Gerbert of Aurillac (c. 940–1003)

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Abstract Gerbert of Aurillac represents one of the most relevant personalities of the European medieval culture, being a prolific scholar as well as an acknowledged teacher especially as tutor of Emperors Otto II and Otto III. A disciple himself of Atto, during his long and successful career, first as a teacher in Reim's Cathedral School, then as Abbot of the monastery of Bobbio, Archbishop of Ravenna and finally as Pope Silvestre II (999–1003), Gerbert always encouraged and promoted the study of the *quadrivium* (arithmetic, geometry, music and astronomy) also through the reintroduction to western Europe of ancient Greek-Roman scientific culture, especially in the augmented Arab versions. Gerbert's influence on western scientific thought refers not only to theory (i.e. the arabs' decimal numeral system or some of Euclid's theorems) being instead always balanced with practical applications that involve instruments (abacus, armillary sphere, astrolabe, etc.) and that immediately affect the lives of common people.

Even though the present study has been developed together by both authors, different authorships can be recognized within the paper. In particular the *Biographical notes* have been written by Luca. J. Senatore while the section dedicated to *Review of Main Works of Gerbertus* has been developed by Carlo Bianchini. All other parts have been written in common.

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Biographical Notes

Born in Auvergne (a mountainous region in central France) in a period spanning from 938 to 950, Gerbert's origins are likely to have been humble. Around 963 he entered the monastery of St. Gerald in Aurillac, a rather strict Benedictine settlement and above all an institution independent from any local control, being subject, like Cluny, only to the Pope.

Here Gerbert starts his training in the *trivium* subjects (grammar, logic, and rhetoric) learning Latin grammar as well. In 967 the first turning point in his life: Count Borrell of Barcelona visits the monastery and, under request of the abbot, decides to take Gerbert back to Spain with him in order to study mathematics there. He had proven to be a particularly brilliant pupil deserving the opportunity of improving his knowledge through the study of the *quadrivium* (arithmetic, geometry, music, and astronomy).

Gerbert moved thus from France to Catalunya where he started to attend the cathedral school of Vic. Catalunya at that time represented, both culturally and politically, the western frontier of Christianity; a stronghold somehow against Arabian penetration but at the same time a privileged point for exchange and communication. Christian Europe was in fact considerably under-developed in comparison with the Muslims of al-Andalus who settled in southern Spain and thus communications (especially in the south-north direction) were generally tolerated and often encouraged.

Books were a highly precious part of this trade: through this channel actually a large part of the ancient Greek/Roman knowledge came back to western Europe together with Persian, Indian and even Chinese achievements, filtered and improved by Arab culture: Muslim "scientists" were highly regarded, and perhaps nowhere in Islam as much as in al-Andalus. Muslim astronomy was the most advanced in the world, and Muslim astronomers proficient in using the astrolabe had done much to map the skies. Although the names of modern planets and constellations are Latin, the names of most major stars—Altair, Deneb, Rigel, Sirius, Fomalhaut, Aldeberan, Betelgeuse—are Arabic as are many of the other terms of astronomy, such as azimuth, almagest, almanac, and the Zodiac. The Arabs were even further advanced in the realm of arithmetic. They had adopted the concept of zero from the Indians and used a positional numeric system much like the modern system—in fact, our numerals are based on the Arabic notation. They had also borrowed the abacus from the Chinese and were proficient in its use. They had gone beyond arithmetic, had established algebra, and were investigating prime numbers and coordinate equations. Their study of proportions made it possible for them to approach music in a quite precise manner, distinguishing accurately between notes, developing theories of harmonies and discords, and constructing musical instruments with quite accurate tuning.

For its geographical position and thanks to the farsighted policy of their bishops, the Vic's Cathedral library became one of the largest and best equipped in Europe, positively influencing the cathedral school itself.

Gerbert took full advantage of these opportunities, quickly showing abilities and knowledge so far beyond average that his name soon became legendary, sometimes also in a negative way as a magician or worse as a devil's disciple.

In 969 Count Borrell and Atto, the bishop of Vic, traveled to Rome on a pilgrimage, taking Gerbert with them. Pope John XIII remained very impressed by the young monk and persuaded emperor Otto I to take Gerbert on as tutor for his young son, the future emperor Otto II. Gerbert attended to his duty for some years and finally the emperor let him move to the famous cathedral school of Reims to study advanced logic. Here he had the opportunity to show all his abilities: not only in the study of arithmetic (mastering arabic numerals he could use quick mind calculation to solve problems extremely difficult with roman notation) but also in applying his wide knowledge to practical issues and mechanical equipment.

A masterpiece from this standpoint is an organ with constant pressure supplied by water power he is said to have built in Reims: not only did Gerbert's machine provide an extended steady level of sound, but its pipes were also mathematically calibrated so that its harmonics were astonishingly superior to anything heard in Western Europe before. The abacus has certainly been one of the main instruments for his work (he actually built a giant one marking out the floor of the nave of the cathedral of Reims) and his book on the subject became a sort of standard in the new cathedral schools that were quickly populating the West. During a visit to Ravenna he met again Otto's son reviving their old acquaintance: a crucial episode for Gerbert, for in 983 his old pupil became Holy Roman Emperor with the name of Otto II. The emperor appointed his teacher as abbot of the famous Abbey of Bobbio (its library was one of the largest in Western Europe) and count of the correspondent district.

As a new abbot, Gerbert tried right from the start to invert the decay tendency of the monastery, too long lacking for strong guidance and under pressure by local nobles eager for its lands. Unfortunately, the death of Otto (983) led to new balances that forced him to move from Bobbio and return once more to Reims: this time, though, as master of the cathedral school and secretary to Adalberon, the archbishop. This new role represents for Gerbert an occasion to get involved in politics that, at that time, meant the contrast between the German Saxons and the Carolingians for the throne of France.

Counting on the weakness of the Empire due to the very young age of emperor Otto III (born in 980) and on the internal contrasts for the regency, Lothair made an attempt to conquer Lorraine (985): on that occasion Gerbert and the archbishop of Reims sided firmly with the count of Paris, Hugh Capet who, after the death of Lothair and of his son, became the first Capetian king of France.

At the death Adalberon (989), even if Gerbert seems to be a natural choice for succession, the king appoints Arnulf, an illegitimate son of Lothair. This episode represents the beginning of a very difficult period: the archbishopric of Reims, with the help of Arnulf, becomes the theatre of the conflict between Duke Charles (the Carolingians' heir) and Hugh Capet and Gerbert himself has to take refuge at the court of the king.

Although in 991 Arnulf's treason was declared and Gerbert appointed as archbishop, his nomination was harshly criticized and finally revoked by Pope Gregory V in 997. The deposed archbishop moves then to the court of Otto II, where he becomes the teacher and advisor of the young Otto III.

Gerbert reaches his new pupil in Ravenna, being appointed archbishop (998) and then, at the death of Gregory V, moves to Rome where the emperor imposes him as Pope Sylvester II in 999.

The Roman populace did not accept a foreign pope, and in 1001 both Otto and Gerbert were forced to return to Ravenna. He will come back only in 1003, the year of his death. Despite his brief reign, Gerbert supported the *renovatio imperii* that intended to regain for Rome the ancient classical values. He sensed the strategic importance of the eastern European countries (in particular Poland and Hungary) promoting their Christianization. Finally, he always defended the autonomy of the Pope from the Emperor's influence.

Gerbert's Work

Gerbert was undoubtedly one of the most relevant writers in the early Middle Ages. Even if is quite difficult to provide in this occasion a systematic classification of his works, we can divide his writings according to the specific character of each one:

- Mathematical writings
  - *Liber abaci*
  - *Libellus de numerorum divisione*
  - *Regula de abaco computi*
  - *De geometria*
  - *Libellus de rationali et ratione uti*
- Ecclesiastical writings
  - *Selecta e concil. Basol., Remens., Masom.,* etc.
  - *De corpore et sanguine Domini*
  - *Sermo de informatione episcoporum*
- Letters
  - *Epistolae ante summum pontificatum scriptae*
  - *218 letters, including letters to the emperor, the pope, and various bishops*
  - *15 letters to various bishops, including Arnulf, and abbots*
  - *Epistolae et decreta pontificia*
  - *5 short poems*
  - *one dubious letter to Otto III.*

- Other
  - *Acta concilii Remensis ad S. Basolum*
  - *Leonis legati epistola ad Hugonem et Robertum reges*

Review of Main Works of Gerbertus

Even if Gerbert's production includes many ecclesiastical writings, the majority of his works actually aim at teaching the *quadrivium* topics: arithmetic, music, geometry and astronomy.

And the transmission of knowledge represents for Gerbert an absolute priority: certainly a means for education but also, more deeply, the privileged tool to penetrate the secret structure of reality and finally to understand the magnificence of the Creation.

In this framework, arithmetic represents the first step in this actual ascent, the key, somehow, to the other three disciplines both from a speculative and practical standpoint. Music cannot be handled without knowing the theory of numbers; on the other hand, proportions and calculations are unavoidable tools both for the problems of the *geometria practica* and for astronomy.

In this period (and for many centuries to come) arithmetic will continue to coincide with the science of numbers in the form handed down by Boethius, while the *practica mathematica* would indicate calculation methods.

This ancient roman philosopher represents Gerbert's principal mathematical source: not only does he adopt the structure and content of Boethius' *corpus*, but also enhances its original goal of preserving the *quadrivium* through its promotion as a "key tool" for the study of Christian theology.

But, at the turn of the first millennium, the boethian *Institutio Arithmetica* hands down to the western medieval culture also the Pythagorean concept of "number" strictly related to the profound structure of the Universe as well as to the most esoteric aspects of Greek culture. The graft of Christian vision on this tradition actually transforms numbers in the ordering principles of the Universe, each one referring to a specific aspect of creation harmony. The disclosure of the properties of numbers actually meant to access the secret rules used by the Creator himself.

Omnia in numero et mensura et ponder (Sapienza Liber XI, 21) is stated in the Bible: an idea already accepted by the patrology (i.e. Augustine of Hippo) to which Gerbert himself refers directly in the *Prologus* of his *Geometria*. Nevertheless arithmetic is also much more than a symbolic representation of reality: it can help in fact in enlightening how nature itself works in physical terms. This level of knowledge, though, is a real challenge for the human mind: it cannot be reached without continuous training and refining. Thus *Arithmetica* becomes also a powerful brain training method for education.

Theory and practice are faces of the same medal for Gerbert: a more profound understanding of numbers cannot be achieved without improving calculation methods: refining procedures but also innovating the instruments used for calculations. From this last standpoint the Scholastic of Reims could count at the same time on two unique opportunities: access to the richest Christian libraries and first-hand information coming from the Arab scientific culture. And Gerbert succeeds in merging together *arithmetica* and the *practica mathematica* introducing and widely promoting the use of the *abacus*, the first calculator the western European culture would have ever counted on for centuries. Even if it cannot be considered a pure invention of the future Pope Sylvester II, nevertheless the instrument described in many writings by Gerbert is quite different from the traditional Latin one essentially because it uses the Arabic positional notation based on 10 Figs. (1, 2, 3, 4, 5, 6, 7, 8, 9 and 10). No Arabic source actually reports a description of such an instrument, reinforcing the hypothesis that a deep renovation both of the calculator and of its use appeared at the end of the first millennium in the Christian mathematicians communities, Gerbert being the creator of this tool or at least its most active "popularizer". (Reims Abbey itself has for long been considered as the center of abacus spreading and its users, the abacisti, at that time were simply called *gerbertisti*).

The use of this new tool not only changed the way calculations were made, but also the way *arithmetica* itself was taught: the abstract study of the *codices* progressively gives place to a more inter-personal relationship between teacher and pupils, allowing these last actual apprentices to learn the use of the abacus through imitation of their teacher's model.

For more than 100 years Gerbert's abacus will represent the most powerful calculation system (hardware and software we would say nowadays) available in Christian western Europe. It will be overcome in the XII century only by the spreading of the algorithm, a method based on the tracing on a table covered with sand or dust of all the calculation steps using all the figures, including zero.

As mentioned above, *arithmetica* provides all necessary fundamentals to the other disciplines of the *quadrivium*. This idea, quite clear in all Gerbert's writings, has actually also been the leitmotif of its teaching activity. Even a discipline apparently quite far from mathematics like Music does perfectly comply with this "quantitative" approach: within the *quadrivium* program, in fact, theory of music and theory of numbers are tightly connected, so that sounds correspond to numbers, and intervals to number ratios. These principles, very well-known in Ancient times since Pythagoras, originated also the metaphysical interpretation of music illustrated by Plato in his Timaeus where the movement of celestial spheres produces the perfect music harmony of the universe.

At the turn of the first millennium, though, this knowledge was essentially lost in the Christian western Europe where the Gregorian chant, a form of monophonic, unaccompanied sacred song, seemed to be the only expression of music. Gerbert radically changes the teaching of Music in his school, again trying to put together theory and practical demonstrations in the same way we have seen about arithmetic calculations.

This time, instead of the abacus, we find a new instrument, the *monochord*: a single string, stretched over a sound box, is fixed at both ends while one movable

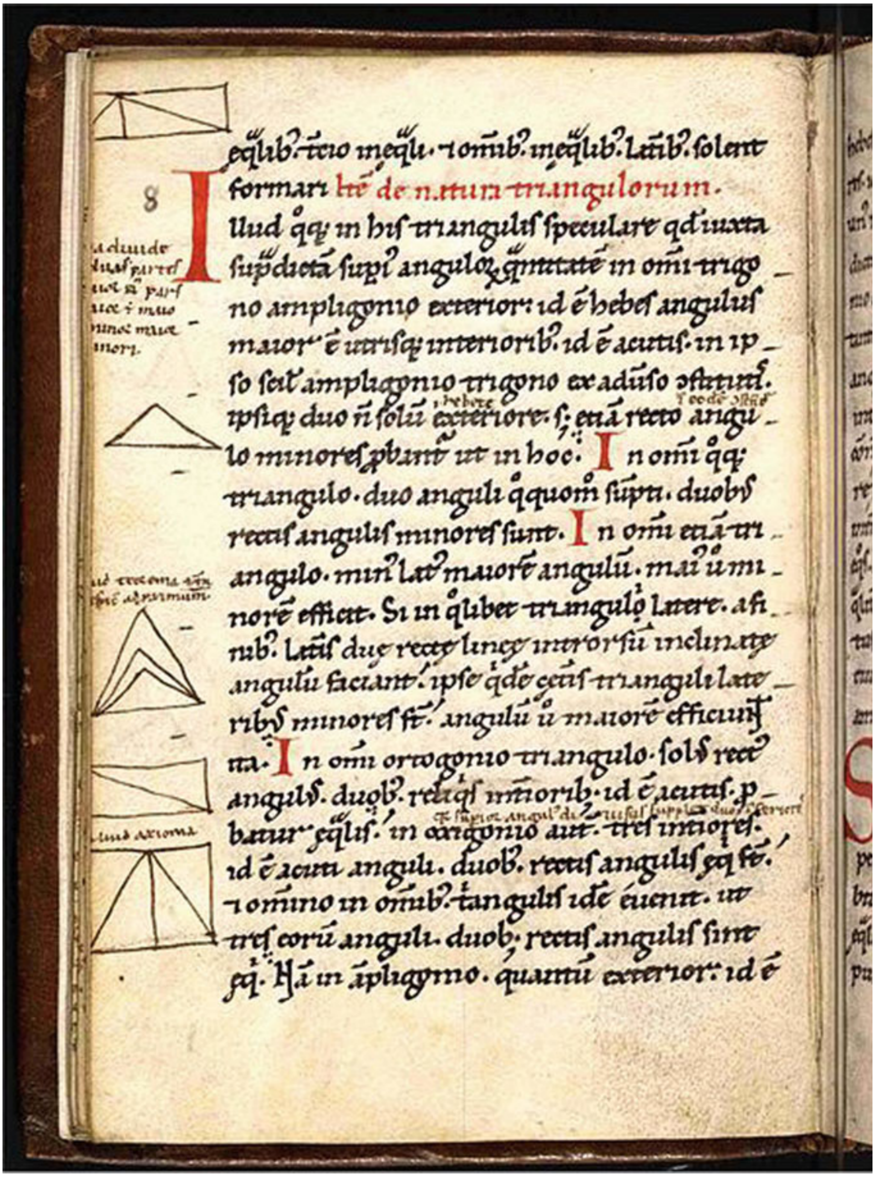

Fig. 1 Gerbert_d'Aurillac, De Geometria

bridge or weight is manipulated to demonstrate mathematical relationships between sounds. The chord can thus be divided according to simple ratios in order to precisely produce the corresponding note which clearly comes as a result of a theoretical reasoning but at the same time becomes immediately audible for any student. Where in a Gregorian chorus the master guided the chant arbitrarily

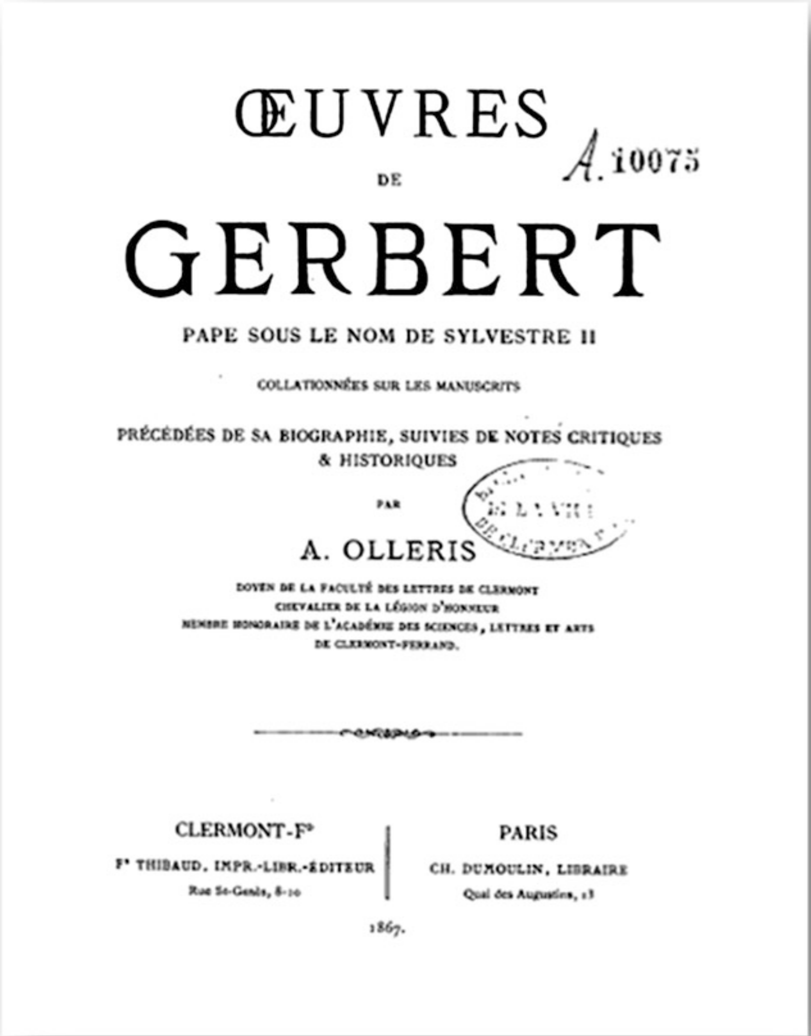

Fig. 2 A. Olleris, Oevres de Gerbert, 1867

representing for all apprentices the reference for any note, the teaching based on the monochord would instead provide an independent and reliable means for tuning.

Unfortunately, only through indirect evidence can we affirm the usage of this object by Gerbert, but this hypothesis is in our opinion at least a quite reasonable one. But the future Pope's reputation in the field of music is mostly related to the construction in Reims of an outstanding pipe organ: an instrument with incomparable steady and harmonious sound thanks to the constant pressure provided by

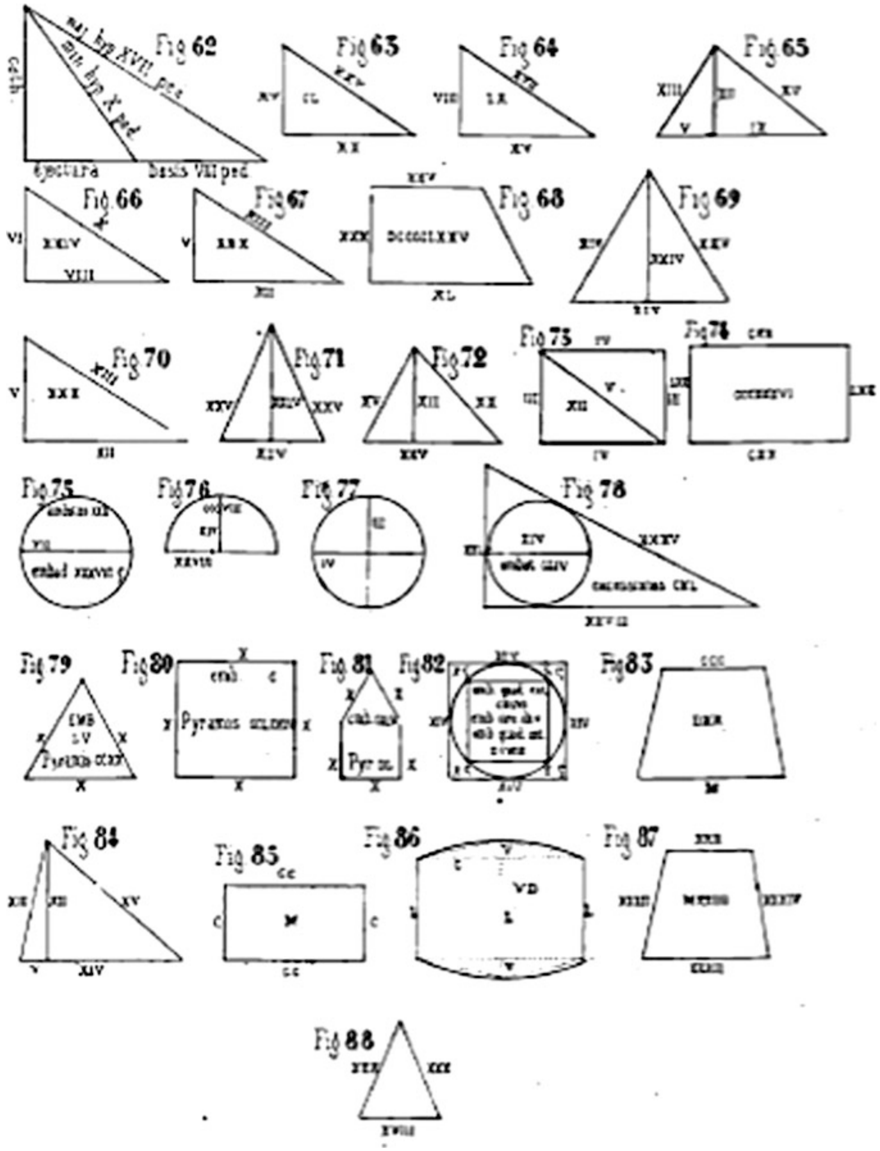

Fig. 3 A. Olleris, Oevres de Gerbert, 1867

water power and the perfect geometry of the pipes. Again it is not a technical problem for Gerbert: better, the technical problem is the final part of a complex process in which solutions are progressively tuned according to theoretical investigations and practical calculations. In the *mensura fistularum*, the work devoted to pipes dimensioning, the Scholastic of Reims discusses the applicability to the organ

Fig. 4 A. Olleris, Oevres de Gerbert, 1867

cylinders of the numerical ratios used for the monochord. There is evidently no automatic correspondence between the two: for a string, its length is the only variable to be considered in order to produce a certain sound, while in a cylinder its

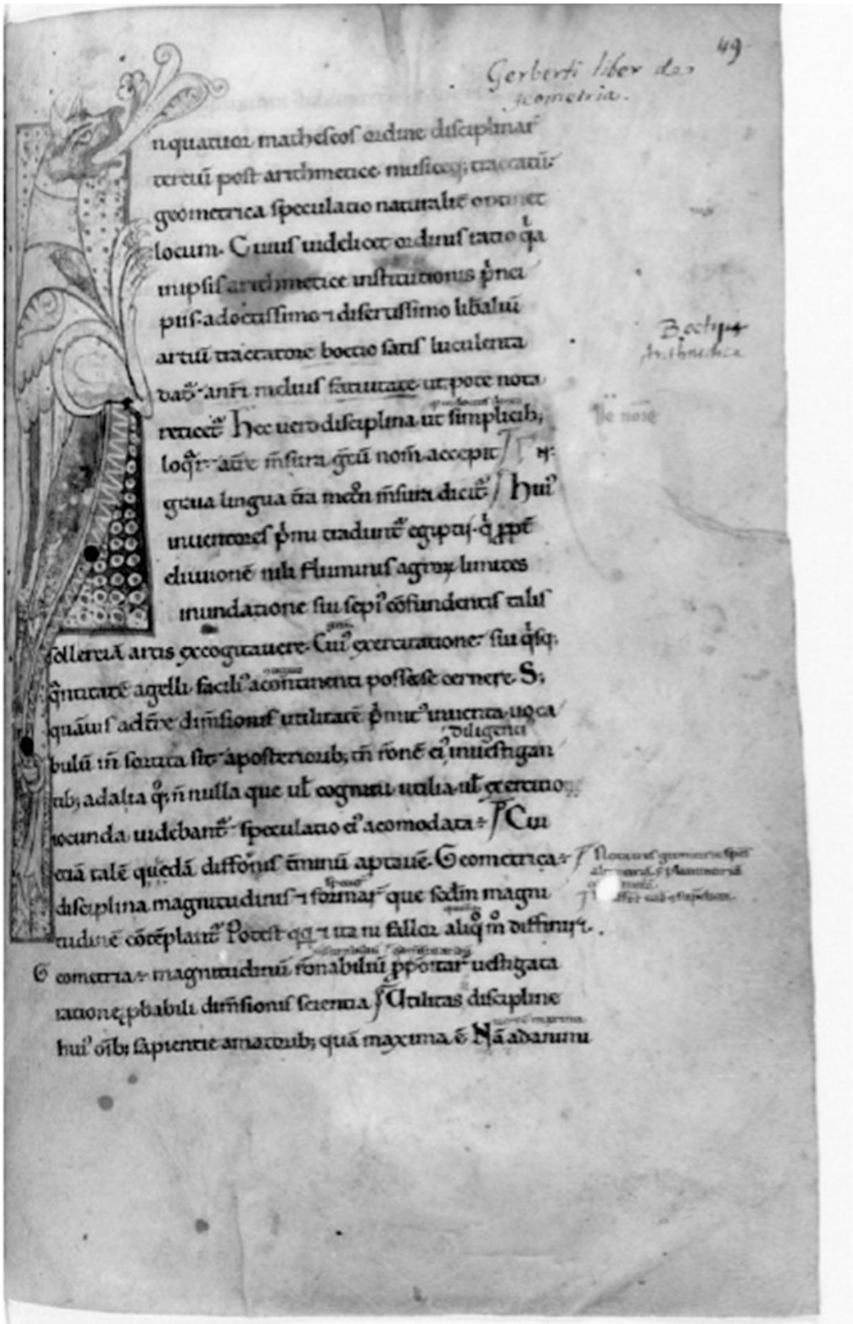

Fig. 5 Gerbert d'Aurillac, Liber des geometria

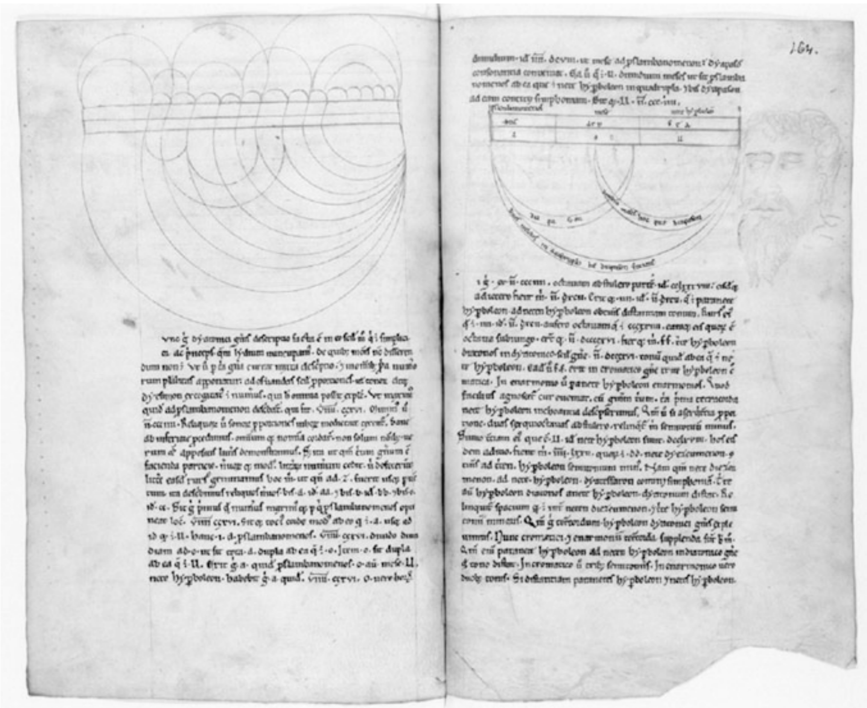

Fig. 6 Gerbert d'Aurillac, De Geometria

volume too must be taken into account. The "practical" rules of medieval *organa* constructors are thus completely bypassed and revised according to this rigorous vision. But Gerbert pushes his research far beyond, demonstrating with complicated calculations that the proportions suitable for cylinders receive the same ratios of the monochord and, finally, of the Universe itself.

Nam tempore nocturno ardentibus stellis operam dabat; agebatque ut eas in mundi regionibus diversis obliquatas, tam in ortu quam in occasu notarent… [In fact, he dedicated himself to the stars twinkling in the night and labored because (students) would notice, both in their rise that in their sunset, those declined in different regions of the sky]. Gerbert's astronomy is then essentially an observation activity. But in a period in which astronomy was often tangled with astrology, the future Pope addresses the discipline coherently with its "scientific" program in which theory and practice are faces of the same medal and instruments are the means to pass from one to the other. In this framework many objects have been traditionally reported as used in Gerbert's teaching. First of all the *spherae*: a *sphere of the stars* (made in solid wood and covered with leather, showing on its surface all constellations); the *armillary sphere*, one of the most famous astronomical objects since Greek antiquity (a spherical framework of rings, centred on Earth, that represent lines of celestial longitude and latitude and other astronomically relevant

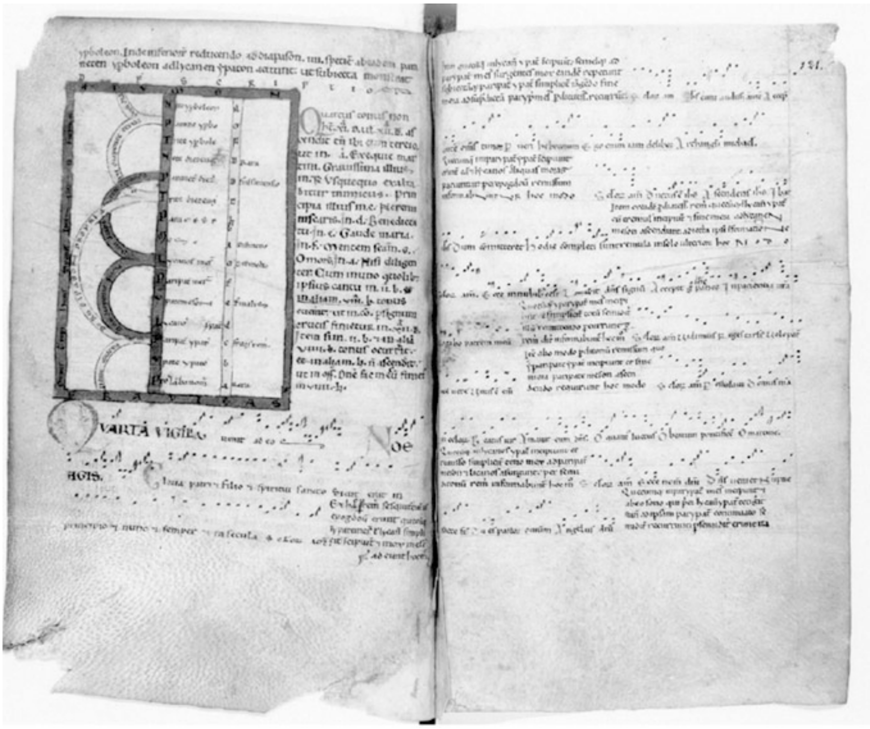

Fig. 7 Gerbert d'Aurillac, De Geometria

elements such as the ecliptic plane); the so-called *half-sphere* (made in solid wood showing on its surface poles, equator, tropics and one meridian) used for observing from its center the celestial bodies through holes (*fistulae*) positioned at the intersection between the parallels and the meridian; and finally a fourth sphere being a sort of fusion between an armillary sphere and the sphere of stars that, thanks again to a pointing system (from the poles to the Ursa Major) could lead to an alignment of the object coherent with the Earth inclination.

Much more debated is what role Gerbert could have played in the adoption and diffusion of the astrolabe in the western European culture. This fundamental astronomical instrument had again a Hellenistic origin but only with the Arabs did it reach its full maturity. At the end of X century it was quite common across the Arab world and it is thus probable that Gerbert too could have seen or even owned one during his stay in the *marca hispanica* or later. Nevertheless the simple possession of such an instrument does not mean the possibility of its comprehensive use: in fact without knowing at least the fundamentals illustrated in the Ptolemy's Planisphere an astrolabe would result in a mysterious object practically useless for most astronomical purposes. As all this information (both on the usage of the astrolabe itself and the Planisphere) would reach Europe only in the XII century

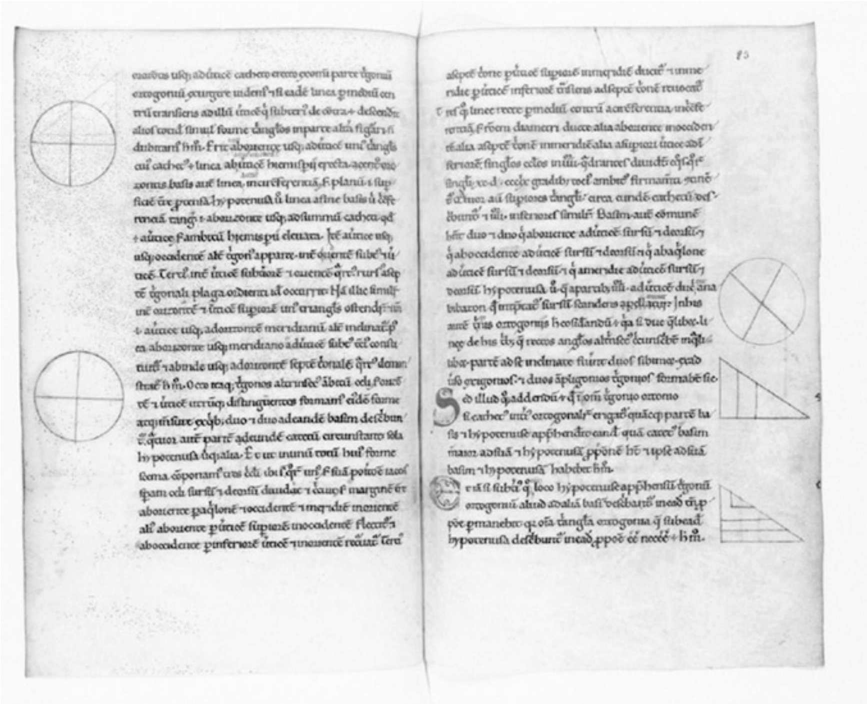

Fig. 8 Anicii Manlii Boëtii, De Arithmetica libri duo

with some systematic translation from the arabic, it is reasonable to imagine that the instrument was known by Gerbert but not completely understood unless for some geometrical application less relevant though for astronomy (i.e. indirect measuring). This hypothesis is also reinforced by the wide use of the *half-sphere* by Gerbert in his teaching activity. This object will in fact rapidly become obsolete with the introduction of the astrolabe (more precise and handy) and it would have been illogic for Gerbert to keep on using it as an alternative to the astrolabe.

Geometry completes the picture of the *quadrivium* disciplines. The content of Gerbert's teaching program actually coincides with the so-called *geometria practica*, an ensemble of geometric principles and propositions addressing the solution of some practical problems. Even if the term appears only in the XII century in the *Didascalicon* written by Hugh de Saint Victor aiming at distinguishing within the tree of medieval geometry two different branches, *geometria theorica* and *geometria practica,* (For more details refer to Bianchini 1994, 1995a, 1995b), nevertheless Gerbert's work represents an important step towards this crucial result. On one side, in fact, he contributes to the conservation and transmission of the roman geometric knowledge in the form established by Boethius to the Christian western culture; on the other, he sows the seeds of a new geometric vision that, trying again

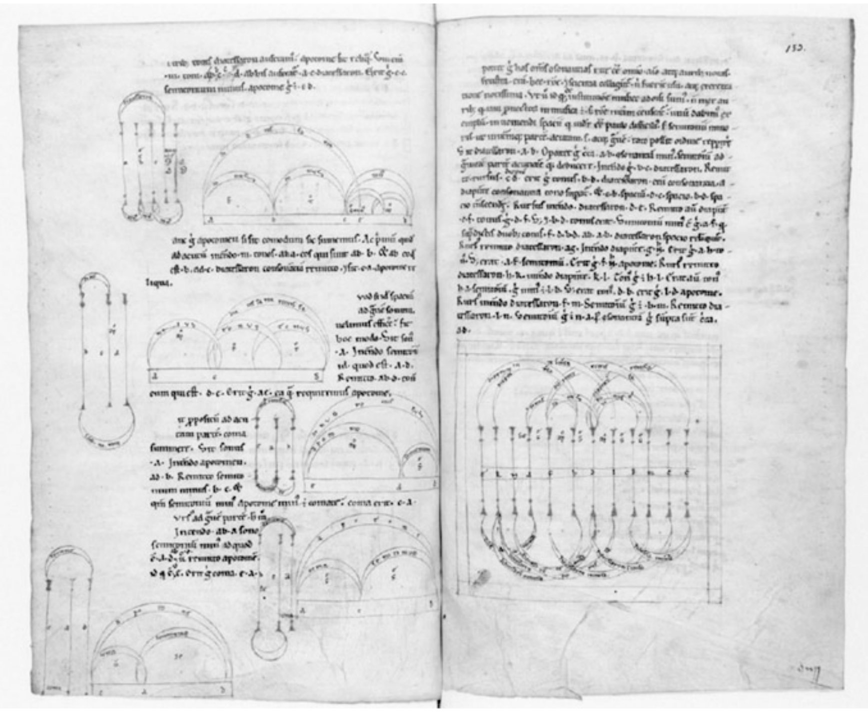

Fig. 9 Gerbert d'Aurillac, De Geometria

to merge together theory and practice, will finally disclose the possibility of repossessing all ancient greek knowledge and especially Euclid's Elements.

Quite apart from the still debated attribution of Gerbert's geometric writings (the so-called *Geometria Gerberti* (Materni 2008) and its probable "collective" character, still this work represents a significant example of that *sub-euclidean geometry* "in use among scholars who have not studied Euclid himself as their immediate source" (Evans 1976, p. 114). The structure of the treatise, quite original for the time, included: an introductive section listing some postulates, definitions and theorems mainly coming from the first four books of the Elements; a metrology section defining a collection of units of measure; methods for calculating the areas of polygons and circles; finally a part—probably apocryphal—concerning solutions to some practical geometric problems (i.e. the direct and indirect measuring of lengths and areas). Many different sources seem thus to have merged together in the *Geometria Gerberti*: certainly the Boethian tradition (cited by Gerbert itself); likely also many of the notions and problems coming from the *corpus agrimensorum romanorum* included in the *codex arcerianus* that the future pope could have come in touch with in the Bobbio Abbey; but certainly the new information he had learned during his spanish stay and, later on, from arabic sources. Actually the treatise reveals the very advanced geometric knowledge of its author: for instance,

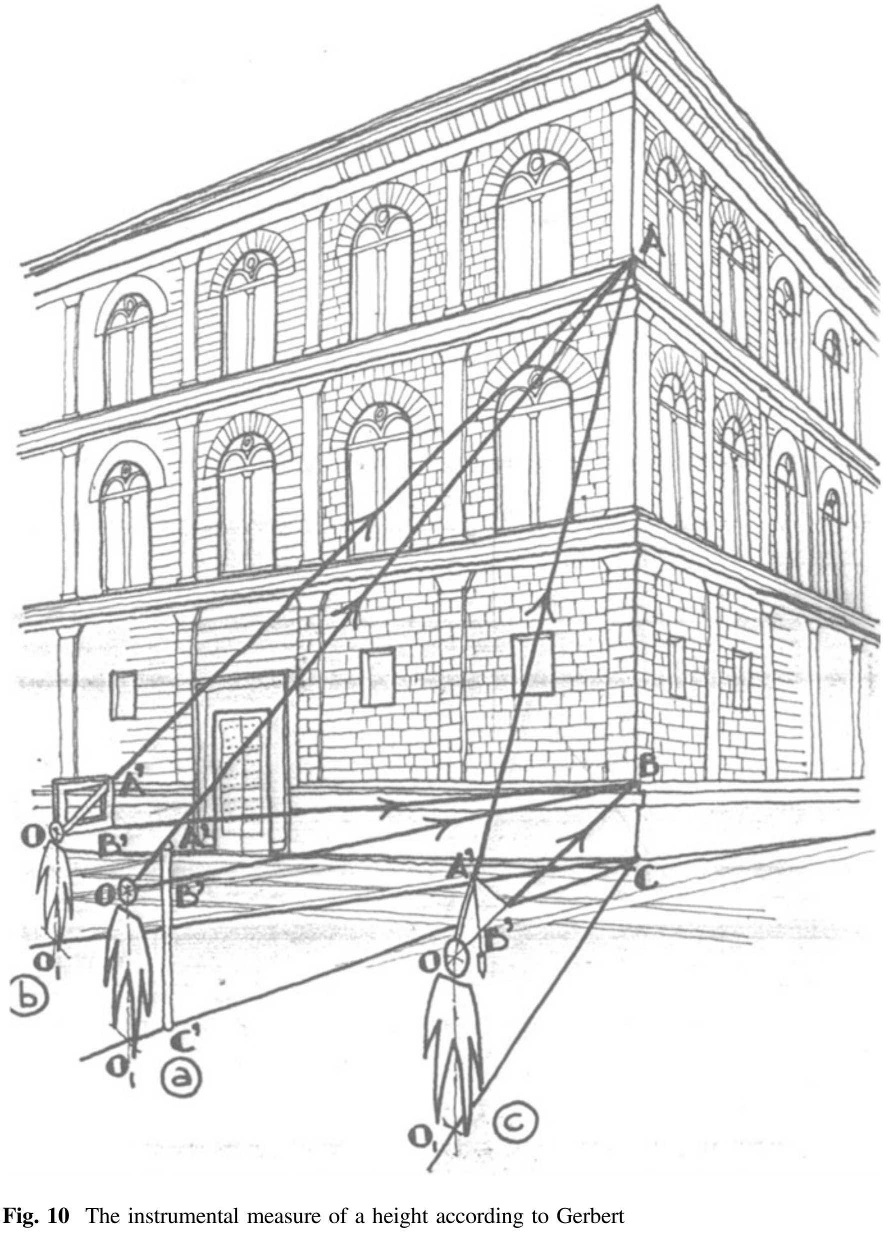

Fig. 10 The instrumental measure of a height according to Gerbert

Gerbert can easily deal with triangles using the Pythagorean theorem (not receding even when fractions are involved in calculations) when his contemporaries had difficulties even in understanding simple geometric definitions like “internal” and “external” angle of a triangle.

But also the probably apocryphal part of the *Geometria*, that recent investigations anyway ascribe to the closer circle of Gerbert's disciples (Jaquemard 2000), shows an original character in comparison with other medieval works. The practical problems illustrated in the different propositions are in fact solved, bending the abstraction of geometric principles to the needs of real life. And this transposition is put in practice thanks to the use of specific measurement instruments and procedures that, as expression of a rigorous geometric background, are capable of leading to more reliable and general results.

From this standpoint, the *Geometria Gerberti* is not only a catalogue of problems, but also an inventory of measurement instruments and techniques. Based on the properties of proportions between similar triangles, some typical survey problems are in fact solved in different ways. More than 20 propositions in *caput IV*, in fact, refer to the "instrumental" measurement of lengths, heights or depths. Instruments, though, can be very simple objects (a stick, arrow and string, a mirror) but also more refined ones, like isosceles, rectangles and magic (3, 4, 5) triangles, squares; finally we find the *horoscopus* (Probably an astrolabe or a quadrant) and the astrolabe.

Quite apart from the clear differences pertaining to instruments, the theory that ensures the correctness of results is more or less the same: the possibility of constructing a proportional relationship between two similar triangles, one on the instrument and the other "on site". In Fig. 10 we can see in fact in O an observer aiming at point A situated, for example, at the top end of the AC height to be measured. The visual ray OA identifies on the instrument the segment A'B' that, together with OB (the horizontal distance between O and the height AC conventionally measured) defines the proportion OB: O′B′ = AB:A′B′ and finally allows the calculation of AB. The final length of AC is then the result of AB + BC, height of point O from the reference plane (the ground). It is worth mentioning the procedure in which a reflecting surface is used (a mirror, a bowl with water). In this case, by choosing a specific point of the mirror to serve as a marker, the observer O reaches a position from which the upper A point is centered on the repair. Two similar triangles are thus constructed also in this situation: the former is made by the unknown height, its distance from the repair on the mirror and by the ray BA; the latter by the height of the observer, its distance from segment BA and the reflected ray. Quite apart from the construction (indeed a little naïve) the procedure implies some knowledge concerning optics (i.e. the linearity of light rays and the law of reflection) not quite usual during the Middle Ages.

Conclusions

Gerbert has clearly devoted his life to the praise of God: in this the enlightening of the perfect mathematical structure of the Creation represents an outstanding component. But his activity was not self-referencing but on the contrary constantly oriented to the spreading of knowledge actuated through the education especially of young generations.

In continuity with the ancient Latin tradition and the Boethius lesson, he devised several means (also practical) for teaching the fundamental *quadrivium* disciplines: the abacus; celestial globes; a hemisphere for observing stars and visualizing imaginary celestial circles; armillary spheres; the astrolabe, even if probably used only for measurements.

With his writings on geometry he tended to merge together the available Euclid's fragments, the knowledge of roman *gromatici* and some new notions acquired from the Arabs.

He showed a very profound acquaintance with music enlightening the close connection between numbers and notes and devising new ways to conceive and build instruments to demonstrate it.

Finally, he coherently went through all disciplines convinced of the substantial unity of knowledge based on mathematics.

For these reasons (probably more than for his ecclesiastic and political career), Gerbert remains a key figure of late 10th century and one of the most relevant scholars in all medieval culture.

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