving problems with separable constraints and a variant of the augmented Lagrangian method in order to minimize a convex quadratic function subject to separable and equality constraints. The effective precision control of the solution of separable problems in the inner loop opened the way to the extension of the optimality results to the problems with separable and linear equality constraints that arise in the dual formulation of the conditions of equilibrium. Apart from the basic SMALSE (Semi-Monotonic Algorithm for Separable and Equality constraints) algorithm, the specialized variants for solving bound and equality constrained quadratic programming problems (SMALBE) and QCQP problems including elliptic constraints with strong curvature (SMALSE-Mw) are considered.

The most important results of the book are presented in the third part, including the scalable algorithms for solving multibody frictionless contact problems, contact problems with Tresca's friction, and transient contact problems.

Chapter 10 presents the basic ideas of the scalable algorithms in a simplified setting of multidomain scalar variational inequalities.

Chapters 11–13 develop the ideas presented in Chap. 10 to the solution of multibody frictionless contact problems, contact problems with friction, and transient contact problems. For simplicity, the presentation is based on the node-to-node discretization of contact conditions. The presentation includes the variational formulation of the conditions of equilibrium, the finite element discretization, some implementation details, the dual formulation, the TFETI (Total finite Element Tearing and Interconnecting) domain decomposition method, the proof of asymptotically linear complexity of the algorithms (numerical scalability), and numerical experiments.

Chapter 14 extends the results of Chaps. 10 and 11 to solving the problems discretized by the boundary element methods in the framework of the TBETI (Total Boundary Element Tearing and Interconnecting) method. The main new features include the reduction of the conditions of equilibrium to the boundary and the boundary variational formulation of the conditions of equilibrium.

Chapters 15 and 16 extend the results of Chaps. 10–14 to solving the problems with varying coefficients and/or with the non-penetration conditions implemented by mortars. It is shown that the reorthogonalization-based preconditioning or the

renormalization-based scaling can relieve the ill-conditioning of the stiffness matrices and that the application of the mortars need not spoil the scalability of the algorithms.

The last part begins with two chapters dealing with the extension of the optimality results to some applications, in particular to contact shape optimization and contact problems with plasticity. The book is completed by a chapter on massively parallel implementation and parallel scalability. The (weak) parallel scalability is demonstrated by solving an academic benchmark discretized by billions of nodal variables. However, the methods presented in the book can be used for solving much larger problems, as demonstrated by the results for a linear benchmark discretized by tens of billions of nodal variables.

Ostrava
January 2016

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Scalable Algorithms for Contact Problems

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2016, XIX, 340 p. 80 illus., 23 illus. in color., Hardcover

ISBN: 978-1-4939-6832-9