# Preface

This book is a collection of original surveys focussing on two very active branches of modern (theoretical and applied) probability, namely the *Malliavin calculus of variations* and *stochastic geometry*. Our aim is to provide (for the first time!) a lively, authoritative and rigorous presentation of the many topics connecting the two fields, in a way that is appealing to researchers from both communities. Each survey has been compiled by leading researchers in the corresponding area. Notation, assumptions and definitions have been harmonized as closely as possible between chapters.

Roughly speaking, stochastic geometry is the branch of mathematics that studies geometric structures associated with random configurations, such as random graphs and networks, random cluster processes, random unions of convex sets, random tilings and mosaics, etc. Due to its strong connections to stereology and spatial statistics, results in this area possess a large number of important applications, e.g. to modelling and statistical analysis of telecommunication networks, geostatistics, image analysis, material science, and many more.

On the other hand, the Malliavin calculus of variations is a collection of probabilistic techniques based on the properties of infinite-dimensional operators, acting on smooth functionals of general point processes and Gaussian fields. The operators of Malliavin calculus typically generalize to an infinite-dimensional setting familiar objects from classical analysis, like, for instance, gradients, difference and divergence operators. When dealing with Malliavin calculus in the context of point processes (as is the case in the present book), one has typically to deal with a number of technical difficulties—related in particular to the intrinsic discrete structure of the underlying objects. As explained in the sections to follow, a crucial tool in partially overcoming these difficulties is given by Wiener–Itô chaotic decompositions, which play a role analogous to that of orthogonal expansions into series of polynomials for square-integrable functions of a real variable.

A fundamental point (which constitutes a strong motivation for the present book) is that, for many prominent models in stochastic geometry, Wiener–Itô chaotic decompositions and associated operators from Malliavin calculus are particularly accessible and amenable to analysis because the involved concepts and expressions have an intrinsic and very natural geometric interpretation.

Of particular interest to us is the application of these techniques to the study of probabilistic approximations in a geometric context, that is, of mathematical statements allowing one to assess the distance between the distribution of a given random geometric object and the law of some target random variable. Probabilistic approximations are naturally associated with variance and covariance estimates, as well as with limit theorems, such as Central Limit Theorems and Laws of Large Numbers, and are one of the leading threads of the whole theory of probability.

The interaction between stochastic geometry and Malliavin calculus is a young and active domain of research that has witnessed an explosion in interest during the past 5 years. By its very nature, such an interaction is a topic that stands at the frontier of many different areas of current research. Investigations gained particular momentum during an Oberwolfach conference in 2013, where many prominent researchers from both fields met for discussions. Since then, an increasing number of collaborations have been initiated or strengthened. Also, although several remarkable results have already been achieved in the field, for instance in the asymptotic study of the Boolean model, random graphs, random polytopes and random k-flats, many questions and problems (e.g. the derivation and use of effective concentration inequalities) remain almost completely open for future investigation.

It is the aim of this book to survey these developments at the boundary between stochastic analysis and stochastic geometry, to present the state of the art in both fields and to point out open questions and unsolved problems with the intention of initiating new research in and between the two areas.

The readership we have in mind includes researchers and graduate students who have a basic knowledge of concepts of probability theory and functional analysis. Most of the fundamental notions that are needed for reading the book are introduced and developed from scratch.

Last but not least, as editors, we would like to thank the numerous colleagues and friends who have been involved in this project: this book would not have been possible without their excellent contributions, as well as their enthusiasm and support.

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