

Preface

Reduced basis (RB) methods represent a very efficient approach for the numerical approximation of problems involving the repeated solution of differential equations arising from engineering and applied sciences. Noteworthy examples include partial differential equations (PDEs) depending on several parameters, PDE-constrained optimization, and optimal control and inverse problems.

In all these cases, reducing the severe computational complexity is crucial. With this in mind, over the past four decades, reduced-order models (ROMs) have been developed aiming at replacing the original large-dimension numerical problem (typically called high-fidelity approximation) by a reduced problem of substantially smaller dimension. Strategies to generate the reduced problem from the high-fidelity one can be manifold, depending on the context.

The strategy adopted in RB methods consists in the projection of the high-fidelity problem upon a subspace made of specially selected basis functions, representing a set of high-fidelity solutions corresponding to suitably chosen parameters. Pioneering works in this area date back to the late 1970s (e.g., B.O. Almroth et al. [5, 6], D. Nagy [193], A.K. Noor and J.M. Peters [201, 202, 203, 204] and address linear and nonlinear structural analysis problems. The first theoretical analysis of RB methods in connection with the use of the continuation method for parametrized equations was presented by J.P. Fink and W.C. Rheinboldt [109, 110] in the mid 1980s. Extensions to problems in fluid dynamics are primarily due to the contributions of Peterson [210] and Gunzburger [124] in the late 1980s.

The method was set on a more general and sound mathematical ground in the early 2000s thanks to the seminal work of A.T. Patera, Y. Maday and coauthors [214, 255]. Their work has led to a decisive improvement in the computational aspects of RB methods owing to an efficient criterion for the selection of the basis functions, a systematic splitting of the computational procedure into an offline (parameter-independent) and an online (parameter-dependent) phase, and the use of a posteriori error estimates that guarantee certified numerical solutions for the reduced problem. These have become the essential constituents of the RB methods now most widely used. Often, they are also embedded into more general reduced-order models.

RB methods have witnessed a spectacular effervescence in the past decade. Additional achievements during that time relate to the treatment of nonlinear and/or parametrically nonaffine problems by the so-called empirical interpolation method and its several extensions. This has substantially improved RB methods, making possible their application to a broad variety of complex problems such as time-dependent problems, optimal control and design problems, and real-time computing.

This is the first textbook to provide a basic mathematical introduction to RB methods. We present a general formulation of RB methods, analyze their fundamental theoretical properties, and discuss their algorithmic and implementation aspects, highlighting their built-in algebraic and geometric structures. More specifically, we carry out both a priori and a posteriori error analysis, formulate strategies for the construction of accurate reduced basis spaces, and analyze offline-online decomposition strategies to ensure the reduction of computational complexity. The entire mathematical discussion is made more stimulating by the use of several representative examples of applicative interest, in the context of both linear and nonlinear PDEs.

The authors are grateful to Charbel Farhat, Yvon Maday, and Anthony Patera for being source of inspiration and for many fruitful discussions on reduced-order models. We also acknowledge David Amsallem, Luca Dedè, Simone Deparis and Toni Lassila for the great amount of time that they have spent with the authors talking about different subjects covered in this book and, last but not least, Gianluigi Rozza for having introduced the last two authors to the subject. In addition, special thanks are due to Francesca Bonadei and Francesca Ferrari of Springer Italia for their invaluable help in the preparation of the manuscript.

Lausanne, Switzerland
May 2015

Alfio Quarteroni
Andrea Manzoni
Federico Negri

Reduced Basis Methods for Partial Differential
Equations

An Introduction

Quarteroni, A.; Manzoni, A.; Negri, F.

2016, XI, 296 p., Softcover

ISBN: 978-3-319-15430-5