

Chapter 2

Background

The experiments reported in this thesis rely on two elements: Josephson superconducting circuits (the processor) and NV center spins in diamond (the memory). In this chapter, we introduce the minimum theoretical background needed to understand the work which follows.

2.1 Superconducting Circuits and Microwave Engineering

In this section, we provide the reader elements of theory on superconducting circuits and microwave engineering which are relevant for this work. We start with the simplest circuit, the superconducting resonator, which is of particular interest for our experiments as it is used as a bus between spins and circuits. We then see how resonators can be turned into active devices, using Josephson junctions. Along these lines, we describe how to implement the qubit concept by pushing Josephson circuits to the level of very large anharmonicity. We finally introduce the field of circuit quantum electrodynamics, which describes the interaction of a qubit with single photons in a resonator.

2.1.1 Superconducting Resonators

2.1.1.1 The LCR Resonator

We start with the quantum treatment of the simplest superconducting circuit that can be built: the LC resonator. Its schematic is given in Fig. 2.1, consisting of an inductance L in parallel with a capacitance C . We note V the voltage across the capacitance and I the current. As explained e.g. in [1, 2], the circuit can be quantized in terms

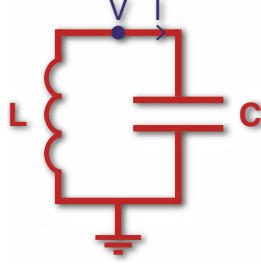


Fig. 2.1 A LC resonator

of two generalized conjugate quantum operators, $\hat{\Phi}$ the flux in the inductance and $\hat{Q}$ the charge accumulated on the capacitor, obeying $[\hat{\Phi}, \hat{Q}] = i\hbar$, with Hamiltonian:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \quad (2.1)$$

This harmonic oscillator Hamiltonian can be written:

$$\hat{H} = \hbar\omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad (2.2)$$

where $\hat{a}^\dagger, \hat{a}$ are respectively the creation and annihilation operators

$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar Z_r}} \left(\hat{\Phi} - iZ_r \hat{Q} \right) \quad (2.3)$$

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_r}} \left(\hat{\Phi} + iZ_r \hat{Q} \right) \quad (2.4)$$

with $\omega_r = 1/\sqrt{LC}$ the resonance frequency and $Z_r = \sqrt{L/C}$ the resonator impedance. The eigenstates $|n\rangle$ of $\hat{H}$ are called Fock states, satisfying $\hat{H}|n\rangle = \hbar\omega_r \left(n + \frac{1}{2} \right) |n\rangle$ with n the number of photons stored in the resonator. The relations Eqs. 2.3 and 2.4 can be inverted to obtain the expression of the flux and charge operators $\hat{\Phi}$ and $\hat{Q}$ and calculate their time derivatives to express the voltage and current operators $\hat{V} = i[\hat{H}, \hat{\Phi}]/\hbar$ and $\hat{I} = i[\hat{H}, \hat{Q}]/\hbar$ in term of the field operators:

$$\hat{V} = iV_{rms}(\hat{a}^\dagger - \hat{a}) \quad (2.5)$$

$$\hat{I} = \frac{V_{rms}}{Z_r}(\hat{a}^\dagger + \hat{a}) \quad (2.6)$$

with $V_{rms} = \omega_r \sqrt{\hbar Z_r/2}$ the root-mean-square vacuum fluctuations of the voltage. The current and voltage in the resonator give rise to associated electric (in the capacitance) and magnetic (around the inductance) fields:

$$\hat{E}(\mathbf{r}) = i\delta E_0(\mathbf{r}) \left(\hat{a} - \hat{a}^\dagger \right), \quad \hat{B}(\mathbf{r}) = \delta B_0(\mathbf{r}) \left(\hat{a} + \hat{a}^\dagger \right) \quad (2.7)$$

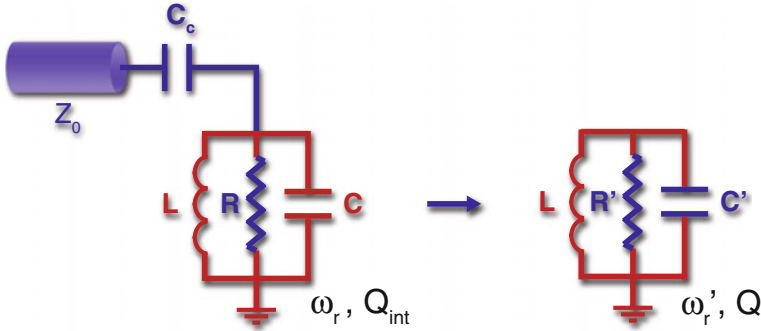


Fig. 2.2 A LCR resonator coupled to an input transmission line with characteristic impedance Z_0 through a gate capacitance C_c . It is equivalent to a resonator with effective resistance R' and capacitance C' . The resonator frequency and characteristic impedance are shifted to ω_r' and Z_r' with respect to that of the decoupled one

at position $\mathbf{r}$, with $\delta E_0(\mathbf{r})$ and $\delta B_0(\mathbf{r})$ their vacuum fluctuations. This electromagnetic field is used to couple the resonator to the qubit and the spins and thus play a major role throughout this manuscript.

In an experiment, a resonator can have internal losses and is coupled to measuring lines. We thus consider the more general case shown in Fig. 2.2 of a resonator with internal losses modelled by a resistance R , and coupled to an input transmission line with characteristic impedance Z_0 through a gate capacitance C_c . The capacitor acts as a semi-reflective mirror in optics and induces a strong impedance mismatch so that the electromagnetic field is confined in it. We want to determine the modified resonator frequency ω_r' and impedance Z_r' . The circuit is equivalently modeled as a modified LCR resonator with an effective resistance R' such that:

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R_{ext}} \quad (2.8)$$

with $R_{ext}/Z_0 = 1/(C_c Z_0 \omega_r)^2 + 1$, and a new capacitance

$$C' = C + \frac{C_c}{1 + (C_c \omega_r Z_0)^2} \approx C + C_c \quad (2.9)$$

This slightly changes the resonance frequency and the impedance to $1/\sqrt{LC'}$ and $\sqrt{L/C'}$ respectively. An important quantity is the resonator quality factor, defined as $Q = R'/Z_r$. This quantity can be separated in two contributions:

$$Q^{-1} = Q_{int}^{-1} + Q_{ext}^{-1} \quad (2.10)$$

where $Q_{int} = R/Z_r$ is the internal quality factor, and $Q_{ext} = R_{ext}/Z_r$ the coupling quality factor. Associated damping rates are defined as $\kappa_L = \omega_r'/Q_{int}$ the rate at which the energy is dissipated in R and $\kappa = \omega_r'/Q_{ext}$ the rate at which the energy leaks out of the resonator into the measurement lines through C_c . In the usual case

where the coupling capacitance is very small compared to the capacitance of the resonator $C_c \ll C'$:

$$\omega'_r \approx \omega_r, \quad Z'_r \approx Z_r, \quad R_{ext} \approx 1/Z_0 C_c^2 \omega_r^2 \quad (2.11)$$

so that the external damping rate writes:

$$\kappa = \omega_r^3 C_c^2 Z_0 Z_r \quad (2.12)$$

and equivalently

$$C_c = 1/\sqrt{Q_{ext} Z_0 Z_r \omega_r^2} \quad (2.13)$$

In the following, we continue noting the resonance frequency ω_r and the impedance Z_r by convenience.

2.1.1.2 Probing the Resonator from the Outside

To probe the resonators, they are connected to voltage sources through measurement lines. In the following, we consider the case often encountered in our experiments of a resonator coupled to two transmission lines (ports 1 and 2) with coupling capacitances $C_{c,1}$ and $C_{c,2}$ yielding the damping rates κ_1 and κ_2 through Eq. 2.12, and a voltage $V_1(t) = V_1 \cos(\omega t)$ applied to port 1 (see Fig. 2.3a). We note I_i the current flowing into the i th port and model the transmission lines by impedances Z_0 , V the voltage across the resonator capacitance, and I the current through the inductance. An equivalent description, but more adapted to our experiments, is to replace the voltage source $V_1(t)$ by a wave $V_{1+}(t)$ incoming onto the resonator input via the transmission line and driving it, generating an intra-cavity field $V(t)$ and $I(t)$, and giving rise to outgoing waves $V_{1-}(t)$ and $V_{2-}(t)$ that describe the leakage of the intra-cavity field towards the measurement apparatus. In this scattering approach (see Fig. 2.3b), the behavior of the resonator is described by the so-called scattering matrix $S_{ij} = a_{out,i}/a_{in}$, with the classical input and output waves $a_{in/out,i}$ defined as

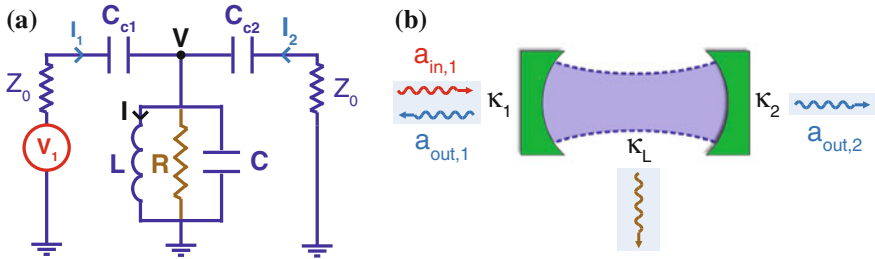


Fig. 2.3 Circuit and its equivalent in the input-output theory framework. **a** Electrical circuit. **b** Input-output theory framework

$$a_{in,i} = \frac{V_i + Z_0 I_i}{2\sqrt{Z_0}} = \frac{V_i^+}{\sqrt{Z_0}} \quad \text{and} \quad a_{out,i} = \frac{V_i - Z_0 I_i}{2\sqrt{Z_0}} = \frac{V_i^-}{\sqrt{Z_0}} \quad (2.14)$$

The quantum-mechanical extension of this classical scattering approach is provided by the input-output theory developed by Gardiner and Collett [3]. There, the intra-resonator field is described by the quantum-mechanical operator $\hat{a}(t)$, whose knowledge directly yields the resonator currents and voltages through Eqs. 2.5 and 2.6. Its evolution (in the Heisenberg representation) obeys to the master equation

$$\partial_t \hat{a}(t) = \frac{[\hat{a}(t), \hat{H}]}{i\hbar} - \left(\sum_i \frac{\kappa_i}{2} \right) \hat{a} + \sum_i \sqrt{\kappa_i} \hat{a}_{in,i}(t) \quad (2.15)$$

This intra-cavity field equation is complemented with a relation stating the continuity of the fields at each port i

$$\hat{a}_{in,i}(t) + \hat{a}_{out,i}(t) = \sqrt{\kappa_i} \hat{a}(t) \quad (2.16)$$

In these equations, the input and output fields $\hat{a}_{in,out}(t)$ are the quantum analog of the classical waves described earlier. In our experiments the drive fields will always be coherent states, so that we will take $\hat{a}_{in,1}(t) = \alpha_{in} e^{-i\omega t}$, with α_{in} normalized such that $|\alpha_{in}|^2$ is the number of photons per second at the resonator input, or equivalently $P = \hbar\omega|\alpha_{in}|^2$. Applying these equations to the linear resonator depicted in Fig. 2.3b yields:

$$\partial_t \alpha(t) = -i\omega_r \alpha(t) - \frac{\kappa + \kappa_L}{2} \alpha(t) + \sqrt{\kappa_1} \alpha_{in}(t), \quad (2.17)$$

with $\alpha(t) = \langle \hat{a} \rangle(t)$ the mean value of the intra-resonator field, $\kappa = \kappa_1 + \kappa_2$ the total resonator external damping rate. In steady-state $\alpha(t) = \alpha e^{-i\omega t}$, yielding

$$\alpha(\omega) = \frac{i\sqrt{\kappa_1}}{(\omega - \omega_r) + i\frac{\kappa + \kappa_L}{2}} \alpha_{in} \quad (2.18)$$

At resonance, the intra-cavity average photon number $\bar{n} = |\alpha|^2$ is

$$\bar{n} = \frac{4\kappa_1}{\hbar\omega_r(\kappa + \kappa_L)^2} P. \quad (2.19)$$

These equations allow us to obtain useful relations linking the resonator maximum voltages and currents V_0 and I_0 defined as $V(t) = V_0 \sin(\omega t)$ and $I(t) = I_0 \cos(\omega t)$ to the input power. Combining Eqs. 2.5, 2.6 and 2.19 yields

$$V_0 = \frac{\sqrt{8Z_r\omega_r\kappa_1}}{\kappa + \kappa_L} \sqrt{P} \quad \text{and} \quad I_0 = \frac{\sqrt{8\omega_r\kappa_1/Z_r}}{\kappa + \kappa_L} \sqrt{P} \quad (2.20)$$

Reflection measurement The reflection coefficient $r = \frac{\alpha_{out,1}}{\alpha_{in}}$ follows from Eqs. 2.16–2.18:

$$r(\omega) = \frac{\sqrt{\kappa_1}\alpha - \alpha_{in}}{\alpha_{in}} = \frac{i\kappa_1}{(\omega - \omega_r) + i\frac{\kappa + \kappa_L}{2}} - 1 \quad (2.21)$$

In the experiments reported in Chap. 5, in particular, we use a resonator with only one port so that $\kappa_2 = 0$ and $\kappa = \kappa_1$ in the above expressions. Depending on the rate between the coupling constant κ and the losses κ_L we can define three regimes characterized by different behaviors of the reflection coefficient (Fig. 2.4a):

- **The over-coupled regime** (red curves) defined by $\kappa \gg \kappa_L$. In this regime $|r| \approx 1$ for all frequencies and the phase ϕ undergoes a 2π shift at resonance

$$\phi = 2 \arctan \left(2 \frac{\omega - \omega_r}{\kappa} \right)$$

- **The critical coupling regime** (green curves) defined by $\kappa = \kappa_L$. For this regime the amplitude of r reaches 0 at resonance, while a discontinuity in its phase brings a phase shift of π .
- **The under-coupled regime** (blue curves) defined by $\kappa \ll \kappa_L$. In this regime the resonance corresponds to a dip in the amplitude of r and a shift $< \pi$ in its phase.

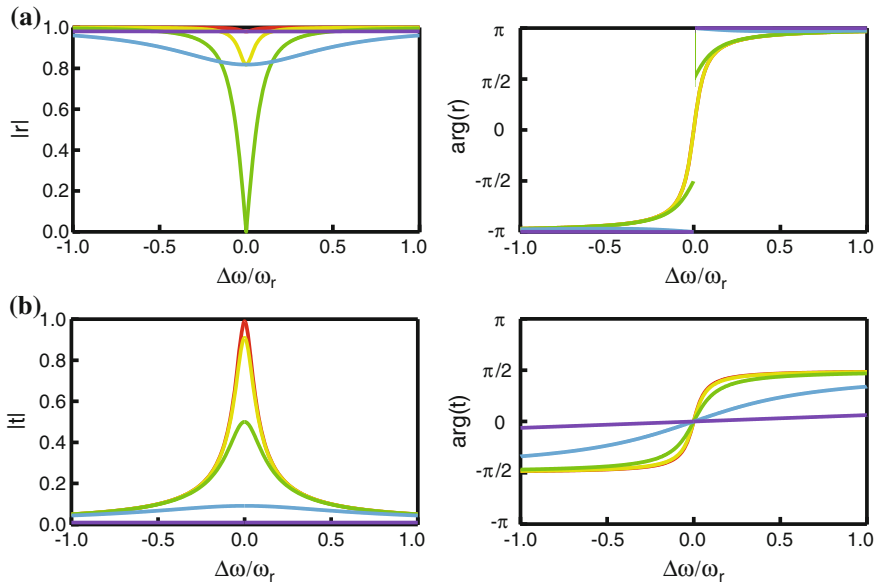


Fig. 2.4 Probing the resonator. **a** Reflection measurement. Frequency dependence of the reflection coefficient r for $\kappa_L = 0.01\kappa$ (red), $\kappa_L = 0.1\kappa$ (yellow), $\kappa_L = \kappa$ (green), $\kappa_L = 10\kappa$ (blue) and $\kappa_L = 100\kappa$ (purple). **b** Transmission measurement. Frequency dependence of the transmission coefficient t for the same values of damping

The width of both the dip and the phase shift decrease when κ_L/κ increases. Both the amplitude and the phase differ very slightly from their out-of-resonance value.

Transmission measurement The transmission coefficient $t = \frac{\alpha_{out,2}}{\alpha_{in}}$ follows from Eqs. 2.16–2.18:

$$t(\omega) = \frac{\sqrt{\kappa_2}\alpha}{\alpha_{in}} = \frac{i\sqrt{\kappa_1\kappa_2}}{(\omega - \omega_r) + i\frac{\kappa + \kappa_L}{2}}. \quad (2.22)$$

In the experiments reported in Chap. 4 we have $\kappa_1 = \kappa_2 = \kappa/2$. As in reflection measurements, depending on the rate between the coupling constant κ and the losses κ_L we can define three regimes characterized by different behaviors of the transmission coefficient (Fig. 2.4b):

- **The over-coupled regime** (red curves) defined by $\kappa \gg \kappa_L$, in which $|t|$ almost reaches 1 at resonances. The phase of t undergoes a π shift at resonance, whose width, as well as the width of $|t|$, is controlled by κ_L .
- **The critical coupling regime** (green curves) defined by $\kappa = \kappa_L$. In this regime, the amplitude of t reaches 1/2 at resonance and the widths are controlled by both κ and κ_L .
- **The under-coupled regime** (blue curves) defined by $\kappa \ll \kappa_L$. In this regime the amplitude of t decreases when κ_L/κ increases and its width is essentially controlled by κ_L .

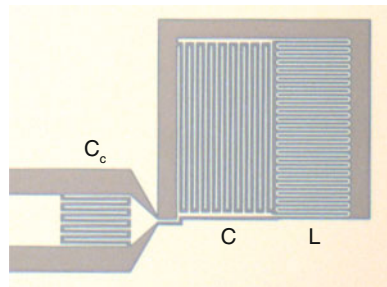
In an experiment, the resonator spectrum is often obtained with a vector network analyzer (VNA). This apparatus measures the S-matrix elements in reflection ($S_{11}(\omega)$) and in transmission ($S_{21}(\omega)$). Note that with respect to the coefficients defined and calculated above, the definitions are different so that $S_{11} = r^*$ and $S_{21} = t^*$.

2.1.1.3 Implementation: Lumped and Distributed-Element

LUMPED- ELEMENT RESONATORS

Resonators can be implemented with lumped capacitors and inductances on chip, provided their dimensions are much smaller than half the wavelength of a 3 GHz signal propagating in silicon (2 cm). An example is shown in Fig. 2.5. The resonator

Fig. 2.5 Picture of a lumped-element resonator



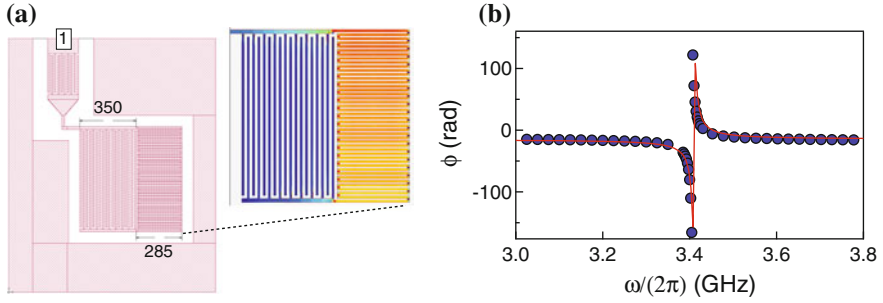


Fig. 2.6 Implementation of a superconducting resonator with lumped electrical elements. **a** Schematic of the simulated chip geometry, showing the resonator and the coupling capacitance to an input port with impedance $50\ \Omega$. The computed alternative current flowing in the inductance is shown in the *inset*. **b** Computed (dots) and fitted (lines) reflected phase ϕ , yielding $\omega_r/(2\pi) = 3.41\ \text{GHz}$ and $Q = 300$

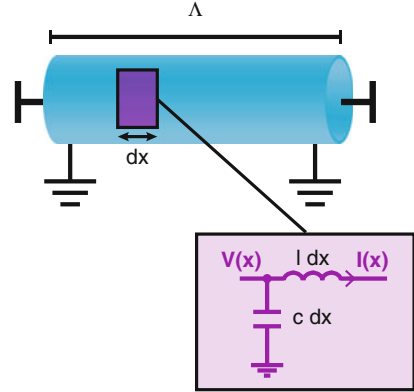
capacitance (C), inductance (L) and coupling capacitance to the external driving line (C_c) are made with interdigitated fingers and meander wires. The geometrical parameters (number of fingers, fingers length, length of the meanders wires, . . .) are chosen using a 2D electromagnetic simulator (Sonnet), to obtain the desired resonator physical parameters (ω_r , Z_r , Q). The software uses the physical description of the circuit (geometry, material properties, . . .), places a voltage source behind the input port of the circuit and solves by finite-element methods the Maxwell's equations to extract the reflection and transmission coefficients.

We show in Fig. 2.6a the Sonnet simulated chip geometry of the resonator pictured in Fig. 2.5, together with the computed alternative current flowing in the resonator inductance I when the drive frequency matches the LC resonance. Note that as expected for a lumped element circuit, the current is approximately constant throughout the wire used as an inductance. The reflected phase ϕ_r on the resonator function of the drive is given in Fig. 2.6c, showing the 2π phase shift at resonance. The fit of the resonance is used to estimate the resonator frequency ω_r and quality factor Q and to correct if needed the design in a next simulation step. To link the oscillating current in the resonator inductance to the magnetic field generated in its surrounding, we use COMSOL Multiphysics. Together with Eq. 2.20, this makes it possible to calculate the magnetic field amplitude $B_1(r)$ to which the spins are coupled, for a given microwave power incident at the resonator input.

DISTRIBUTED- ELEMENT RESONATORS

A resonator can also be built using a piece of transmission line of length Λ with an open circuit at each end (Fig. 2.7). A transmission line is characterized by its capacitance c and inductance l per unit length, and characteristic impedance $Z_0 = \sqrt{l/c}$. Adding boundary conditions creates stationary modes which propagate with phase velocity $\bar{c} = 1/\sqrt{lc}$. The details of the quantum treatment of the field in such resonator can be found e.g. in [4]. We give here the relevant results for the design of

Fig. 2.7 Distributed-element resonators. A transmission line, with open circuits at each end, constitutes a microwave resonator



our experiments. The different stationary modes are equivalent LC resonators k with frequency $\omega_k = \frac{k\pi\bar{c}}{\Lambda}$ and impedance $Z_r = \frac{2}{\pi} Z_0$. In our experiments, the fundamental mode $k = 1$ is the only one of interest, with frequency

$$\omega_r = \frac{\pi \bar{c}}{\Lambda} \quad (2.23)$$

This equivalent resonator [5] can be put in the form of the LCR resonator model with capacitance $C = \frac{\pi}{2} \cdot \frac{1}{Z_0 \omega_r}$, inductance $L = \frac{2}{\pi} \cdot \frac{Z_0}{\omega_r}$, resistance $R = \frac{2}{\pi} \cdot Q_{int} Z_0$ and voltages and currents inside the resonator expressed in the form of Eqs. 2.5 and 2.6 with $V_{rms} = \omega_r \sqrt{\frac{\hbar Z_0}{\pi}}$. We use a particular type of transmission line geometry: the coplanar waveguide (CPW). It consists of a center strip conductor (of width S) that is separated by a gap (of width W) from ground planes (of width b) on either sides, as shown in Fig. 2.8a. To realize the resonator, the transmission line is terminated at one end by an open gap and connected at the other end to external line through coupling capacitance C_c . The effective dielectric constant felt by the electromagnetic mode propagating in the resonator derives from the relative dielectric constant of the substrate ϵ_r (Silicon in our experiment) as $\epsilon_{eff} = \epsilon_0(1 + \epsilon_r)/2$ yielding a phase velocity $\bar{c} = c/\sqrt{\epsilon_{eff}}$. The value of $\bar{c}$ is used with formula Eq. 2.23 to determine the resonator length Λ required to obtain the desired resonance frequency. The coupling capacitance C_c sets the resonator quality factor according to Eq. 2.10.

The configuration of the electromagnetic field generated by the CPW resonator is shown in the inset of Fig. 2.8. There exist analytical expressions linking the voltage $V(z)$ and current $I(z)$ in the CPW to the electromagnetic field of Eq. 2.7. These expressions are used to calculate the magnetic field to which the spins are coupled in our experiments.

According to [6], the rms vacuum fluctuations of the magnetic field created by a wave traveling in the z direction along the resonator are:

$$\delta B_x(x, y, z) = -\frac{2\mu_0}{\eta b} \sqrt{\epsilon} \sum_{n=1}^{\infty} \frac{1}{F_n} \left[\frac{\sin(n\pi\delta/2)}{n\pi\delta/2} \sin \frac{n\pi\bar{\delta}}{2} \right] \cos\left(\frac{n\pi x}{b}\right) e^{-\gamma_n y} \cdot V(z)$$

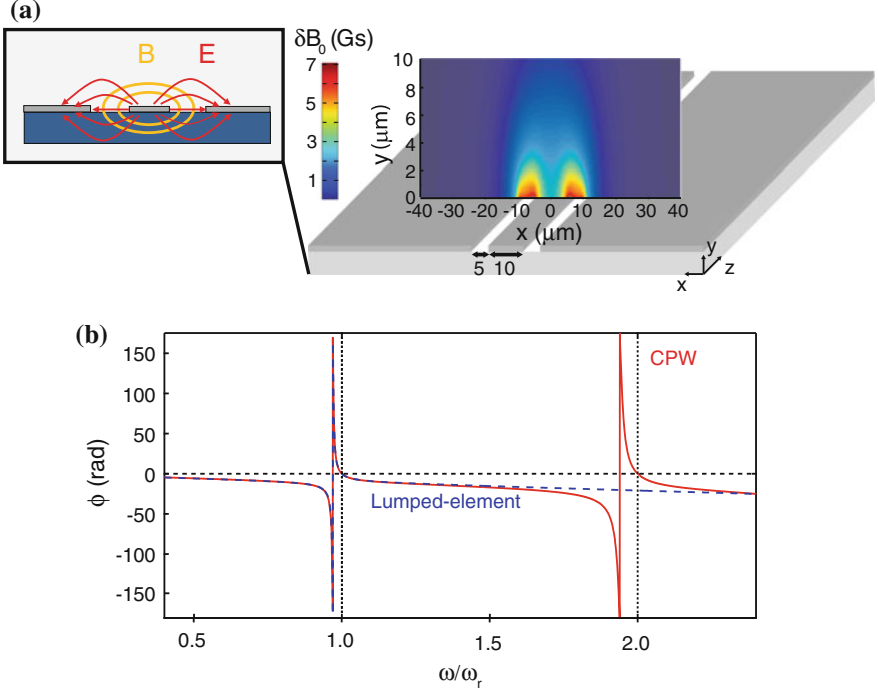


Fig. 2.8 Implementation of a superconducting resonator with a section of CPW line. **a** 3D schematic of the resonator showing the electromagnetic field. The rms vacuum fluctuations of the magnetic field δB_0 is computed for a typical resonator geometry ($S, W = 10, 5 \mu\text{m}$). **b** Reflected phase as a function of the reduced frequency ω/ω_r for a $\lambda/2$ resonator (red solid line). The first resonance frequency of the loaded resonator is shifted down with respect to that of the decoupled one. The fundamental mode of the distributed resonator is very close to those of a lumped element LCR resonator with the same C and Z_r (blue dashed lines)

$$\begin{aligned} \delta B_y(x, y, z) &= -\frac{2\mu_0}{\eta b} \sqrt{\epsilon} \sum_{n=1}^{\infty} \left[\frac{\sin(n\pi\delta/2)}{n\pi\delta/2} \sin \frac{n\pi\bar{\delta}}{2} \right] \sin \left(\frac{n\pi x}{b} \right) e^{-\gamma_n y} \cdot V(z) \\ \delta B_z(x, y, z) &= -i \frac{2\mu_0}{\eta b} \epsilon \left(\frac{4\pi cb}{\omega_r} \right) \sum_{n=1}^{\infty} \frac{1 - \epsilon_{eff}}{nF_n} \left[\frac{\sin(n\pi\delta/2)}{n\pi\delta/2} \sin \left(\frac{n\pi\bar{\delta}}{2} \right) \right] \\ &\quad \sin \frac{n\pi x}{b} e^{-\gamma_n y} \cdot V(z) \end{aligned} \quad (2.24)$$

with

$$V(z) = V_{rms} \cos(\pi z / \Lambda) \quad (2.25)$$

and a geometrical parameter

$$\gamma_n = \sqrt{\left(\frac{n\pi}{b} \right)^2 + \left(\frac{4\pi cb \sqrt{\epsilon_{eff} - 1}}{n\omega_r} \right)^2} \quad (2.26)$$

In this expression, μ_0 is the vacuum permeability, $\eta = 376.7$ the vacuum impedance and $\delta = W/b, \bar{\delta} = (S + W)/b, F_n = \frac{b\gamma_n}{n\pi}$ are determined by the resonator geometry.

2.1.2 Josephson Junction Based Circuits

A non-linear and lossless element can be introduced in superconducting circuits to turn them into active devices. This element is the Josephson tunnel junction, which consists of two superconducting electrodes coupled through a thin layer of insulating material. In this section, we introduce the reader to Josephson junction based circuits. We discuss two particular implementations, the bistable and the frequency-tunable resonator and see that when pushed to the level of very large anharmonicity, Josephson circuits can be used to build qubits.

2.1.2.1 The Josephson Junction and Its Derivatives

The Josephson junction used together with inductances and capacitances to build superconducting circuits is depicted in Fig. 2.9. Its physics is based on the Josephson effect [7], which states that between two closely spaced superconducting electrodes separated by an insulating barrier, a supercurrent I_J flows according to the classical equations:

$$I_J = I_c \sin \varphi \quad (2.27)$$

$$V = \varphi_0 \frac{\partial \varphi}{\partial t} \quad (2.28)$$

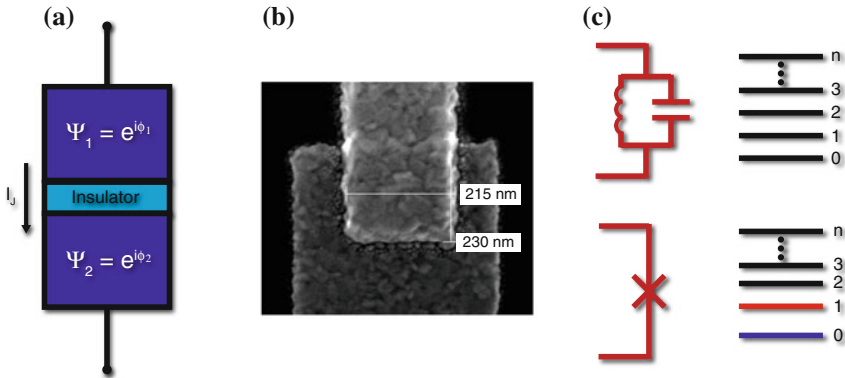


Fig. 2.9 The Josephson junction, a non-linear electrical element. **a** A Josephson junction is composed of two superconducting electrodes connected through an insulating barrier. **b** Scanning electron micrograph of a Josephson junction used in this thesis work: the electrodes are made of aluminum and the insulator of aluminum oxide. **c** A LC resonator has equally spaced energy levels which prevents any individual transition from being selectively addressed. A Josephson junction (represented by the *X* symbol) has unequally spaced energy levels, a property that can be used to bring non-linearity to a circuit

where $\varphi_0 = \hbar/2e$ is the reduced superconducting flux quantum, I_c the critical-current of the junction—that is the maximum supercurrent that the junction can support—and $\varphi = \varphi_1 - \varphi_2$ and V respectively the superconducting phase difference and voltage across the junction. This system of nonlinear equations represents the main results of the general theory of the Josephson junction which have many applications such as SQUID magnetic field detectors [8] and quantum limited oscillators, mixers and amplifiers [9, 10]. The dynamical behavior of the junction can be outlined in the expression of the derivative of the supercurrent I_J :

$$\frac{dI_J}{dt} = \frac{I_c \cos \varphi}{\varphi_0} V = \frac{I_c \sqrt{1 - (I_J/I_c)^2}}{\varphi_0} V \quad (2.29)$$

The Josephson junction appears equivalent to a nonlinear inductance:

$$L_J(\varphi) = \frac{\varphi_0}{I_c \sqrt{1 - (I_J/I_c)^2}} \approx \frac{\varphi_0}{I_c} \left(1 + \frac{(I_J/I_c)^2}{2} + O((I_J/I_c)^4) \right) \quad (2.30)$$

Note that for $I_J \ll I_c$, the junction behaves as a point-like inductance $L_J = \varphi_0/I_c$ whose value is entirely governed by the Josephson critical current. The energy associated with the phase difference φ across the Josephson junction writes:

$$E = E_J(1 - \cos \varphi) \quad (2.31)$$

where $E_J = I_c \varphi_0$ is called the Josephson energy. In addition, charge can accumulate on the capacitor C formed by the junction, giving rise to an electrostatic energy $E_C = Q^2/2C$. A quantum treatment of the Josephson junction is obtained by treating the flux $\Phi = \varphi_0 \varphi$ and charge Q as conjugate operators with commutation relation $[\hat{\Phi}, \hat{Q}] = i\hbar$. The Hamiltonian writes:

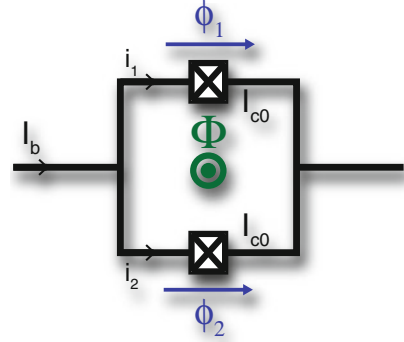
$$\hat{H} = \frac{1}{2C} \hat{Q}^2 + E_J (1 - \cos \hat{\varphi}) \quad (2.32)$$

Here the nonlinearity present in the system is a key ingredient for realizing Josephson junction based circuits since it breaks the degeneracy of the energy level spacing in comparison to LCR resonator (Fig. 2.9c).

The SQUID: a tunable inductor Another element that we extensively use in our experiment is the SQUID that we describe here. It consists in a superconducting loop interrupted by two Josephson junctions (Fig. 2.10). We consider a balanced SQUID of same critical currents $I_{c,1} = I_{c,2} = I_c$ and note ϕ_1 and ϕ_2 the superconducting phase difference across each of the junctions. Due to flux quantization, a magnetic flux applied through the loop produces a difference Φ between these phases:

$$\Phi = \frac{\Phi_0}{2\pi} (\phi_1 - \phi_2) \quad (2.33)$$

Fig. 2.10 Electrical scheme of a SQUID



where $\Phi_0 = 2\pi\varphi_0$. When the self-inductance of the loop remains negligible compared to the inductance of the SQUID, the bias current through the SQUID writes:

$$I_b = 2I_c \cos\left(\pi \frac{\Phi}{\Phi_0}\right) \sin\left(\frac{\phi_1 + \phi_2}{2}\right) \quad (2.34)$$

which shows that a SQUID behaves as a Josephson junction with a critical current $I_c(\Phi) = 2I_c \cos\left(\pi \frac{\Phi}{\Phi_0}\right)$, and therefore as a flux-tunable inductor with an inductance composed of a linear term

$$L_J(\Phi) = \frac{\varphi_0}{I_c(\Phi)} \quad (2.35)$$

with $I_c(\Phi) = 2I_c |\cos(\pi \frac{\Phi}{\Phi_0})|$ the flux-dependent critical current of the SQUID. Note that in addition, as already mentioned there exists a non-linear term dependent on the bias current I_b across the SQUID $\frac{\varphi_0}{2I_c^3(\Phi)} I_b^2$. This unwanted non-linear term arises when the bias current I_b is comparable to the critical current $I_c(\Phi)$.

2.1.2.2 Josephson Superconducting Resonators

Josephson junctions can be embedded in superconducting resonators to endow them with new functionalities. In our experiments, we are interested in two types of Josephson junctions based resonators. The first one, with a single junction, acts as a bistable hysteretic detector with the applied microwave field, called cavity Josephson bifurcation amplifier (CJBA) and will be used in our hybrid circuits to optimize the superconducting qubit measurement process. The second, with a SQUID, acts as a frequency flux-tunable resonator when supplied to an external magnetic flux Φ threading the SQUID loop, and as a quantum bus, will be engineered to transfer quantum states from the superconducting circuits processor to the spins memory.

THE CAVITY JOSEPHSON BIFURCATION AMPLIFIER

We present briefly here the physics of cavity Josephson bifurcation amplifier, that we use to realize a single-shot readout scheme for superconducting qubits in our hybrid circuits. A detailed discussion of the device can be found e.g. in [5, 9–11]. The CJBA is shown in Fig. 2.11, consisting of a transmission line resonator with a Josephson junction embedded in its central conductor. As discussed in Sect. 2.1.1.3, it can be modeled equivalently as a lumped elements resonator with capacitance C and inductance L plus a Josephson junction with inductance L_J . Its Hamiltonian, sum of harmonic oscillator Eq. 2.2 and Josephson junction terms Eq. 2.32, can be rewritten in the form of a Kerr birefringence Hamiltonian:

$$\hat{H} = \hbar\omega_r \left(\hat{a}^\dagger \hat{a} + \frac{K}{2} \hat{a}^{\dagger 2} \hat{a}^2 \right) \quad (2.36)$$

with resonance frequency

$$\omega_r = \frac{1}{\sqrt{(L_J + L)C}} \quad (2.37)$$

and Kerr constant

$$K = -\pi Z_r \frac{e^2}{h} \left(\frac{L_J}{L_J + L} \right)^3 \quad (2.38)$$

When driven under classical field $\beta_d(t) = \beta_d e^{-i\omega_d t}$ at frequency ω_d , the intra-cavity field α of Eq. 2.18 in the CJBA satisfies:

$$i \left(\Omega \frac{\kappa}{2} \alpha + K |\alpha|^2 \alpha \right) + \frac{\kappa}{2} \alpha = -i\beta_d \quad (2.39)$$

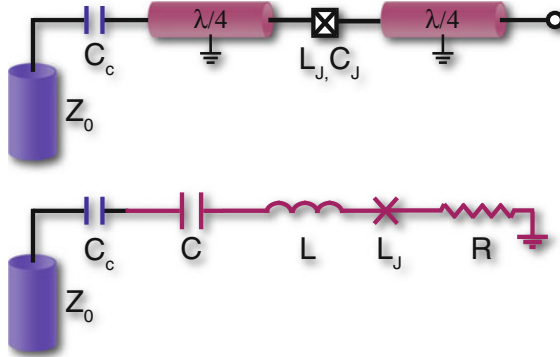


Fig. 2.11 The cavity Josephson bifurcation amplifier. The CJBA fabricated in this thesis work are nonlinear, open-end $\lambda/2$ resonator, consisting of two sections of transmission line joined by a Josephson junction. Its equivalent lumped elements circuit is a LC resonator with capacitance C and inductance L , plus a Josephson junction with inductance L_J

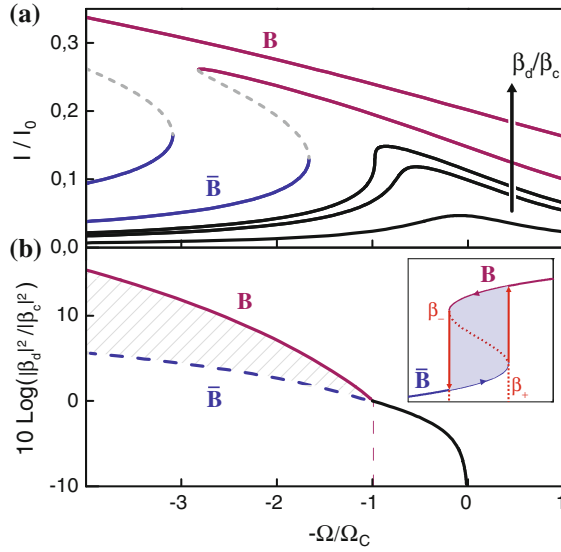


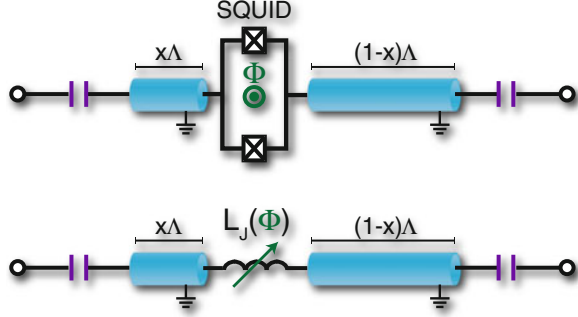
Fig. 2.12 The CJBA: a hysteretic detector for the drive amplitude. **a** Current in a CJBA as a function of the reduced drive detuning, plotted for various drive amplitude β_d . For drive detuning Ω_d larger than a threshold value Ω_r , the resonator becomes bistable above a certain power threshold $\beta_d > \beta_c$. **b** Phase diagram in the $\Omega - \beta_d$ plane, showing the region of low intracavity field ($\bar{\mathbf{B}}$), the region of high intracavity field ($\mathbf{B}$), and the bistable region (*dashed*) where both $\bar{\mathbf{B}}$ and $\mathbf{B}$ dynamical states can exist. The intracavity field bifurcates (see *inset*) from one states to the other when increasing (decreasing) the drive until reaching β_+ (β_-)

where $\Omega = 2Q(1 - \frac{\omega_d}{\omega_r})$ is the reduced detuning, which accounts for off-resonance drive. For certain couple of drive parameters (β_d, ω_d) , this equation has two solutions $\alpha_{i=1,2}$. Figure 2.12a shows the intracavity field α as a function of Ω , plotted for several values of the drive amplitude β_d . These double solutions appear when increasing the drive amplitude, and is conditioned by the drive frequency such that $\Omega > \Omega_r = \sqrt{3}$, the bistable region, in which we typically operate the CJBA to readout superconducting qubits. In Fig. 2.12b, we call $\bar{\mathbf{B}}$ and $\mathbf{B}$ respectively the low (α_1) and high (α_2) intracavity field state, and $\beta_{\pm}$, the bifurcation points, i.e. the points at which the system can change state. Starting with the resonator in $\bar{\mathbf{B}}$, the intra-cavity field remains in the lower branch while increasing the drive until reaching β_+ , where it suddenly grows to reach the upper branch $\mathbf{B}$ (the bifurcation). Operated in this bistable region, the CJBA acts as a hysteretic detector for the drive amplitude.

THE FREQUENCY TUNABLE RESONATOR

Being a flux-tunable inductor as seen above, a SQUID loop can be inserted in a resonator to make it tunable in frequency with an applied local magnetic field. Such resonators are useful to transfer quantum states between two systems at different frequencies and are extensively used in our hybrid circuits between the circuits and

Fig. 2.13 The frequency tunable resonator



the spins. Its implementation with distributed element resonator is schematized in Fig. 2.13. A SQUID is inserted in the central conductor of a $\lambda/2$ coplanar waveguide resonator at the position x ($x \in [0, 1]$, with respect to Λ the length of the resonator). As shown in [12], the insertion of the SQUID is equivalent to the insertion of the flux tunable inductance $L(\Phi)$ given by Eq. 2.35. This modifies the resonator frequency, with a dependence on the position x of the insertion. When introduced in the middle of the resonator ($x = 1/2$), the resonance frequency is changed from ω_0 (without SQUID) to:

$$\omega_r\left(\frac{\Lambda}{2}, \Phi\right) = \frac{\omega_0}{1 + L_J(\Phi) \frac{\omega_0}{Z_r}} \quad (2.40)$$

and the quality factor Q_0 to

$$Q_c(\Phi) \approx Q_0 \left(1 + 4L_J(\Phi) \frac{\omega_0}{Z_r} \right) \quad (2.41)$$

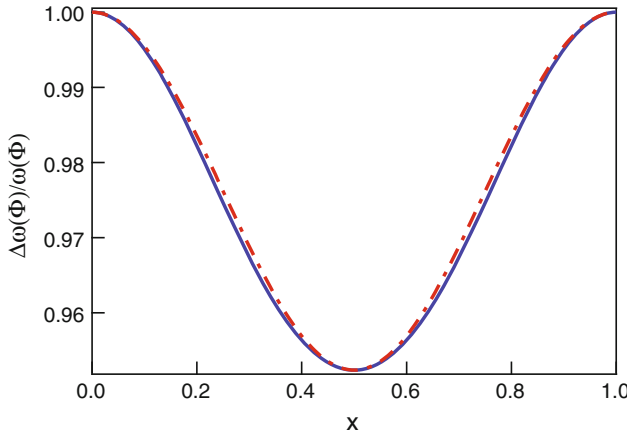


Fig. 2.14 Dependence of resonance frequency on the position of the SQUID in the resonator. $\Delta\omega_r(x\Lambda, \Phi_f)$ is shown at fixed applied magnetic flux Φ_f as a function of x . The line is fitted with Eq. 2.42 (dashed and dotted line)

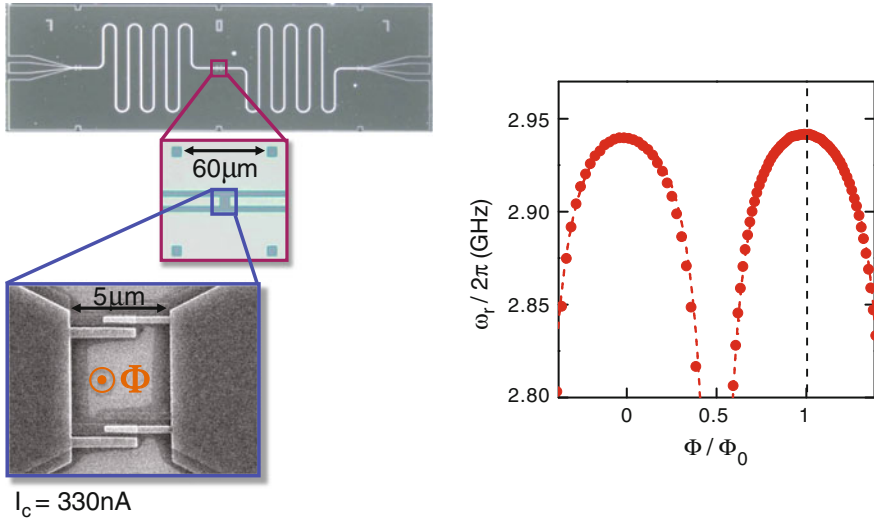


Fig. 2.15 The tunable resonator. The resonance frequency of a resonator containing a SQUID is tuned by varying the flux applied to the SQUID loop

In our experiments, for design reasons we have found preferable that the SQUID is not located at $x = 1/2$ but closer to the resonator end. We have thus extended the analysis to the case of an arbitrary value of x . To do so, we computed numerically the value of the resonance frequency as a function of x in the case where the ratio between the SQUID inductance $L_J(\Phi)$ and the resonator inductance Z_r/ω_0 is small compared to 1. The result is shown in Fig. 2.14. We find that it is well approximated by the intuitive formula:

$$\omega_r(x\Lambda, \Phi) = \omega_0 \left[1 - \sin^2(\pi x) \frac{L_J(\Phi)}{\frac{Z_r}{\omega_0} + L_J(\Phi)} \right] \quad (2.42)$$

The characterization of the resonator requires a probe power low enough to remain in the linear regime, i.e. $I_b \ll I_c(\Phi)$. Figure 2.15 shows the resonance frequency period with the variation of the flux through the SQUID loop operated in the linear regime, demonstrating that the resonator can be effectively tuned over hundreds of MHz.

2.1.2.3 Superconducting Artificial Atoms

To build an artificial atom, the non-linearity of Josephson junction based circuit has to be pushed to the level of very large anharmonicity so that individual levels can be addressed separately. In this case, the $|0\rangle$ to $|1\rangle$ transition can be used to implement the qubit concept. The specific Josephson circuit we used to implement artificial atoms

in our experiments is the transmon qubit [13] pioneered by Schoelkopf's group at Yale, a variant of the Cooper Pair Box (CPB) developed in 1996 in the Qnantronics group [14].

THE TRANSMON QUBIT

The Cooper-pair box artificial atom (CPB) consists in a superconducting island connected via a Josephson junction to a grounded reservoir (Fig. 2.16). The island is coupled to an input voltage source through a gate capacitance C_g . Cooper-pairs tunnel on and off the island via the Josephson junction. The gate circuit can be used to induce an excess charge $N_g = C_g V_g / (2e)$ on the island. The total number of Cooper-pairs having tunneled through the junction is described by the operator $\hat{N}$ and equivalently with its canonical conjugate, the superconducting phase difference $\hat{\theta}$ across the junction. We recall here the properties of the CPB that is needed to design and understand our experiment. An in-depth treatment of the Cooper-pair box can be found in A. Cottet's thesis [15]. The Hamiltonian of the CPB writes:

$$\hat{H} = E_C (\hat{N} - N_g)^2 - E_J \cos \hat{\theta} \quad (2.43)$$

with $E_C = (2e)^2 / 2C_\sigma$ and $C_\sigma = C_J + C_B + C_g$. Diagonalization of this Hamiltonian yields the circuit eigenenergies $\hbar\omega_i$ and eigenstates $|i\rangle$ so that the Hamiltonian can be recast in the form

$$\hat{H} = \hbar \sum_i \omega_i |i\rangle \langle i| \quad (2.44)$$

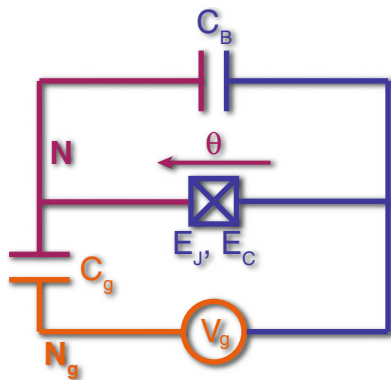


Fig. 2.16 Circuit schematic of a Cooper Pair Box. A superconducting island (*purple*) is connected to a reservoir (*blue*) through a Josephson junction with Josephson energy E_J , and a capacitance C_J . This island is also electrostatically coupled to ground through a geometric capacitor C_B , and to a voltage source V_g through a capacitor C_g . The gate circuit (*yellow*) can be used to induce an offset charge on the island

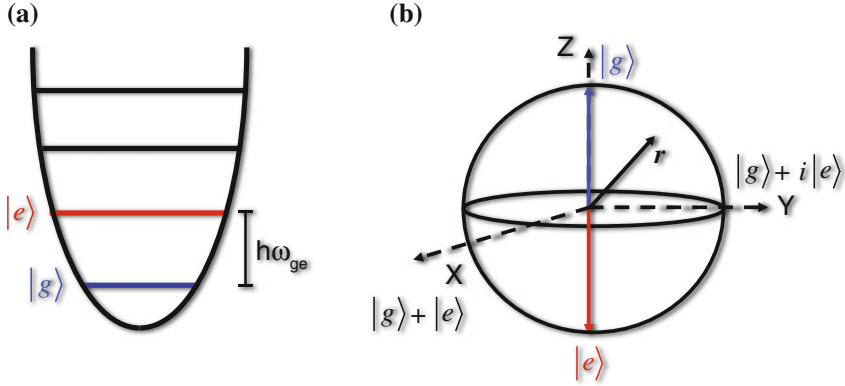


Fig. 2.17 Superconducting qubits. **a** The two lowest levels $|g\rangle$ and $|e\rangle$ are separated by a transition at ω_{ge} , that can be used to implement the qubit concept. **b** The Bloch vector of the qubit state ($\mathbf{r}$) represents a pure quantum state if it falls on the surface of the Bloch sphere or a mixed quantum state if it falls inside the sphere. The north pole corresponds to the state $|g\rangle$, the south pole to $|e\rangle$. The intersections of the unit sphere with the x , y axes corresponds respectively to the state $|g\rangle + |e\rangle$ and $|g\rangle + i|e\rangle$

We use the two lowest levels, that we will call in the following $|g\rangle$ and $|e\rangle$ to implement the qubit concept. Any pure state of the qubit can be written in a linear combination $|\psi\rangle = \alpha|g\rangle + \beta|e\rangle$ with α and β complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$. The state of the qubit can be mapped to the points on a sphere of radius 1, the Bloch sphere, with north pole corresponding to the state $|g\rangle$ and south pole to $|e\rangle$ and re-written in the spherical coordinates

$$|\psi\rangle = \cos(\theta/2)e^{-i\varphi/2}|g\rangle + \sin(\theta/2)e^{i\varphi/2}|e\rangle \quad (2.45)$$

with $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$, the polar and azimuth angles. The corresponding position $\mathbf{r}$, also called the Bloch vector, is defined in Fig. 2.17b and can represent a pure or mixed state.

In our experiments, we use a split Cooper pair box with a balanced SQUID loop instead of a single Josephson junction to allow for tunability of the qubit frequency ω_{ge} . The Hamiltonian of this circuit shown in Fig. 2.18a recasts in the form of Eq. 2.43 of the Cooper pair box with Josephson energy $E_J^*(\Phi)$ depending of the applied flux Φ to the SQUID loop:

$$E_J^*(\Phi) = E_J \sqrt{\frac{1 + \cos(\Phi/\varphi_0)}{2}} \quad (2.46)$$

In addition, the charging energy has been strongly reduced by designing a large geometrical capacitance C_B such that the qubit operates in the Josephson regime $E_J \gg E_c$ (Transmon). As shown in Fig. 2.18c, in this regime the charge dispersion of the energy levels of the Cooper pair box becomes extremely weak, thus rendering the qubit frequency practically insensitive to the value of the gate charge N_g .

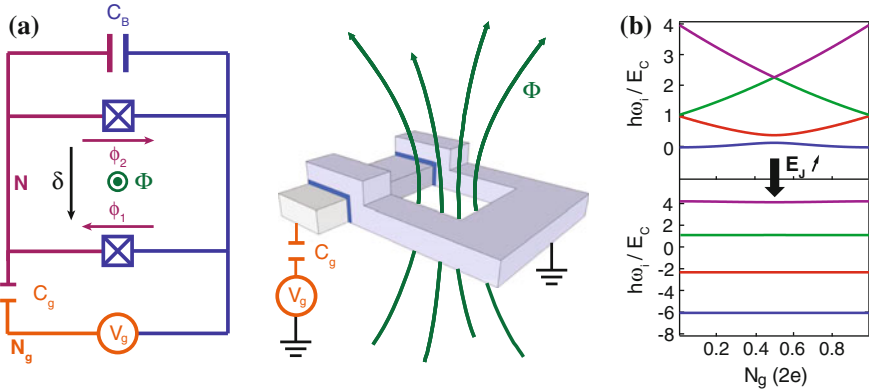


Fig. 2.18 Implementation of a superconducting qubit with a Transmon. **a** The Transmon qubit used in this thesis work is a split CPB with the Josephson junction replaced by a balanced SQUID loop and **b** operated in the regime $E_J \gg E_C$. The first energy levels of a Transmon qubit show a charge-dispersion curve almost completely flat

DECOHERENCE

Independently of the physical implementation (superconducting qubits, spins, atoms, . . .), the coupling to the environment deteriorates the quantum state supported by a qubit. This is the decoherence, which materializes in two processes:

- **Energy relaxation** The qubit in the excited state $|e\rangle$ decays to the ground state $|g\rangle$ by dissipating energy into its environment. This process involves the emission a photon with $\hbar\omega_{ge}$ and is characterized by an exponential decay time T_1 .
- **Dephasing** The qubit transition frequency ω_{ge} randomly fluctuates due to interactions with the environment. This process involves loss of the phase coherence φ of the quantum state and is characterized by a decay time T_φ .

The Bloch sphere is a useful tool to visualize the states and the decoherence processes (Fig. 2.19). Relaxation precipitates the Bloch vector towards the ground state; Dephasing shrinks the Bloch vector towards the center of the sphere. Since energy relaxation also causes loss of phase coherence, the two times are often expressed as one characteristic decoherence timescale, the free induction decay time T_2^* , with

$$\frac{1}{T_2^*} = \frac{1}{2T_1} + \frac{1}{T_\varphi} \quad (2.47)$$

There is usually a tradeoff between protecting the qubit from decoherence on the one hand and being able to easily control, readout and couple it. Superconducting qubits can be easily manipulated but have typically coherence properties far lower than the one of isolated microscopic systems such as nuclear and electronic spins in crystals. One of the major challenges in superconducting circuits has actually been to improve the coherence properties from the first qubit operation in 1999 with

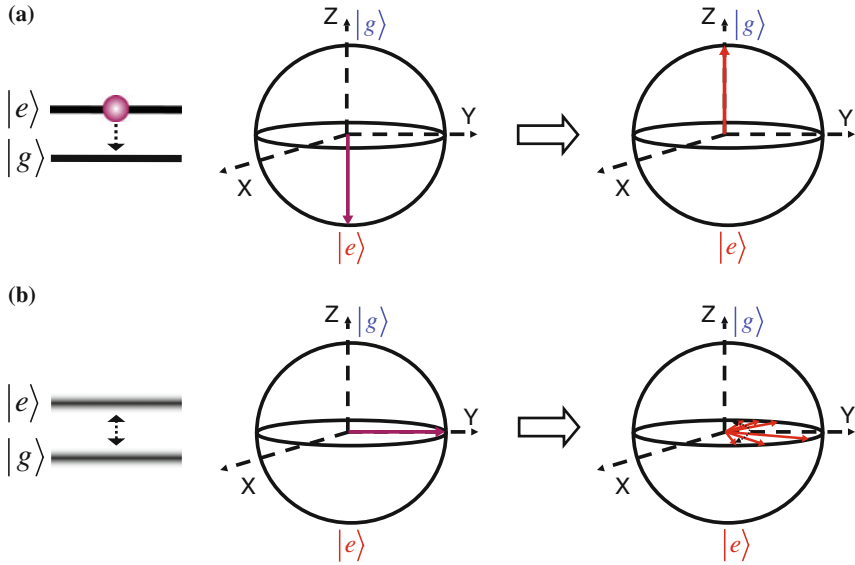


Fig. 2.19 Decoherence mechanisms. **a** A qubit in the excited state decays to the ground state via energy relaxation. **b** Interaction with the environment causes the energy level spacing between the qubit states to jitter leading to a loss of phase coherence called dephasing

5–10 ns range [16] to allow the implementation of quantum gate operation in multi-qubit small-scale processors. Progress in circuit design [17–20] and fabrication techniques has led to longer coherence times, up to tens of microseconds nowadays for transmon qubits [21, 22].

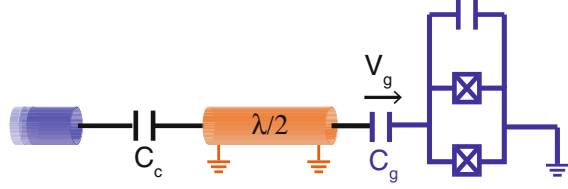
2.1.3 Circuit Quantum Electrodynamics

In this thesis, we use a resonator for coupling spins and superconducting qubits. Both are two-level systems (TLS) that can be described within the same framework when interacting with a planar resonator, referred as circuit quantum electrodynamics. In this section, we focus on the coupling between a resonator and a superconducting qubit on chip. Two regimes are of interest for our experiments: the resonant regime for quantum state transfer between the qubit and the bus resonator, and the dispersive regime for non-destructive measurement of the qubit state.

2.1.3.1 Qubit-Resonator Coupling

We consider the system shown in Fig. 2.20, in which a qubit of the transmon type, is capacitively coupled (C_g) to a CPW resonator, itself coupled (C_c) to the $50\ \Omega$ input transmission line. The Hamiltonian of the system is the sum of the Hamiltonians of the resonator (Eq. 2.2) and the qubit (Eq. 2.43), plus an interaction term $\hat{H}_{int}$ between

Fig. 2.20 Schematic of a typical CQED circuit, consisting of a Transmon qubit capacitively coupled to a transmission line resonator



them. For small couplings $C_g \ll C_c$, this interaction Hamiltonian approximates as the energy stored in the capacitance between the qubit and the resonator:

$$\hat{H}_{int} = \frac{1}{2} C_g \hat{V}_g^2 = \frac{1}{2} C_g \left(V_{rms} (\hat{a}^\dagger + \hat{a}) - \hat{V} \right)^2 \quad (2.48)$$

where $\hat{V}_g$ is the bias voltage at the coupling capacitance, V_{rms} the root-mean-square fluctuations of the voltage of the resonator introduced in Eqs. 2.5 and 2.6 and $\hat{V} = 2e/C_\Sigma \cdot (\hat{N}_g - \hat{N})$ the voltage across the Transmon electrodes (see Fig. 2.16). In the limit where $\beta = C_g/C_\Sigma \ll C_\Sigma$, we can restrict ourselves to the first order coupling term in V_{rms} such that¹:

$$\hat{H}_{int} = 2e\beta V_{rms} \hat{N} (\hat{a}^\dagger + \hat{a}) \quad (2.49)$$

The coupling strength g between the qubit and the resonator can be identified by rewriting the interaction Hamiltonian in term of $\hat{\sigma}_+ = |e\rangle\langle g|$ and $\hat{\sigma}_- = |g\rangle\langle e|$ the raising and lowering operators of the qubit:

$$\hat{H}_{int} = \hbar g (\hat{\sigma}_+ + \hat{\sigma}_-) (\hat{a} + \hat{a}^\dagger) \quad (2.50)$$

with g given by

$$\hbar g = 2\beta V_{rms} \langle g | \hat{N} | e \rangle \quad (2.51)$$

In the case where the coupling between the resonator and the qubit is such that $g \ll \omega_r, \omega_{ge}$, we can ignore the terms in the interaction Hamiltonian that describe simultaneous excitation or desexcitation of the qubit and the resonator, and rewrite the total Hamiltonian in the rotating wave approximation:

$$\hat{H} = \hbar\omega_r \left(\hat{a}^\dagger \hat{a} + 1/2 \right) - \hbar \frac{\omega_{ge}}{2} \hat{\sigma}_z + \hbar g \left(\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_- \right) \quad (2.52)$$

This is the Jaynes Cumming Hamiltonian. In an experiment, both the cavity and the qubit have losses, characterized by damping rates κ (cavity) and γ (qubit). The regime of interest, in which the coupling strength is much larger than all losses in

¹The terms in N_g which do not corresponds to coupling term but to a renormalization of the resonator capacitance are omitted.

the system, is called the strong coupling regime ($g \gg \kappa, \gamma$). In this case, losses are invisible on the time scale of the qubit-resonator interaction and the Jaynes-Cumming Hamiltonian can be analytically diagonalized. The two eigenstates of the individual systems (the intracavity field state $|n\rangle$ for the cavity and the ground state $|g\rangle$ and excited state $|e\rangle$ for the atom) are no longer eigenstates of the Hamiltonian Eq. 2.52 but instead a coherent superposition of both system states [14]:

$$|+, n\rangle = \cos \theta_n |e, n\rangle + \sin \theta_n |g, n+1\rangle \quad (2.53)$$

$$|-, n\rangle = -\sin \theta_n |e, n\rangle + \cos \theta_n |g, n+1\rangle \quad (2.54)$$

with θ_n a mixing angle defined as

$$\tan(2\theta_n) = -2g \frac{\sqrt{n+1}}{\Delta} \quad (2.55)$$

where $\Delta = \omega_{ge} - \omega_r$ is the frequency difference between the cavity and the qubit. The energies corresponding to these states are

$$\hbar\omega_{\pm,n} = \hbar\omega_r(n+1) \pm \hbar \frac{\Omega_{n,\Delta}}{2} \quad (2.56)$$

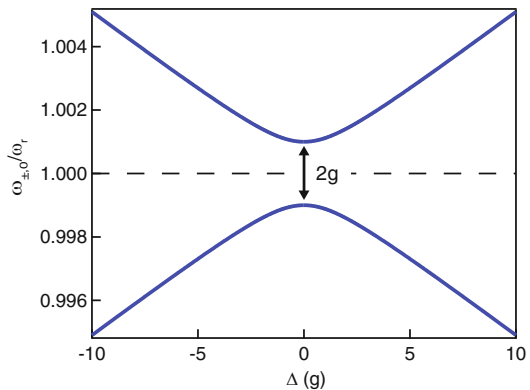
with $\Omega_{n,\Delta}$ the vacuum Rabi frequency given by

$$\Omega_{n,\Delta} = \sqrt{4g^2(n+1) + \Delta^2} \quad (2.57)$$

In the experiment reported in Chap. 4, we exchange a single photon between the qubit and the resonator. This corresponds to the case $n = 0$. We show in Fig. 2.21 the frequencies $\omega_{\pm,0}$ as a function of the detuning Δ between the resonator and the qubit.

An avoided crossing of width $2g$ appears between the two eigenstates of the coupled system (see Fig. 2.21). This phenomenon is known as the vacuum Rabi splitting. If the qubit is prepared in state $|e\rangle$ at $t = 0$ with the cavity empty ($|e, 0\rangle$),

Fig. 2.21 Vacuum Rabi splitting. The frequency of the resonator shows an anti-crossing when approaching the qubit frequency ($\Delta = 0$)



the probability to find the system in state $|g, 1\rangle$ is:

$$P(t) = \frac{4g^2}{\Omega_{0,\Delta}^2} \sin^2 \left(\frac{\Omega_{0,\Delta}}{2} t \right). \quad (2.58)$$

Resonant regime: vacuum Rabi oscillations When the cavity and the qubit are resonant such that $\Delta \ll g$, the frequency $\Omega_{0,\Delta} \approx 2g$. If the qubit is prepared in $|e\rangle$ at $t = 0$ with the cavity in $|0\rangle$, the probability to find the resonator in $|1\rangle$ at time t becomes:

$$P(t) = \sin^2(gt) = \frac{1 - \cos(2gt)}{2} \quad (2.59)$$

These oscillations are known as vacuum Rabi oscillations. After a time $\pi/2g$, the resonator is prepared in state $|1\rangle$ with probability 1; we will use this phenomenon in the experiments reported in Chap. 4 to transfer a quantum state from the qubit into the spin ensemble via a resonator.

Dispersive regime: cavity-pull When the cavity is far detuned from the TLS transition frequency such that $\Delta \gg g$, the probability (Eq. 2.58) to exchange a photon becomes negligible. This regime has been discussed e.g. in [5] and is called the dispersive regime. In this case, the Hamiltonian is approximated by an effective Hamiltonian:

$$\hat{H}_{disp} = \frac{1}{2} \hbar \omega'_{ge} \hat{\sigma}_z + \underbrace{\hbar(\omega'_r + \chi \hat{\sigma}_z) \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right)}_{\text{cavity pull}} \quad (2.60)$$

with the dispersive shift $\chi = g^2/\Delta$. The term labeled as cavity pull remains the Hamiltonian of a harmonic oscillator, but with frequency $\omega'_r + \chi$ when the TLS is in its ground state and $\omega'_r - \chi$ when in its excited state. This qubit-state dependent frequency shift is the basis for qubit readout in cQED. We place ourselves in this dispersive regime to measure the superconducting qubit.

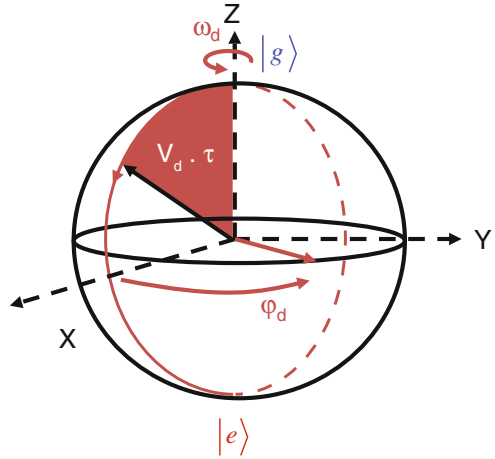
2.1.3.2 Resonant Qubit Manipulation

To drive the qubit, a microwave pulse at a frequency ω_d close to ω_{ge} is sent onto the resonator input. It generates a field of amplitude $|\alpha|e^{-i(\omega_d t + \phi_d)}$, whose action on the qubit is given by the Hamiltonian term $g|\alpha| [e^{-i(\omega_d t + \phi_d)} \hat{\sigma}_+ + e^{i(\omega_d t + \phi_d)} \hat{\sigma}_-]$. In the rotating frame at ω_d , the complete qubit Hamiltonian can then be rewritten as:

$$\hat{H}_d = \frac{\hbar \delta \omega}{2} \hat{\sigma}_z + \hbar \frac{\Omega_R}{2} \hat{\sigma}_n \quad (2.61)$$

with $\delta \omega = \omega_{ge} - \omega_d$ the qubit-drive detuning, $\Omega_R = 2g|\alpha|$ the Rabi frequency, and $\hat{\sigma}_n = \cos \phi_d \hat{\sigma}_x + \sin \phi_d \hat{\sigma}_y$. In the Bloch sphere representation (Fig. 2.22), the

Fig. 2.22 Qubit operations represented in the Bloch sphere rotating at the microwave frequency ω_d



qubit Bloch vector precesses around an axis making an azimuthal angle ϕ_d with the x -axis, and an angle θ such that $\tan \theta = \delta\omega / \Omega_R$ with the x - y plane. By appropriate selection of the pulse duration and detuning, arbitrary rotations can be achieved on the Bloch sphere.

2.1.3.3 Non-destructive Qubit Readout with Josephson Bifurcation Amplifier

To measure the qubit state, we use the cavity pull effect seen in Sect. 2.1.3.1. The Transmon-resonator system is in the dispersive regime, where the resonator frequency is shifted up or down depending on the qubit state as expressed by Eq. 2.60. The common measurement technique consists in sending a microwave pulse at ω_r and measuring the relative phase reflected onto the resonator. At resonance, the reflected

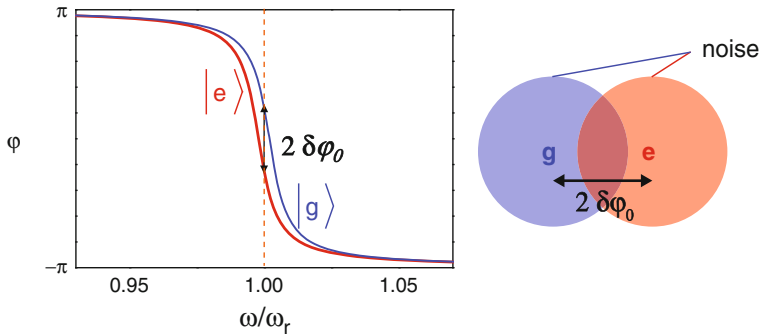


Fig. 2.23 Standard dispersive measurement technique. The phase of a reflected probe pulse is measured. The noise coming mainly from the cryogenic amplifier, introduces an uncertainty on the discrimination between $|g\rangle$ and $|e\rangle$ qubit states (red and blue disks), which depends on the averaging time, and which can be of the same order or even larger than the separation between the two vectors to be discriminated

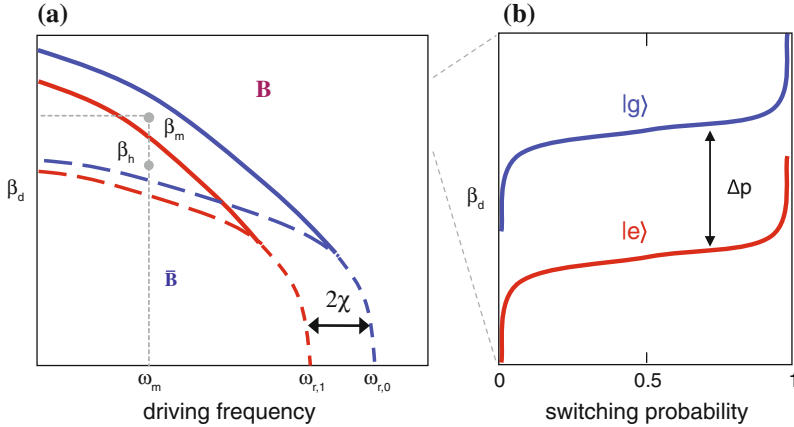


Fig. 2.24 Single-shot qubit readout. **a** Same CJBA phase diagram as on Fig. 2.12 for a CJBA embedding a qubit that shifts the diagram by $\pm\chi$ when being in its $|g\rangle$ and $|e\rangle$ states, respectively. **b** Probability of switching from $\bar{\mathbf{B}}$ to $\mathbf{B}$ as a function of the CJBA input drive amplitude β_d , shown for an embedded qubit in state $|g\rangle$ (blue) or $|e\rangle$ (red)

phase of the microwave signal onto the cavity undergoes a π shift (see Sect. 2.1.1.2). When coupled to the qubit, the frequency at which occurs this π -shift is dependent on the qubit state which defines a phase difference $\delta\varphi_0$ shown in Fig. 2.23. The discrimination between the two qubit states is obtained by averaging out to reduce the noise on the measured reflected phase.

However, as noise can be of the same order or even larger than $\delta\varphi_0$, requirement on the averaging is tight to obtain good signal to noise ratio. In our experiments, we optimize the measurement process by replacing the resonator by a cavity Josephson bifurcation amplifier (R), the hysteretic detector that we have seen Sect. 2.1.1.2. Figure 2.24 shows the phase diagram of the CJBA, indicating the stability regions of the different solutions $\bar{\mathbf{B}}$ (low-amplitude) and $\mathbf{B}$ (high-amplitude) of R for the two different qubit states $|g\rangle$ and $|e\rangle$. We take advantage of its hysteretic behavior in a measurement process separated in two steps:

- **Map the qubit state to the resonator:** at fixed drive frequency ω_m , the amplitude of the measurement pulse is quickly ramped from 0 to the value β_m as indicated in Fig. 2.24a. If the qubit is in state $|g\rangle$, R remains in the low-amplitude state $\bar{\mathbf{B}}$, while switches to the high-amplitude state $\mathbf{B}$ if in $|e\rangle$. In this way, the state of the qubit is mapped to one of the two intra-cavity field states of R, resonator which can be easily measured by standard microwave techniques;
- **Maintain and measure the resonator until its discrimination:** the measurement of the resonator has to be well averaged out, thus during a long time interval (typically 1–2 μs) to discriminate between the two oscillator states with certainty. To avoid further switching processes during the time needed to measure, the drive amplitude is lowered to a value β_h at which the switching probability of R is very small. At this point, we can measure R for an arbitrary long time without being limited by spurious switching events.

This measurement technique is qualified of single-shot readout and well-known for its robustness with readout contrast demonstrated up to 93 % in past experiments [23]. In our hybrid circuit experiments, we will use the CJBA as a tool for efficient qubit readout.

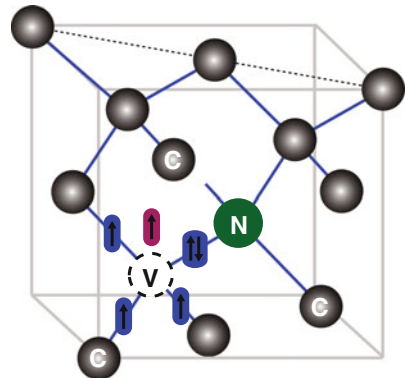
2.2 NV Center Spins in Diamond

We now turn to the second component of our hybrid circuits: the NV center spins in diamond. The Nitrogen-Vacancy center (NV) is an impurity in diamond which has drawn an increasing amount of interest in the quantum optics community for its spin and optical properties. The reason is that it is paramagnetic [24, 25], has good coherence properties [26, 27], and using Optically Detected Magnetic Resonance (ODMR) it is possible to optically read out and polarize the electron spin state [28]. We give in the following more details on its spin and optical properties which will be exploited to implement memory operations and explicit the specific sample properties required for our hybrid circuit implementations.

2.2.1 Structure

The NV center consists of a substitutional nitrogen atom (N) next to a vacancy (V) in an adjacent lattice site of the diamond crystalline matrix (Fig. 2.25). It has a trigonal symmetry around the crystallographic direction connecting the nitrogen and the vacancy that we refer to as the NV axis and note Z in the following. The NV axis coincides with $\langle 111 \rangle$ directions of the diamond lattice which implies four different possible NV axis orientations. The electronic structure of the NV center determines its optical and spin properties. Three electrons are provided by the dangling bonds of the vacancy to neighboring carbon atoms, two by the dangling bonds of the nitrogen atom itself. This configuration forms the first kind of NV center, the NV^0 , which is

Fig. 2.25 Schematic representation of the NV center in diamond



neutral and has electron spin $S = 1/2$. There are also cases where the NV center has captured an additional electron from other Nitrogen donors in the diamond: the negatively charged NV centers (NV^-). In this thesis work we concentrate exclusively on the latter one which has electron spin $S = 1$ and will omit ‘-’ sign from now.

2.2.2 The NV Center Spin Qubit

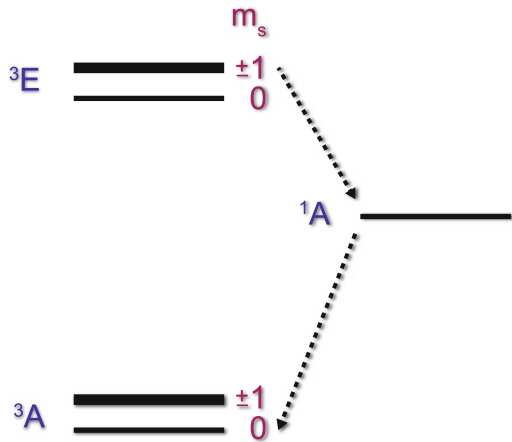
Symmetry and structure determine the electronic properties of the NV center. In the case of NV, the electronic configuration leaves two unpaired electrons which couple together to form either triplet or singlet states. The determination of the exact energy level structure resulting of this coupling has combined many efforts both from the theoretical and experimental side including optical [29], electron [30] paramagnetic resonance. For the work we discuss in this thesis, we limit ourselves to relevant states, the electronic ground (3A) and the first excited state (3E) which both are spin triplet states, plus a singlet level 1A present between the ground and excited states [28]. The schematic of the energy level diagram is shown in Fig. 2.26.

2.2.2.1 NV Qubit Transition: Spin Properties

The electronic ground state 3A is a spin triplet ($S = 1$). The system can be described in the basis $m_S = 0, \pm 1$ (the spin quantization along Z, the NV axis) using the dimensionless spin-1 vectorial operator $\mathbf{S} = (\hat{S}_X, \hat{S}_Y, \hat{S}_Z)$, with

$$\hat{S}_X = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \hat{S}_Y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \hat{S}_Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (2.62)$$

Fig. 2.26 Level diagram of the NV center, showing ground (3A) and first excited (3E) electronic states, as well as the 1A state. A transition in the optical domain separates the two electronic states, which are both spin triplet ($S = 1$)



Note that restricting ourselves to a 2-level subspace, one obtains the relations $\hat{S}_{X,Y} = \frac{1}{\sqrt{2}}\hat{\sigma}_{x,y}$ between the spin-1 operators and the usual spin-1/2 Pauli matrices which will be used later in this manuscript. The spin Hamiltonian is a sum of the zero-field splitting ($\hat{H}_{ZF}$), the electron Zeeman shift ($\hat{H}_B$) and the hyperfine interaction with the nitrogen nucleus ($\hat{H}_{HF}$). These three contributions are described below.

Zero-field splitting The zero-field splitting $\hat{H}_{ZF}$ results from the dipole-dipole magnetic coupling between the two unpaired electron spins forming $S = 1$. As the name suggests it leads to a level splitting even in the absence of external magnetic field. The zero-field Hamiltonian can be written using the zero-field splitting tensor $\bar{\mathbf{D}}$:

$$\hat{H}_{ZF}/\hbar = \mathbf{S} \cdot \bar{\mathbf{D}} \cdot \mathbf{S} \quad (2.63)$$

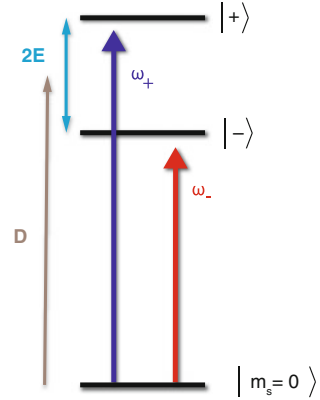
$$= D_X \hat{S}_X^2 + D_Y \hat{S}_Y^2 + D_Z \hat{S}_Z^2 \quad (2.64)$$

which can be equivalently rewritten as²:

$$\hat{H}_{ZF}/\hbar = D \hat{S}_Z^2 + E (\hat{S}_X^2 - \hat{S}_Y^2) \quad (2.65)$$

where we defined $D = 3D_Z/2$ the zero-field splitting and $E = (D_X - D_Y)/2$ the strain induced splitting. Due to the axial symmetry of the NV center, D_X and D_Y should be identical. In our experiment however, a distortion of the trigonal symmetry appears due to the effect of strain and local electric fields on the orbital energy yielding $D_X \neq D_Y$ (and thus $E \neq 0$). As a result, the energy eigenstates $|\pm\rangle$ are not identical to the two pure spin eigenstates $|m_S = \pm 1\rangle$. The state $|\pm\rangle$ are linear combinations of $|m_S = \pm 1\rangle$, with at zero magnetic field $|\pm\rangle = (|m_S = +1\rangle \pm |m_S = -1\rangle)/\sqrt{2}$ separated in frequency by $2E$. Microwave transitions are induced between the $|m_S = 0\rangle$ ground state and either the $|+\rangle$ or the $|-\rangle$ excited states with transition frequency $\omega_{\pm}$ (Fig. 2.27). For our hybrid circuits, we use an ensemble of NV centers

Fig. 2.27 Electron spin levels of the electronic ground state



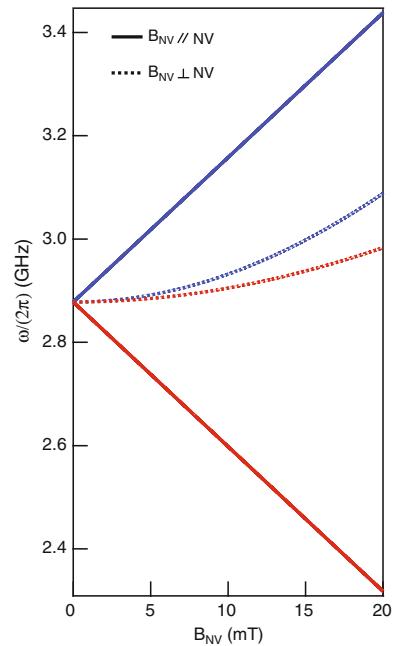
²The constants terms are omitted.

which involves a distribution of both the zero-field splitting $\rho(D)$ and strain induced splitting $\rho(E)$ Hamiltonian parameters. Average values are $D/2\pi = 2.878$ GHz and $E/2\pi$ between 1–5 MHz from sample to sample.

Zeeman interaction The Zeeman term appears upon the application of an external magnetic field $\mathbf{B}_{NV}$. It writes $\hat{H}_B/\hbar = -\gamma_e \mathbf{B}_{NV} \cdot \mathbf{S}$, with $\gamma_e = -g_e \mu_B/\hbar = -2\pi \times 2.8$ MHz/Gs the gyromagnetic moment of the NV electron spin ($g_e = 2$ [24] the NV Landé factor and μ_B the Bohr magneton). The Hamiltonian $\hat{H}_{ZF} + \hat{H}_B$ can be diagonalized. In Fig. 2.28, we show the spin energy states evolution with the external magnetic field $\mathbf{B}_{NV}$ applied parallel and perpendicular to the NV center axis for 2 MHz strain coefficient. When $B_{NV} \gg E/|\gamma_e|$, the strain-induced fine structure splitting becomes negligible compared to the Zeeman splitting, the states $|\pm\rangle$ are well approximated by the pure spin states $|m_S = \pm 1\rangle$ and the resonant frequencies $\omega_{\pm}$ tend towards their asymptotes $D \mp \gamma_e B_{NV}$. In our experiments, the magnetic field $\mathbf{B}_{NV}$ is applied parallel to superconducting bus resonator on a collection of NV center spins yielding different Zeeman shifts for the four spin groups of crystallographic NV orientations.

Hyperfine interaction The NV center is also coupled by hyperfine interaction to nearby nuclear spins. One can distinguish the contribution of the hyperfine coupling to the ^{14}N nuclear spin, which is present in each NV center and modifies appreciably its spectrum, from the hyperfine coupling to the ^{13}C spins in the diamond lattice, which has a different effect on each NV center and contributes to decoherence. We thus include in the NV Hamiltonian only the coupling to the nuclear spin-1 ($I = 1$)

Fig. 2.28 Energy states dependence of the microwave transitions $\omega_{\pm}$ under a magnetic field, applied parallel (perpendicular) to the NV center axis showing a linear (quadratic) Zeeman effect



of the nitrogen atom ^{14}N . The hyperfine term can be written as $\hat{H}_{HF}/\hbar = \mathbf{S} \cdot \bar{\mathbf{A}} \cdot \mathbf{I}$ with $\mathbf{I}$ the dimensionless operator of the nitrogen nuclear spin and $\bar{\mathbf{A}}$ the hyperfine interaction tensor. The hyperfine tensor is given by [30]:

$$\bar{\mathbf{A}} = \begin{pmatrix} A_{\perp} & & \\ & A_{\perp} & \\ & & A_{\parallel} \end{pmatrix} \quad (2.66)$$

with $A_{\perp}/2\pi = 2.7\text{MHz}$ and $A_{\parallel}/2\pi = 2.14\text{MHz}$. In addition, the nitrogen atom has a quadrupole moment yielding a term $\hat{H}_P = P\hat{I}_Z^2$ with $P = -2\pi \times 5\text{MHz}$. In total, the NV Hamiltonian is:

$$\hat{H}/\hbar = \underbrace{\mathbf{S} \cdot \bar{\mathbf{D}} \cdot \mathbf{S}}_{ZF} - \underbrace{\gamma_e \mathbf{B}_{NV} \cdot \mathbf{S}}_B + \underbrace{\mathbf{S} \cdot \bar{\mathbf{A}} \cdot \mathbf{I}}_{HF} + P\hat{I}_Z^2 \quad (2.67)$$

The NV eigenstates are displayed in Fig. 2.29b, showing the effect of the strain as a lifting of degeneracy at zero field and the hyperfine interaction with ^{14}N as a splitting by approximately 2.17MHz of the resonance into a triplet structure. In practice, one of the two transitions at frequencies $\omega_{\pm}$ between the spin ground state $|m_S = 0\rangle$ and the excited states $|\pm\rangle$ is used, equivalent to a two-level system with ground state $|g\rangle$ and excited state $|e\rangle$ and transition frequency ω_S . In this two level system description, the Hamiltonian of the NV center spin (Eq. 2.67) reduces to

$$\hat{H}_a = -\hbar \frac{\omega_{\pm}}{2} \hat{\sigma}_z \quad (2.68)$$

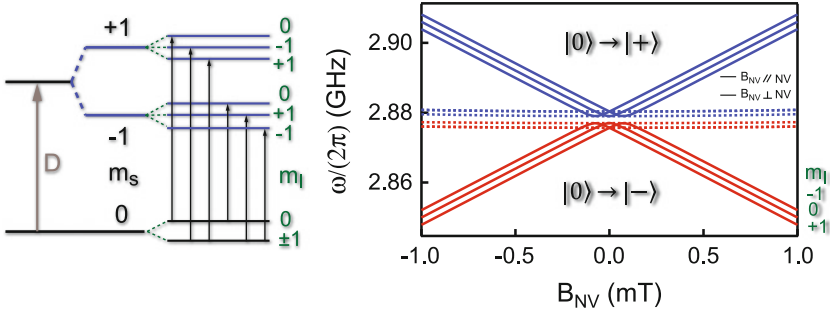


Fig. 2.29 NV energy diagram showing the Zeeman and hyperfine structure of the electronic ground state under a magnetic field applied parallel (perpendicular) to the NV center axis. Six microwave transitions are allowed. Under magnetic field applied perpendicular to the NV axis, the transitions $|m_S = 0, m_I = \pm 1\rangle \rightarrow |m_S = +1, m_I = +1\rangle$ and $|m_S = 0, m_I = \pm 1\rangle \rightarrow |m_S = +1, m_I = -1\rangle$ (respectively $|m_S = 0, m_I = \pm 1\rangle \rightarrow |m_S = -1, m_I = +1\rangle$ and $|m_S = 0, m_I = \pm 1\rangle \rightarrow |m_S = -1, m_I = -1\rangle$) are degenerated

An oscillating magnetic field $\mathbf{B}_d(t) = B_d \cos(\omega_d t) \cdot \mathbf{e}_x$ can be applied to drive the spin transitions. The corresponding Hamiltonian is $\hat{H}_d = -\gamma_e \mathbf{S} \cdot \mathbf{B}_d(t)$ and can be rewritten in the frame rotating at ω_d

$$\hat{H}_d/\hbar = -\gamma_e \frac{\hat{B}_d \cdot \hat{S}_X}{2} = -\gamma_e \frac{B_d}{2\sqrt{2}} \hat{\sigma}_x \quad (2.69)$$

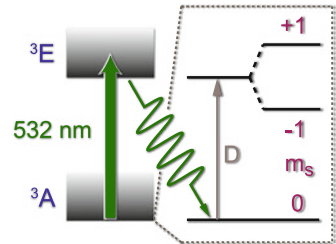
which corresponds to a Rabi frequency $\Omega_R = \gamma_e B_d / \sqrt{2}$. In our experiments, the spins are driven via a resonator, with the oscillating magnetic field generated by the oscillating current in the resonator inductance. As explained earlier, it is possible to link the incoming microwave power P to this current, and the current to a spatially-dependent magnetic field $B_d(\mathbf{r})$. The last equation shows how to determine the Rabi frequency $\Omega_R(\mathbf{r}, P)$ of a spin located at position $\mathbf{r}$, which will be used in Chap. 5.

2.2.2.2 NV Qubit Initialization: Optical Properties

Spin state initialization can be obtained through the use of an optical transition to the electronic excited state which has the property to relax selectively in $|m_S = 0\rangle$ of the electronic ground state. For this, drive of the NV centers to the electronic excited state has to be implemented. The electronic ground and excited states of the NV centers are coupled through an electrical dipolar transition with zero phonon line (ZPL) at 1.945 eV (Fig. 2.30), corresponding to emission at $\lambda_{ZPL} = 638$ nm. This radiative transition is coupled to the phonons of the diamond matrix which allows for vibronic sideband excitations. Due to the availability, usually 532 nm laser light is used to excite from the triplet ground state to the excited one [31]. The optical transitions between 3A and 3E are spin conserving ($\Delta m_S = 0$). As a result, NV centers initially in the $|m_S = 0\rangle_{\text{ground}}$ (respectively in $|m_S = \pm 1\rangle_{\text{ground}}$) end up in the $|m_S = 0\rangle_{\text{excited}}$ (respectively in the $|m_S = \pm 1\rangle_{\text{excited}}$) excited state and can decay back through the same radiative transition.

There is however a second possibility given by intersystem crossing (ISC): once in the excited electronic state the NV center can return into the triplet ground state via the singlet metastable level 1A . The decay back from the metastable state into the triplet ground state occurs preferentially into the $|m_S = 0\rangle_{\text{ground}}$ [32, 33]. The point is that ISC is strongly spin state dependent so that there is high probability from $|m_S = \pm 1\rangle_{\text{excited}}$ state to decay back via the metastable state while a very low

Fig. 2.30 Optical repumping of NV centers in their $m_S = 0$ ground state by application of green (532 nm) laser pulses exciting the 3E - 3A transition



one from $|m_S = 0\rangle_{excited}$ state. Hence, after several optical cycles, the NV-centers are mainly polarized in the ground state $|m_S = 0\rangle_{ground}$ regardless of their former state: this is the optical repumping. This process occurs in few tens of nanoseconds at low temperature [34], sufficiently fast to be included in our memory protocol. The maximum spin polarization reachable using optical repumping is $\sim 90\%$, according to [35].

For experiments which do not require such fast pumping, or alternatively in which the amount of excited spins is intrinsically very low (for example the experiment 1 of this thesis work), cooling down in a dilution fridge at milliKelvin (mK) temperatures via the surrounding bath temperature is sufficient to ensure large polarization in $|m_S = 0\rangle$. Indeed for NV center at 30 mK, $\hbar\omega_{ge} \gg k_B T$ and the probability for a spin to be excited is $p_{\pm} = e^{-(\hbar\omega_{ge}/k_B T)} \approx 0.01$. Note however that the NV energy relaxation time is of order 5 ms [36] at room temperature, and even longer at low temperature. This implies that once excited, the relaxation of the spin polarization towards the equilibrium is long. Hence, it is possible that the temperature of the spins is different than the one of the cryostat, due to thermal excitation, i.e. via measurement transmission lines.

2.2.3 Coherence Times

The coherence properties of NV centers are characterized by the free-induction decay time T_2^* (measured by Ramsey fringes), the Hahn-echo decay time T_2 (measured by a spin-echo sequence), and the coherence time under dynamical decoupling sequences such as Carr-Purcell-Meiboom-Gil T_{2CPMG} (see Fig. 2.31). The value found for these

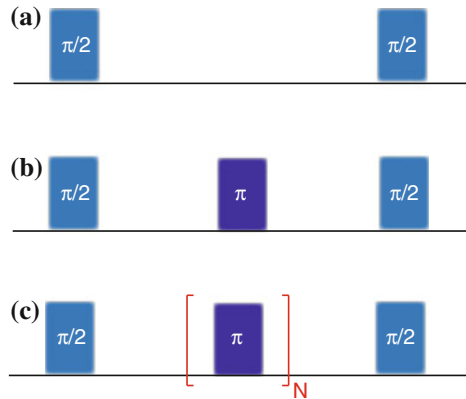


Fig. 2.31 Measuring the coherence times. **a** Ramsey sequence. Two $\pi/2$ rotations are performed, separated by a delay during which the spins precess freely. The envelope of the oscillations for increasing τ is characterized by the dephasing time T_2^* , associated to fluctuations of the environment. **b** Spin echo sequence. An intermediate π rotations around x is performed to refocus the dephasing due to static fluctuations. **c** Dynamical decoupling sequence. Successive π rotations are performed to refocus the dephasing due to dynamic fluctuations

times depends crucially on the local magnetic environment of each spin in a sphere of radius few tens of nanometers. In diamond, the main magnetic impurities are neutral nitrogen atom (the P1 centers) which have an electronic spin $1/2$ and carbon ^{13}C nuclei with their nuclear spin $1/2$ present to 1.1 % abundance in natural carbon. The longest echo coherence times $T_2 = 2\text{ms}$ were therefore measured in ultra-pure samples growth by Chemical-Vapor Deposition (CVD) with very low nitrogen concentration as well as isotopically enriched carbon source. In such samples, the coherence time has been extended out up to $T_{2\text{CPMG}} = 0.5\text{s}$ under Carr-Purcell-Meiboom-Gil dynamical decoupling sequences at low (100 K) temperatures [27]. This is five orders of magnitude longer than superconducting qubits, which motivates the idea of using NV centers for storing quantum information.

The crystals used in our experiments are however not as pure. Indeed, we need relatively large concentrations of NV centers of orders $\sim 1.76 - 17.6 \cdot 10^5 \mu\text{m}^3$ (1–10 ppm) to efficiently absorb microwave photon in superconducting resonator. These concentrations are not easily reached with sample grown by CVD. Our crystals instead are grown by a method called High-Pressure-High-Temperature (HPHT). HPHT diamond usually have a large nitrogen concentration of 1–100 ppm. NV centers are created from this nitrogen doped diamond in two steps: (i) irradiation with protons or with electrons of a diamond crystal to produce vacancies (ii) annealing at $800\text{--}1000^\circ\text{C}$ to allow the vacancies to migrate and form the NV defect. This method unavoidably leaves a significant residual concentration of P_1 centers (1–100 ppm), which limits the spin coherence time (both T_2^* and T_2) to lower values than reported with CVD diamonds. In the experiments discussed here, these residuals P1 centers are the main cause of decoherence. The contribution of P1 centers to the decoherence has been thoroughly studied [37–40]. In particular, it was shown [41] that T_2^* and T_2 are inversely proportional to the P1 center concentration ($[P1]$), with the relations $1/T_2^* = -\gamma_e \sqrt{1.2 \cdot 10^{-4} [P1]^2 + 6.4 \cdot 10^{-4} \text{ s}^{-1}}$ and $1/T_2 = 1.4 \cdot 10^4 \times [P1] \text{ s}^{-1}$ (see Fig. 2.32). At low concentration, the contribution of the ^{13}C becomes dominant as evidenced by the saturation.

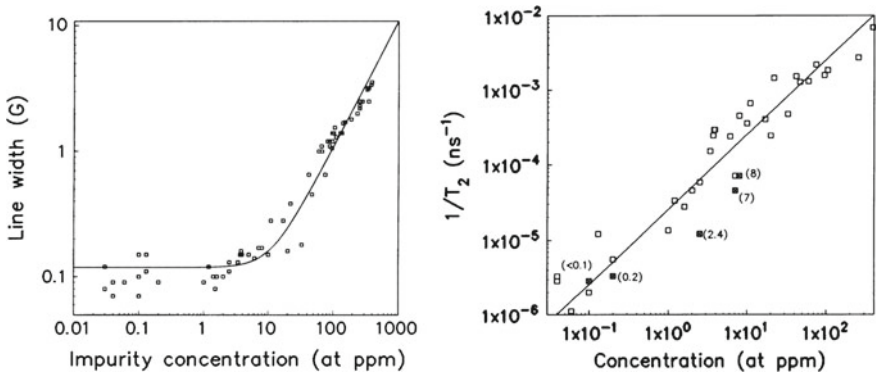


Fig. 2.32 Dependence of the coherence times on the P1 concentration [41]. Dependence of the linewidth $\gamma = 1/T_2^*$ (left) and spin-spin relaxation rate $1/T_2$ (right)

2.3 Coupling Ensembles of NV Center Spins to Superconducting Circuits

In our experiments, a superconducting resonator is used to mediate the interaction between the NV center spins and the rest of the circuit. In the following, we describe the spins-resonator system. We first evaluate the interaction strength of a single NV center spin to a superconducting resonator for typical cQED parameter. We will see that this coupling strength is too weak to allow for coherent exchange of quantum information, but instead an ensemble of such spin benefit from a collective enhancement. We will consider our experimental conditions in which the spin ensemble has both static distributions of spin-resonance frequency and coupling strength to the resonator and treat the dynamics of the spins-resonator system.

2.3.1 Single Spin-Resonator Coupling

Strictly speaking, NV centers are spin-1 systems so that it is not possible to write the interaction of the electronic spin with a resonator field on the Jaynes Cummings form of Sect. 2.1.3.1. However by applying a static bias on the spins, one can operate on one of these transitions only: the NV center is reduced to a two-level system with ground state $|g\rangle = |m_S = 0\rangle$, excited state $|e\rangle = |\pm\rangle$ and transition frequency $\omega_s = \omega_{\pm}$. The Hamiltonian of the single spin-resonator system can be written:

$$\hat{H} = \hat{H}_r + \hat{H}_a + \hat{H}_{int} \quad (2.70)$$

with $\hat{H}_r$ the free-field Hamiltonian given by Eq. 2.2, $\hat{H}_a$ the Hamiltonian of the NV center spin given by Eq. 2.68 and a coupling term function of $\mathbf{S}$ the magnetic dipole of the NV and $\mathbf{B}$ the magnetic field sustained by the resonator:

$$\hat{H}_{int}/\hbar = -\gamma_e \mathbf{S} \cdot \mathbf{B} \quad (2.71)$$

$$= -\frac{\gamma_e}{\sqrt{2}} [\hat{\sigma}_x \delta B_x(\mathbf{r}) + \hat{\sigma}_y \delta B_y(\mathbf{r})] (\hat{a} + \hat{a}^\dagger) \quad (2.72)$$

$$= -\frac{\gamma_e}{\sqrt{2}} [\hat{a} \hat{\sigma}_+ \{\delta B_x(\mathbf{r}) - i \delta B_y(\mathbf{r})\} + \hat{a}^\dagger \hat{\sigma}_- \{\delta B_x(\mathbf{r}) + i \delta B_y(\mathbf{r})\}] \quad (2.73)$$

$$= g^* \hat{a} \hat{\sigma}_+ + g \hat{a}^\dagger \hat{\sigma}_- \quad (2.74)$$

with the (complex) spin-resonator coupling constant defined as

$$g = -\frac{\gamma_e [\delta B_x(\mathbf{r}) + i \delta B_y(\mathbf{r})]}{\sqrt{2}} \quad (2.75)$$

The modulus of this coupling constant can be estimated numerically for typical parameters of resonators in circuits, using the analytical expression for the magnetic field generated by a CPW waveguide Eq. 2.24. For a 50Ω resonator on Silicon with

geometrical parameters $S, W = 10, 5 \mu\text{m}$ and resonance frequency $\omega_r/(2\pi) = 2.88 \text{ GHz}$, the magnetic field generated at the surface is $\sim 450 \text{ pT}$ yielding a single-spin-resonator coupling constant

$$g \sim 2\pi \cdot 10 \text{ Hz}. \quad (2.76)$$

This value is four orders of magnitude smaller than resonator linewidth κ reachable in circuit QED. The coupling strength of an individual NV center spins to one electromagnetic mode is thus too weak to allow for strong coupling ($g \gg \kappa, \gamma$) and subsequent coherent exchange of quantum information. This issue is overcome by using large ensembles of spins, as explained in the following.

2.3.2 Spin Ensemble-Resonator Coupling: Collective Effects

New effects appear when an ensemble of N spins is collectively coupled to a resonator. The goal of this section is to give the elements of theory relevant for understanding our quantum memory protocol and experiments, and to derive a number of formulas for the analysis.

2.3.2.1 The Tavis-Cummings Model

The Tavis-Cummings model [42] considers N identical spins-1/2 with frequency ω_s coupled with the same coupling constant g to a single cavity mode at frequency ω_r (see Fig. 2.33). The Hamiltonian writes:

$$\hat{H}_{TC}/\hbar = \omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{\omega_s}{2} \sum_{j=1}^N \hat{\sigma}_z^{(j)} + g \sum_{j=1}^N \left(\hat{a} \hat{\sigma}_+^{(j)} + \hat{a}^\dagger \hat{\sigma}_-^{(j)} \right), \quad (2.77)$$

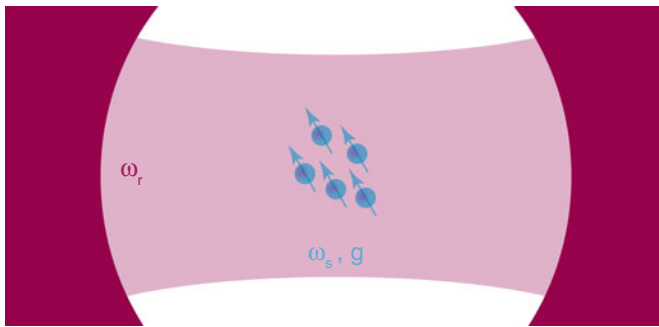


Fig. 2.33 The Tavis-Cummings model. N identical two-level systems of frequency ω_s are identically coupled to a single cavity mode with coupling strength g

with $\hat{\sigma}_{z,\pm}^{(j)}$ the Pauli spin operators of spin j . The internal states of the ensemble can be written in the 2^N -dimensional basis spanned by the states $\Pi_{j=1,\dots,N}|i\rangle_j$, with $i = g, e$. As a useful notation, we refer in the following to the collective ground state $|G\rangle \equiv |g_1 \dots g_N\rangle$, and the state with only spin j excited $|E_j\rangle \equiv |g_1 \dots e_j \dots g_N\rangle$. Introducing the collective spin operators $\hat{S}_{x,y,z} = \sum_{j=1}^N \hat{\sigma}_{x,y,z}^{(j)}/2$ and $\hat{S}_{\pm} = \sum_{j=1}^N \hat{\sigma}_{\pm}^{(j)}$, $\hat{H}_{TC}$ can be rewritten as:

$$\hat{H}_{TC}/\hbar = \omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \omega_s \hat{S}_z + g \left(\hat{a} \hat{S}_+ + \hat{a}^\dagger \hat{S}_- \right), \quad (2.78)$$

called the Tavis-Cummings Hamiltonian.

THE COLLECTIVE BASIS

One key property of this Hamiltonian is that it commutes with the total spin $\hat{\mathbf{S}}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$, implying that its eigenvalue³ $\mathcal{S}(\mathcal{S}+1)$ is a good quantum number whose value is constant in time. The system dynamics is thus restrained to subspaces with fixed $\mathcal{S}$, and it is therefore interesting to describe it in the basis of the simultaneous eigenstates of $\hat{\mathbf{S}}^2$ and $\hat{S}_z$:

$$\hat{\mathbf{S}}^2 |\mathcal{S}, m\rangle = \mathcal{S}(\mathcal{S}+1) |\mathcal{S}, m\rangle \quad (2.79)$$

$$\hat{S}_z |\mathcal{S}, m\rangle = m |\mathcal{S}, m\rangle \quad (2.80)$$

where m can take any of the $(2\mathcal{S}+1)$ values $-\mathcal{S}, -\mathcal{S}+1, \dots, \mathcal{S}-1, \mathcal{S}$. Note that these states are highly degenerated since there are only $N(N+3)/2$ possible values for the couple $(\mathcal{S}, m)$ whereas there are 2^N possible spin states. The degeneracy of the states $|\mathcal{S}, m\rangle$ is given by $\frac{N!(2\mathcal{S}+1)}{(N/2+\mathcal{S}+1)!(N/2-\mathcal{S})!}$ [43], implying that states with low values of $\mathcal{S}$ are much more numerous than states with large $\mathcal{S}$. At the extreme, states with $\mathcal{S} = N/2$ are non-degenerate.

Another relevant quantity is the degeneracy of states having a well-defined value of m , which is simply given by $\frac{N!}{(N/2+m)!(N/2-m)!}$, the number of different ways to flip $N/2+m$ spins among N . One sees that there is only one ground state corresponding to $|N/2, -N/2\rangle$, N states with a single excitation $m = -N/2 + 1$, among which 1 state $|N/2, -N/2 + 1\rangle$ and $(N-1)$ of the form $|N/2 - 1, -N/2 + 1\rangle$, $N(N-1)/2$ states with 2 excitations ($m = -N/2 + 2$), among which 1 with $\mathcal{S} = N/2$, $(N-1)$ with $\mathcal{S} = N/2 - 1$, and $N(N-3)/2$ with $\mathcal{S} = N/2 - 2, \dots$. A schematic description of the collective states summarizing their degeneracy is in Fig. 2.34.

³ $\mathcal{S}$ takes any integer or half-integer value between 0 and $N/2$.

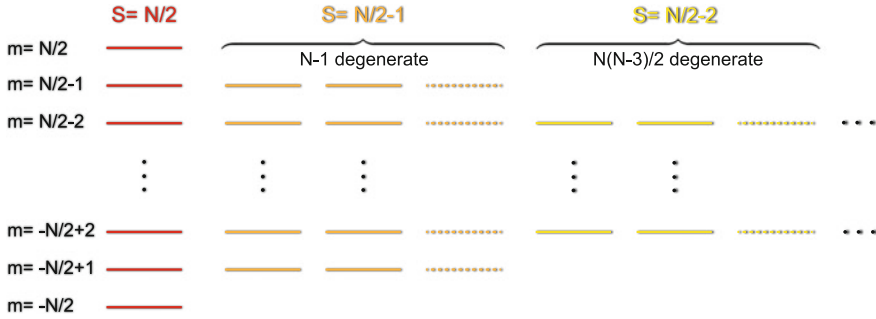


Fig. 2.34 Energy level diagram of an ensemble of N spins- $1/2$. States $S \neq N/2$ are highly degenerate. There is one ground state corresponding to $|N/2, -N/2\rangle$, N states with a single excitation $m = -N/2 + 1$, among which 1 state $|N/2, -N/2 + 1\rangle$ and $(N - 1)$ of the form $|N/2 - 1, -N/2 + 1\rangle$

COLLECTIVE ENHANCEMENT OF THE COUPLING

Of particular interest is the situation where the initial state is the collective ground $|G\rangle$, corresponding to $m = -N/2$ and hence $S = N/2$. Since the Tavis-Cummings Hamiltonian preserves S and since the states $|N/2, m\rangle$ are non-degenerate, the system dynamics is restricted to the $(N + 1)$ -dimensional manifold of perfectly symmetric states for which $S = N/2$, instead of the full 2^N -dimensional Hilbert space. Putting one excitation in the spin system can be done by absorbing a photon from the cavity, which excites the state $|B\rangle = |N/2, -N/2 + 1\rangle \equiv S_+|G\rangle/|S_+|G\rangle| = \sum_k |E_k\rangle/\sqrt{N}$. The $N - 1$ other states with one excitation in the spin ensemble (i.e. with $m = -N/2 + 1$) can be written as $|D_j\rangle = \sum_{k=0}^{N-1} \exp^{ijk2\pi/N} |E_k\rangle/\sqrt{N}$ (with $j = 1, \dots, N - 1$). It is straightforward to see that $\langle D_j | B \rangle = 0$, which implies that all $|D_j\rangle$ states are of the form $|N/2 - 1, -N/2 + 1\rangle$. Since the $|D_j\rangle$ states are states with $S = N/2 - 1$, they cannot be coupled to $|G\rangle$ by the Tavis-Cummings Hamiltonian, and one gets

$$\langle E, 0 | \hat{H} | G, 1 \rangle = (1/\sqrt{N}) \sum_i g = g\sqrt{N} \quad (2.81)$$

$$\langle D_j, 0 | \hat{H} | G, 1 \rangle = 0. \quad (2.82)$$

By describing the spin-cavity coupling in the collective basis, we thus come to the important conclusion that (for single-excitation states) only one collective state (*bright*) $|B\rangle$ is coupled to the cavity mode with a strength enhanced by a factor $\sqrt{N}$ compared to the single spin case, whereas $(N - 1)$ collective spin states (*dark*) $|D_j\rangle$ are decoupled from the radiation field.

LOW- EXCITATION APPROXIMATION

In the limit of small excitation numbers, i.e. where $m + N/2 \ll N$, a useful approximation (called the Holstein-Primakoff approximation) is possible [44–46]. It consists

in replacing each spin operator $\hat{\sigma}_{\pm}^{(j)}$ by bosonic operators $\hat{s}_j, \hat{s}_j^{\dagger}$ that verify $[\hat{s}_j, \hat{s}_j^{\dagger}] = 1$ using the following rules

$$\hat{\sigma}_{-}^{(j)} \rightarrow \hat{s}_j \quad (2.83)$$

$$\hat{\sigma}_{+}^{(j)} \rightarrow \hat{s}_j^{\dagger} \quad (2.84)$$

$$\hat{\sigma}_z^{(j)} \rightarrow -1 + 2\hat{s}_j^{\dagger}\hat{s}_j. \quad (2.85)$$

In doing so, one neglects all effects linked to the saturation of a spin since a harmonic oscillator can have an arbitrary number of excitations contrary to a spin. This approximation is thus only valid when the system dynamics is restricted to states for which $\langle \hat{s}_j^{\dagger}\hat{s}_j \rangle \ll 1$ or equivalently $\langle \hat{\sigma}_z^{(j)} \rangle + 1 \ll 1$. In our situation where the spin ensemble can only be collectively excited and thus where a large number of spins have the same average excitation, this is indeed equivalent to the condition stated at the beginning of this section that the total number of excitation of the ensemble verifies $m + N/2 \ll N$.

In the same way as previously, it is useful to describe the spins in the collective basis in the Holstein-Primakoff approximation, by defining the collective bosonic operators

$$\hat{b} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{s}_k \quad (2.86)$$

$$\hat{d}_j = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \exp^{ijk2\pi/N} \hat{s}_k \quad (2.87)$$

and their conjugate. It is then straightforward to show that the Tavis-Cummings Hamiltonian can be rewritten as

$$\hat{H}_{HP}/\hbar = \omega_r \left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2} \right) + \omega_s (\hat{b}^{\dagger}\hat{b} + \sum_{j=1}^{N-1} \hat{d}_j^{\dagger}\hat{d}_j) + g_{ens} (\hat{a}^{\dagger}\hat{b} + \hat{a}\hat{b}^{\dagger}). \quad (2.88)$$

In the Holstein-Primakoff approximation, the coupling of N spins to a cavity with constant g can thus be described as a collection of N harmonic oscillators: one *bright mode* b coupled to the cavity mode with an enhanced coupling constant

$$g_{ens} = g\sqrt{N}, \quad (2.89)$$

and $N - 1$ *dark modes* d_j (with $j = 1, \dots, N - 1$) completely decoupled from the cavity field. From the ground state $|G\rangle \equiv |0, \dots, 0\rangle$, one can generate N different single-excitation states by applying one of the collective creation operators $\hat{b}^{\dagger}$ or $\hat{d}_j^{\dagger}$, and then $N(N + 1)/2$ two-excitation states by applying a second time one of these operators, and so forth. The single-excitation manifold is strictly identical to the

$m = -N/2 + 1$ collective spin states described earlier. The two-excitation manifold contains an excess of N states in the Holstein-Primakoff description (the states with 2 excitations in the same mode and 0 in the others), but for N large this is a small error compared to the much larger number of states with one excitation in two different modes, which are identical to the $m = -N/2 + 2$ collective spin states.

At this point it is interesting to make a side remark regarding thermal equilibrium of the spin ensemble at finite temperature T . In the limit where $k_B T \ll \hbar\omega_s$ the mean number of excited spins at equilibrium is approximately given by $N \exp(-\hbar\omega_s/k_B T) \ll N$, so that the Holstein-Primakoff approximation is valid. According to the previous discussion, the thermal state of the spin ensemble is thus well described by the thermal state of a collection of N independent harmonic oscillators. Even though the total number of excited spins $N \exp(-\hbar\omega_s/k_B T)$ can be very large, we thus come to the conclusion that each of the collective modes actually has a large probability $(1 - \exp(-\hbar\omega_s/k_B T))$ to be found in its ground state $|0\rangle$. This means that in order for our quantum memory to be initialized, the requirement is simply that the mean excitation of the collective bright mode should be small ($\exp(-\hbar\omega_s/k_B T) \ll 1$), and not that the total number of excited spins should be small ($N \exp(-\hbar\omega_s/k_B T) \ll 1$), which would be an impossible condition to fulfill in any realistic experiment.

Note however that if the memory uses n modes for storage, the requirement that each of these modes be initialized in its ground state becomes more stringent ($n \exp(-\hbar\omega_s/k_B T) \ll 1$), but for realistic values of $n = 10^2$ – 10^3 , this still seems achievable experimentally.

EXCITATION SPECTRUM

The Hamiltonian $\hat{H}_{HP}$ can be very simply diagonalized by introducing two new operators which are linear combinations of the bright mode and the cavity field, called polaritons, and defined as

$$\hat{p}_+ = \cos \theta \hat{a} + \sin \theta \hat{b} \quad (2.90)$$

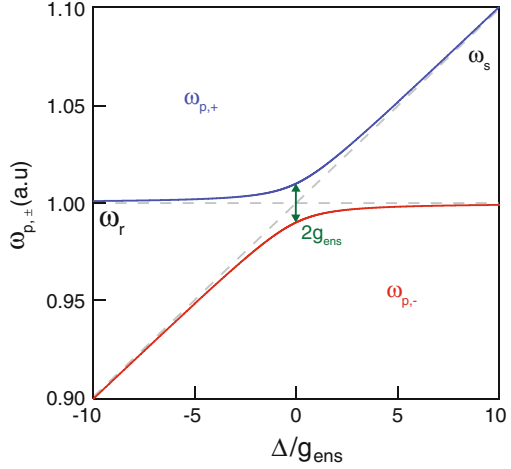
$$\hat{p}_- = -\sin \theta \hat{a} + \cos \theta \hat{b}, \quad (2.91)$$

with $\tan(2\theta) = -2g_{\text{ens}}/\Delta$, $\Delta = \omega_s - \omega_r$ being the frequency difference between the cavity and the spins. These bosonic operators (since $[\hat{p}_+, \hat{p}_+^\dagger] = [\hat{p}_-, \hat{p}_-^\dagger] = 1$) describe coupled spin-photon excitations. Introducing $\omega_{p,\pm} = \omega_r \pm \frac{1}{2}\sqrt{4g_{\text{ens}}^2 + \Delta^2}$, the Hamiltonian can then be written as

$$\hat{H}_{HP}/\hbar = \omega_s \sum_{j=1}^{N-1} \hat{d}_j^\dagger \hat{d}_j + \omega_{p,+} \hat{p}_+^\dagger \hat{p}_+ + \omega_{p,-} \hat{p}_-^\dagger \hat{p}_-. \quad (2.92)$$

which shows that the Hamiltonian eigenstates are tensor products of Fock states of the dark modes (at frequency ω_s) and the polaritonic modes (at frequencies $\omega_{p,\pm}$).

Fig. 2.35 Polaritonic modes. The system excitation spectrum consists of only the two polaritonic modes



Probing the system with a microwave tone coupled to the cavity input, the system excitation spectrum consists of only the two polaritonic modes since the dark modes are decoupled to the cavity and thus invisible. Sweeping the spin frequency across the cavity, one should thus observe an avoided level crossing (see Fig. 2.35), with minimal peak separation of $2g_{ens}$ obtained at resonance ($\Delta = 0$), where $\hat{p}_+ = (\hat{a} + \hat{b})/\sqrt{2}$ and $\hat{p}_- = (\hat{a} - \hat{b})/\sqrt{2}$. The experimental observation of such an anticrossing in the transmission spectrum of a resonator coupled to an ensemble of NV centers is reported in Chap. 4 and is the starting point of this thesis work.

DYNAMICS

Coming back to the Hamiltonian Eq. 2.88, we see that the interaction between the bright mode and the cavity is of the beam-splitter type $\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger$. It is well-known [47] that suddenly switching on resonantly this type of interaction for a well-chosen duration produces a dynamics corresponding to a SWAP operation, exchanging the two quantum states of the cavity and the bright mode. For instance, if the cavity is prepared at $t = 0$ in a single-photon Fock state $|1\rangle$ and the bright mode in its ground state $|0_b\rangle$, the coupled system quantum state $|\psi(t)\rangle$ will evolve at $t > 0$ as

$$|\psi(t)\rangle = \cos(g_{ens}t)|1, 0_b\rangle + \sin(g_{ens}t)|0, 1_b\rangle \quad (2.93)$$

which shows that the single-photon state will indeed be transferred from the resonator into the spin ensemble after an interaction time $\pi/(2g_{ens})$. This vacuum Rabi oscillation between the resonator and the bright mode of the spin ensemble is the basis for the *write* experiment that will be related in Chap. 4.

SUPERRADIANCE

Besides the collective enhancement of the coupling of the bright mode to the cavity field, another interesting phenomenon arises from the Tavis-Cummings Hamiltonian when the initial state is the fully excited state $|e_1 \dots e_N\rangle$. This state is also symmetrical and can be identified as $|N/2, N/2\rangle$ in the collective spin basis. Being the state of maximal energy it eventually has to relax by spontaneous emission of photons into the electromagnetic mode defined by the cavity. The Tavis-Cummings Hamiltonian shows however that its relaxation should still take place in the basis of fully symmetrical states $S = N/2$, going down the energy ladder from $|N/2, N/2\rangle$ until it reaches the ground state $|N/2, -N/2\rangle$, via states $|N/2, m \approx 0\rangle$, by steps of $\Delta m = -1$ corresponding to successive photon emissions.

Defining the photon spontaneous emission rate of an isolated spin as γ_1 , the spin ensemble collective radiation rate (assuming that the cavity is rapidly damped so that the intra-cavity photon number stays ≈ 0) is given by $\gamma_1 |\langle m-1, N/2 | S_- | m, N/2 \rangle|^2$, which was shown by Dicke to be equal to $I_0(N/2+m)(N/2-m+1)$. For $m = N/2$, this rate is $N\gamma_1$ as would be the case for independent emitters, but this quantity rapidly increases with decreasing m , reaching a maximum of $(N/2)^2\gamma_1$ when $m = 0$, after which it goes down again to $N\gamma_1$ when the system approaches its ground state. This implies that the photon emission of a collection of symmetrically excited spins does not take place as a simple exponential, as would be the case if each spin would radiate individually, but instead consists of a coherent pulse of light, which takes place in an overall much shorter time given by γ_1^{-1}/N and with a much stronger peak intensity of $N^2\gamma_1$.

This enhanced collective spontaneous emission is called superradiance and has been widely studied both theoretically and experimentally. In our context superradiance is relevant because it complicates the *read step* of our memory protocol, since as explained in Chap. 3 this step involves the application of a π pulse to the spins which precisely excites them into $|e_1 \dots e_N\rangle$. The desexcitation of this state by emission of a superradiant pulse would be disastrous for the fidelity of the memory, and is prevented in the protocol described in Chap. 3.

2.3.2.2 The Realistic Model

In our experiments, several hypotheses of the Tavis-Cummings model are not satisfied (Fig. 2.36):

1. The spin-resonator coupling is far from being the same for each spin. Indeed, in our experiments (see Chaps. 4 and 5), the resonator is implemented by a superconducting planar circuit, on top of which the diamond crystal is fixed, and the magnetic field it generates (as well of course as its quantum fluctuations) is spatially inhomogeneous. This results in a position-dependent spin-resonator coupling constant $g_i = g(\mathbf{r}_i)$, which now becomes a complex number since the field direction may vary between different spins. The distribution $\rho(g) = \sum_i \delta(g - |g_i|)$ characterizes the amplitude variations of the coupling constant in the continuous limit.

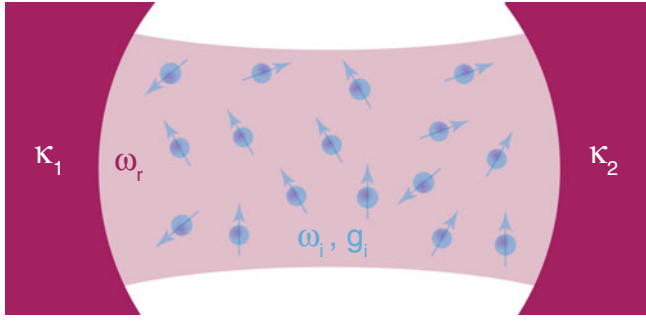


Fig. 2.36 The realistic model. The cavity is coupled to external transmission lines through coupling capacitances and has a finite damping rate $\kappa = \kappa_1 + \kappa_2$. The spin ensemble has both static distributions of spin-resonance frequency $\rho(\omega)$ and coupling strength $\rho(g)$

2. The NV centers throughout the ensemble have slightly different frequencies due to their different local magnetic environments (see Sect. 2.2.3). This inhomogeneous broadening is described by a (static) distribution of frequency

$$\rho(\omega) = \sum_i |g_i|^2 \delta(\omega - \omega_i) \quad (2.94)$$

around the central frequency ω_s .

3. The cavity has a finite damping rate κ due to coupling to the line and internal losses, implying that the quantum states inside the resonator have a finite lifetime.

COUPLING CONSTANT INHOMOGENEITY

The coupling constant inhomogeneity would not alter any of the conclusions reached in the previous section, provided all spins have the same frequency ω_s . Indeed one can redefine the bright mode in the Holstein-Primakoff approximation as $\hat{b} = 1/g_{ens} \sum_{k=1}^N g_k \hat{s}_k$, with

$$g_{ens} = \left(\sum_{j=1}^N |g_j|^2 \right)^{1/2} = \left(\int g^2 \rho(g) \cdot dg \right)^{1/2} \quad (2.95)$$

the collective coupling constant, as well as $N - 1$ orthogonal dark modes d_j ($j = 1, \dots, N - 1$) (without giving their explicit expression, which would be more tedious than in the previous section). It is then straightforward to see that the Tavis-Cummings Hamiltonian can be rewritten exactly in the form of Eq. 2.88, implying that all the previous conclusions about collective effects would be completely unchanged, apart from a re-definition of the ensemble coupling constant provided by Eq. 2.95. In the remaining of this work we will use these new definitions of the bright mode b , and of g_{ens} . Note that with these definitions of $\rho(\omega)$ and g_{ens} , one finds that $\int \rho(\omega) d\omega = g_{ens}^2$.

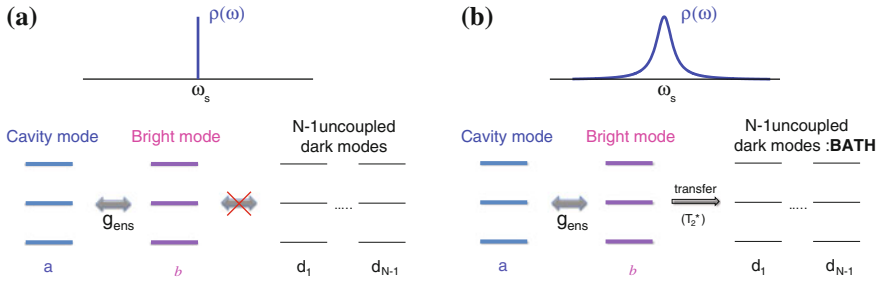


Fig. 2.37 Transfer from the bright mode to the dark mode subspace enabled by inhomogeneous broadening. In the homogeneous case (a), the cavity and bright mode constitutes a closed system. The bright mode is coupled to the bath of dark modes in case of inhomogeneous broadening (b), which make it possible the transfer to the multimode dark mode subspace

INHOMOGENEOUS BROADENING

The inhomogeneity in resonance frequency of the NV centers has more profound consequences. This can be seen qualitatively by noting that if the ensemble is prepared in the state $|1_b\rangle \equiv \hat{b}^\dagger|0_b\rangle = (\sum_{k=1}^N g_k|E_k\rangle)/g_{\text{ens}}$ at $t = 0$, it will evolve at later time into $(\sum_{k=1}^N g_k e^{-i(\omega_k - \omega_s)t}|E_k\rangle)/g_{\text{ens}}$. The overlap of this state with $|1_b\rangle$ varies in time as the Fourier transform of $\rho(\omega - \omega_s)$. It therefore decays in a time $T_2^* = 2/\Gamma$, Γ being the characteristic width of the $\rho(\omega)$ distribution (see below for a more quantitative definition).

At times $t \gg T_2^*$, the system is thus in a state with $m = -N/2 + 1$ but orthogonal to $|1_b\rangle$, which therefore belongs to the sub-space spanned by the $N - 1$ dark states $|1_{d_j}\rangle$. This brings us to the important conclusion that in the presence of inhomogeneous broadening, the cavity and bright mode do not constitute a closed system any more because the bright mode is now coupled to the large ensemble of dark modes (Fig. 2.37), which act as a bath because of their extremely large number (of order 10^{11} in our experiments). As we will see in Chap. 3, this quantum state transfer from the bright mode into the dark mode subspace is a key resource of our quantum memory protocol because it can be reversed with the appropriate pulse sequence.

2.3.2.3 Coupling Regimes

We now address the issue of whether the coherent collective effects discussed in the idealized model (vacuum Rabi oscillations, . . .) pertain in the realistic model. Somewhat qualitatively, one can expect two different situations, depending on the relative values of g_{ens} , κ , Γ :

- If $g_{\text{ens}} \gg \kappa, \Gamma$ (*strong coupling regime*), one can expect that the dynamics will be dominated by the collective radiative effects seen previously. The low-energy excitation spectrum should display a visible avoided level crossing [45, 46]; it

should be possible to coherently transfer a quantum state between the resonator and the bright mode of the spin ensemble (even though this state will eventually leak out into the dark modes); and if all the spins are inverted, they should emit a superradiant pulse [48]. This regime is called the *strong coupling regime*. Being able to reach it is a key requirement in some steps of our quantum memory protocol. It requires samples with a large number of spins, but with narrow linewidth.

- If $g_{ens} \ll \kappa, \Gamma$ (*weak coupling regime*), no anticrossing is visible; no coherent state transfer is possible; and as discussed in [48], superradiance is suppressed.

The cross-over between weak and strong coupling can also be quantified using a dimensionless number, the so-called cooperativity defined as

$$C = \frac{2g_{ens}^2}{\kappa\Gamma}. \quad (2.96)$$

If $C \gg 1$, the system is in the strong coupling regime, and in the weak coupling regime if $C \ll 1$.

THE KEY PARAMETERS

The ensemble coupling constant g_{ens} The ensemble coupling constant g_{ens} is a central quantity to describe the spin-resonator coupling. We demonstrate here a formula that provides a simple way to estimate g_{ens} as a function of basic experimental parameters, and which will be used extensively throughout this thesis. We have seen in Sect. 2.3.1 that a single NV center spin located at position $\mathbf{r}$ is coupled to the rms vacuum magnetic field fluctuations of the resonator $\delta\mathbf{B}_0(\mathbf{r})$ with a coupling constant of modulus $|g(\mathbf{r})| = (|\gamma_e|/\sqrt{2})|\delta\mathbf{B}_0(\mathbf{r})| \sin\theta(\mathbf{r})$, $\theta(\mathbf{r})$ being the angle between $\delta\mathbf{B}_0(\mathbf{r})$ and the NV axis. To proceed, we note that the energy E_n of a n -photon Fock state in the cavity is equal to $(n + 1/2)\hbar\omega_r$, but can also be written as $E_n = \frac{2}{2\mu_0} \int d\mathbf{r} \langle n | \hat{B}(\mathbf{r})^2 | n \rangle = \frac{2n+1}{\mu_0} \int d\mathbf{r} |\delta B_0(\mathbf{r})|^2$, so that

$$\int d\mathbf{r} |\delta B_0(\mathbf{r})|^2 = \frac{\mu_0 \hbar \omega_r}{2}. \quad (2.97)$$

For a sample of volume V with a homogeneous concentration of NV centers ρ_{NV} , this allows us to rewrite

$$g_{ens} = \frac{|\gamma_e|}{\sqrt{2}} [\rho_{NV} \int_V d\mathbf{r} |\delta B_0(\mathbf{r}) \sin\theta(\mathbf{r})|^2]^{1/2} = -\frac{|\gamma_e|}{\sqrt{2}} \sqrt{\frac{\rho_{NV} \mu_0 \hbar \omega_r}{2}} \sqrt{\alpha\eta}, \quad (2.98)$$

where η and α are dimensionless numerical factors defined as

$$\eta = \frac{\int_V d\mathbf{r} |\delta B_0(\mathbf{r})|^2}{\int d\mathbf{r} |\delta B_0(\mathbf{r})|^2} \quad (2.99)$$

and

$$\alpha = \frac{\int_V d\mathbf{r} |\delta B_0(\mathbf{r}) \sin \theta(\mathbf{r})|^2}{\int_V d\mathbf{r} |\delta B_0(\mathbf{r})|^2}. \quad (2.100)$$

The first parameter η is called the filling factor, and describes what fraction of the magnetic mode volume is occupied by the spins. In our experiments, the diamond is often glued on top of the resonator. If the resonator is of the coplanar waveguide type of total length L , with the diamond symmetrically covering a section of length l as in Chap. 4, it is straightforward to see that $\eta = \int_{(L-l)/2}^{(L+l)/2} dx \sin^2(\frac{\pi x}{L}) / L$. If the diamond covers a section of length l of a lumped-element inductance of length L , then $\eta = l/(2L)$. The second parameter, α , should be evaluated numerically. It is of order 1 and accounts for the fact that the resonator magnetic field is not necessarily transverse to the NV axis. Overall, we get

$$g_{ens} = |\gamma_e| \sqrt{\frac{\mu_0 \hbar \omega_r \rho_{NV} \alpha \eta}{4}}. \quad (2.101)$$

It is interesting to note that all the geometric factors are included in the dimensionless parameters η and α . This can be intuitively understood by the fact that reducing the transverse dimensions of a CPW resonator by a factor β would enhance the vacuum fluctuations of the magnetic field by the same factor β , thus increasing the single-NV-resonator coupling constant also by β , but it would also reduce by a factor β^2 the total number of spins N , which exactly compensates the previous gain since g_{ens} scales like $\sqrt{N}$. Provided the filling factor is maximized, the only way to increase further g_{ens} is thus to increase the sample concentration ρ_{NV} .

The characteristic width Γ The definition of Γ , which was introduced as the characteristic width of $\rho(\omega)$, can be precised. If the spin distribution is well-behaved (a single Lorentzian or Gaussian peak), its definition is straightforward as the peak width. But if the ensemble contains spins with different frequencies, due to either hyperfine coupling with a nucleus, or to a different Zeeman shift, $\rho(\omega)$ then may consist of a sum of peaks with possibly widely different center frequencies. Note that in the definition above, g_{ens} includes a sum over all the spins regardless of their resonance frequency, with a contribution from spins that are completely off-resonance with the cavity. This issue is solved by using the following definition [48] in the above criteria for strong / weak coupling:

$$\Gamma^{-1} \equiv \frac{1}{g_{ens}^2} \int_{-\infty}^{+\infty} \frac{\rho(\omega) d\omega}{\gamma/2 + i(\omega - \omega_s)}, \quad (2.102)$$

with $\omega_s = \int_{-\infty}^{+\infty} \rho(\omega) \omega d\omega / g_{ens}^2$ being the average spin frequency. That definition makes the criteria above well-defined regardless of the detailed shape of $\rho(\omega)$.

In the following, we illustrate these qualitative statements by explicitly computing several physical quantities useful in our experiments for given spin and coupling

constant densities $\rho(\omega)$ and $\rho(g)$, and show how they evolve between the weak and strong coupling regimes.

2.3.3 The Resonator-Spins System in the Low-Excitation Regime

In this section we restrict ourselves to calculating quantities in the weak excitation regime, where the Holstein-Primakoff approximation holds. We first derive analytical expressions for the system spectrum, as probed by measuring its transmission or reflection coefficient (see Fig. 2.38). We then compute the system dynamics with the cavity initialized in a single-photon Fock state, a situation implemented in the experiments described in Chap. 4. Here we rely on the work performed by our collaborators at Institut Néel I. Diniz and A. Auffèves described in detail in [46], and the closely related work performed at Aarhus University [45].

2.3.3.1 Master Equation

Our starting point is the cavity—spin ensemble Hamiltonian in the Holstein-Primakoff approximation, taking into account the inhomogeneity of spin frequency, and written in the individual spin basis using the bosonic operators $\hat{s}_j$:

$$\hat{H}_{HP2}/\hbar = \omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \sum_{j=1}^N \omega_j \hat{s}_j^\dagger \hat{s}_j + \sum_{j=1}^N \left(g_j^* \hat{s}_j^\dagger a + g_j \hat{s}_j a^\dagger \right) + i\sqrt{\kappa_1}(\beta a^\dagger - \beta^* a), \quad (2.103)$$

where the last term describes the cavity drive through port 1, with $\beta(t) = \beta_0 e^{-i\omega t}$, $|\beta_0|^2$ being the number of photons per second at the cavity input. In order to apply input-output theory and to obtain tractable expressions, damping phenomena are described in the Markov approximation. For the cavity field this is well justified,

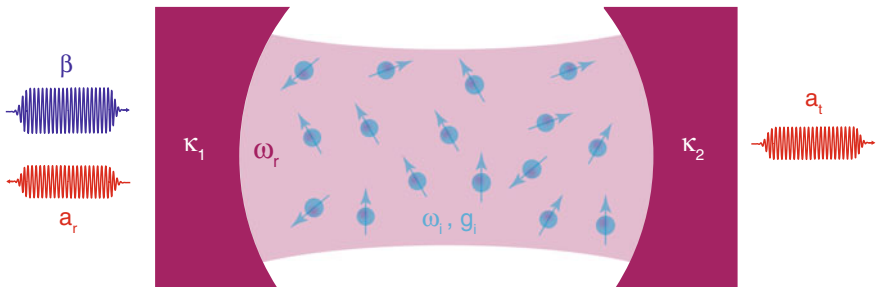


Fig. 2.38 Probing the resonator-spins system

as known from quantum optics. For the spins, the situation is more complex. Spins have negligibly low energy relaxation rates at cryogenic temperatures, as will be showed in the following chapters. Spin dephasing is in general non-Markovian, since as explained in the previous section it is caused by a spin bath which has a slow and complex dynamics. Nevertheless, since in this work we are mainly interested in treating quantitatively the static effects of inhomogeneous broadening, we will make a crude approximation and take as a transverse damping rate the Hahn-echo decoherence rate T_2^{-1} (which implies treating the corresponding bosonic operator as having its energy damped at a rate $\gamma = 2T_2^{-1}$). Note that in all the results of this section, this decoherence rate simply adds up to the width Γ of the spin frequency distribution $\rho(\omega)$; since $\gamma \ll \Gamma$ the exact modelling of decoherence is not very critical. These approximations yield the following Lindblad master equation for the system density matrix ρ

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{1}{i\hbar} [\hat{H}_{HP2}, \hat{\rho}] + \sum_k \Omega[\hat{c}_k] \hat{\rho} \quad (2.104)$$

where $\Omega[\hat{c}_k] \hat{\rho} \equiv -\frac{1}{2} \hat{c}_k^\dagger \hat{c}_k \hat{\rho} - \frac{1}{2} \hat{\rho} \hat{c}_k^\dagger \hat{c}_k + \hat{c}_k \hat{\rho} \hat{c}_k^\dagger$ describes each damping phenomenon by a jump operator $\hat{c}_k$. For cavity damping, $\hat{c}_1 = \sqrt{\kappa + \kappa_L} \hat{a}$, and for spin damping $\hat{c}_{2,j} = \sqrt{\gamma} \hat{s}_j$. In this way the following equations are obtained:

$$\partial_t \langle \hat{a} \rangle = -[(\kappa + \kappa_L)/2 + i\omega_r] \langle \hat{a} \rangle - i \sum_{j=1}^N g_j \langle \hat{s}_j \rangle + \sqrt{\kappa_1} \beta(t) \quad (2.105)$$

$$\partial_t \langle \hat{s}_j \rangle = -(\gamma/2 + i\omega_j) \langle \hat{s}_j \rangle - i g_j^* \langle \hat{a} \rangle \quad (2.106)$$

Looking for steady-state solutions of the form $\langle \hat{a} \rangle(t) = a_0 e^{-i\omega t}$ and $\langle \hat{s}_j \rangle(t) = s_{j0} e^{-i\omega t}$, we get

$$s_{j0} = -\frac{i g_j^*}{\gamma/2 + i(\omega_j - \omega)} a_0 \quad (2.107)$$

which yields

$$a_0 = \frac{i \sqrt{\kappa_1} \beta_0}{\omega - \omega_r + i \frac{\kappa + \kappa_L}{2} - K(\omega)}, \quad (2.108)$$

where we have introduced the function

$$K(\omega) \equiv \sum_j \frac{|g_j|^2}{\omega - \omega_j + i \frac{\gamma}{2}} = \int_{-\infty}^{+\infty} \frac{\rho(\omega') d\omega'}{\omega - \omega' + i \frac{\gamma}{2}}. \quad (2.109)$$

From the previous equations it appears that all the information about the spins is included in the function $K(\omega)$ which is therefore an important quantity on which we will give more details later. In an experiment, one measures the field either transmitted by the cavity or reflected on it. Using input-output theory as explained in the beginning of this chapter, one obtains the transmitted field as $a_t(\omega) = \sqrt{\kappa_2}a_0(\omega)$, so that the transmission coefficient $t(\omega) = a_t(\omega)/\beta_0$ writes

$$t(\omega) = \frac{i\sqrt{\kappa_1\kappa_2}}{\omega - \omega_r + i\frac{\kappa + \kappa_L}{2} - K(\omega)}. \quad (2.110)$$

In the same manner, the reflection coefficient $r(\omega) = \sqrt{\kappa_1}a_0(\omega)/\beta_0 - 1$ is found to be given by

$$r(\omega) = \frac{i\kappa_1}{\omega - \omega_r + i\frac{\kappa + \kappa_L}{2} - K(\omega)} - 1. \quad (2.111)$$

2.3.3.2 Measuring the Parameters of the System

The spin susceptibility The function $K(\omega)$ contains all the information that can be accessed by measuring the microwave transmission or reflection coefficients of the spin-resonator system. It should thus not be a surprise that this function is directly linked to the quantity measured in dc magnetic resonance, namely the spin susceptibility $\chi(\omega)$, defined as the ratio of the induced magnetization $M_x(t)$ and the applied microwave field $H_x(t)$. More precisely, for an applied field $H_x(t) = 2H_1 \cos(\omega t)$, the induced magnetization is

$$M_x(t) = 2H_1(\chi'(\omega) \cos(\omega t) + \chi''(\omega) \sin(\omega t))$$

with $\chi = \chi' - i\chi''$ [49]. This changes the resonator inductance L into $L(1 + 4\pi\eta\chi(\omega))$, η being the filling factor and χ the complex spin susceptibility in cgs units. The resonator frequency is therefore shifted by $-2\pi\eta\omega_r \text{Re}(\chi)$, and the extra field damping rate is $-2\pi\eta\omega_r \text{Im}(\chi)$. This yields the following direct link between $K(\omega)$ and $\chi(\omega)$:

$$\chi(\omega) = -K^*(\omega)/(2\pi\eta\omega_r). \quad (2.112)$$

Another important point is that in the limit where $\gamma \ll \Gamma$ (which is always satisfied in our systems), $K(\omega)$ is directly linked to the spin density function by the relation [45]

$$\rho(\omega) = -\frac{1}{\pi} \text{Im}[K(\omega)]. \quad (2.113)$$

The cooperativity Given the definition of Γ by Eq. 2.102, $\Gamma^{-1} = iK(\omega_s)/g_{ens}^2$. This implies that the cooperativity verifies

$$C = \frac{2g_{ens}^2}{\kappa\Gamma} = \frac{2iK(\omega_s)}{\kappa}. \quad (2.114)$$

Since on the other hand $K(\omega)$ is directly linked to the reflection and absorption coefficients at frequency ω , this implies that the cooperativity can be directly accessed by measuring $r, t(\omega_s)$. This relation is most useful in the case of reflection on a lossless ($\kappa_L = 0$) one-sided ($\kappa_2 = 0$) cavity at resonance with the spins ($\omega_s = \omega_r$), in which case:

$$r(\omega_s) = \frac{i\kappa_1}{i\frac{\kappa_1}{2} - K(\omega_s)} - 1 = \frac{1 - C}{1 + C}. \quad (2.115)$$

This relation shows in particular that at resonance, perfect absorption of an incoming microwave by the spins is achieved for the impedance matching condition $C = 1$, which determines the threshold between the weak and the strong coupling regimes. It also offers a convenient way of determining the system parameters in the weak coupling regime, since measuring the reflection coefficient at resonance yields the cooperativity which straightforwardly leads to the value of g_{ens} . In the strong coupling regime, this relation is still valid but inconvenient since $r(\omega) \approx -1$ as long as $C \gg 1$; in that limit g_{ens} is much more conveniently determined by the polaritonic peak separation.

2.3.3.3 Spectroscopy and Dynamics

SPECTROSCOPY

It is possible to obtain analytical expressions of $K(\omega)$ for typical distributions. For a Lorentzian $\rho(\omega) = g_{ens}^2 \frac{w/2\pi}{(\omega - \omega_s)^2 + (w/2)^2}$, one can show [45, 46] that

$$K(\omega) = \frac{g_{ens}^2}{(\omega - \omega_s) + i\frac{w+\gamma}{2}}. \quad (2.116)$$

Using the definition of the characteristic width Γ introduced earlier, one gets as expected $\Gamma = (w + \gamma)/2$. In the case of three hyperfine components (as for NV centers), the spin distribution consists of a sum of 3 Lorentzian peaks centered on frequencies $\omega_{s,j} = \omega_s + j\Delta_{hf}$ with $\Delta_{hf}/2\pi = 2.17\text{ MHz}$ and $j = -1, 0, +1$ yielding

$$\rho(\omega) = \sum_{j=-1,0,+1} \frac{g_{ens}^2}{3} \frac{w}{2\pi} \frac{1}{[\omega - (\omega_s + j\Delta_{hf})]^2 + (w/2)^2}. \quad (2.117)$$

The corresponding K function is

$$K(\omega) = \sum_{j=-1,0,+1} \frac{g_{ens}^2/3}{\omega - (\omega_s + j\Delta_{hf}) + i\frac{w+\gamma}{2}}, \quad (2.118)$$

with a characteristic width

$$\Gamma = \left(\frac{w+\gamma}{2}\right) \cdot \frac{(\frac{w+\gamma}{2})^2 + \Delta_{hf}^2}{(\frac{w+\gamma}{2})^2 + \frac{1}{3}\Delta_{hf}^2}. \quad (2.119)$$

Note that in the limit $w \ll \Delta_{hf}$, one finds $\Gamma \approx 3(w+\gamma)/2$. If the cavity is resonant with only one of the three peaks, the cooperativity would therefore be the same, should one consider the three peaks as making part of the ensemble, or only this resonant peak and disregard the two others (since the collective coupling would then be reduced by $\sqrt{3}$, but the width Γ by 3).

Figure 2.39 shows the single- and triple-Lorentzian $\text{Re}[K(\omega)]$ and $\text{Im}[K(\omega)]$, for $g_{ens}/2\pi = 1$ MHz. When increasing Γ , the three peaks corresponding to the hyperfine components disappear and merge in a single peak, a case sometimes encountered in our experiment.

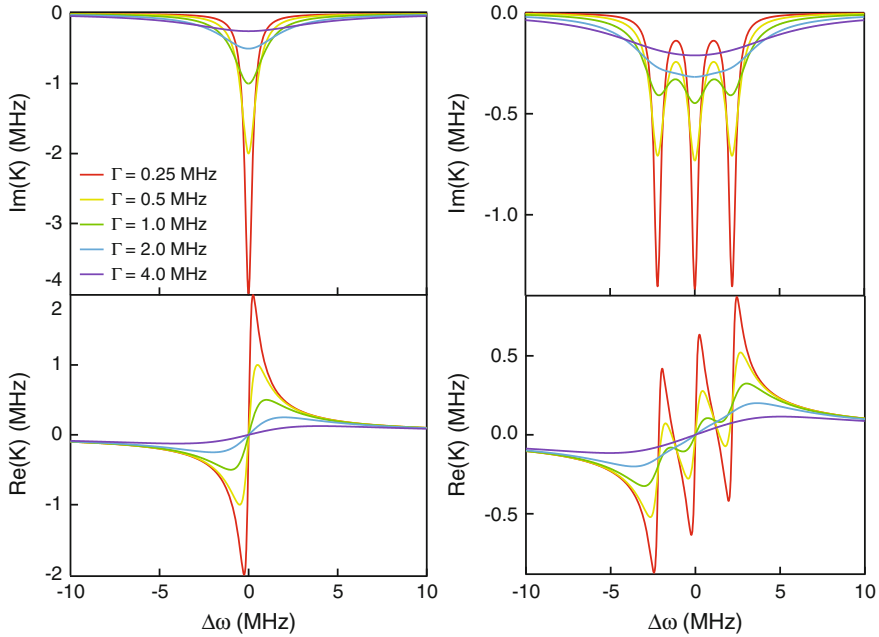


Fig. 2.39 The K-function. The imaginary and real part of the function $K(\omega)$ are displayed respectively on the *top* and on the *bottom* for a Lorentzian (*left*) and sum of three Lorentzian (*right*) distributions

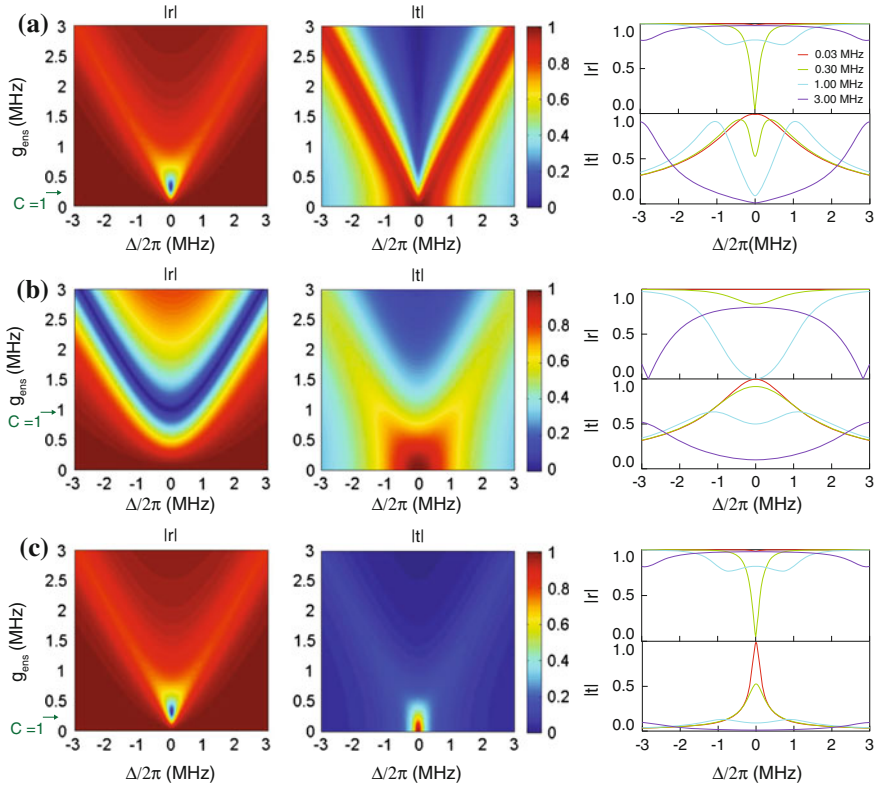


Fig. 2.40 Reflection and transmission of a resonator resonantly coupled to a single Lorentzian distribution of spins. The strong coupling is manifested by the appearance of two polaritonic signal separated by $2g_{\text{ens}}$ when $g_{\text{ens}} > \max(\kappa/2, \Gamma) = 2\pi \times 1$ MHz. In the *left panel*, the graphs for $g_{\text{ens}}/2\pi = 0.03$ MHz (red), 0.3 MHz (green), 1 MHz (light blue) and 3 MHz (blue)

We can now explicitly evaluate the expressions Eqs. 2.22 and 2.111 giving the resonator transmission and reflection coefficient. We show in Fig. 2.40 a two-dimensional plot of the modulus of the reflection and transmission coefficients of a resonator resonantly coupled ($\omega_r = \omega_s$) to a spin ensemble with a single Lorentzian distribution, as a function of the collective coupling strength g_{ens} .

The reflection coefficient (on the left) is computed for the condition of the experiment reported in Chap. 5 in which the resonator has only one port ($\kappa = \kappa_1$). The transmission coefficient (on the right) for the condition of the experiment reported in Chap. 4 in which the resonator has two identical ports ($\kappa_1 = \kappa_2 = \kappa/2$). In both cases we assume a lossless cavity $\kappa_L = 0$.

Starting from the decoupled cavity reflection / transmission coefficients ($g_{\text{ens}} = 0$), the first effect of the spins (for low values of g_{ens} , i.e. in the weak coupling regime) is to produce absorption dips. This is the situation commonly encountered in magnetic resonance, where in general $C \ll 1$ so that the spins only bring minor

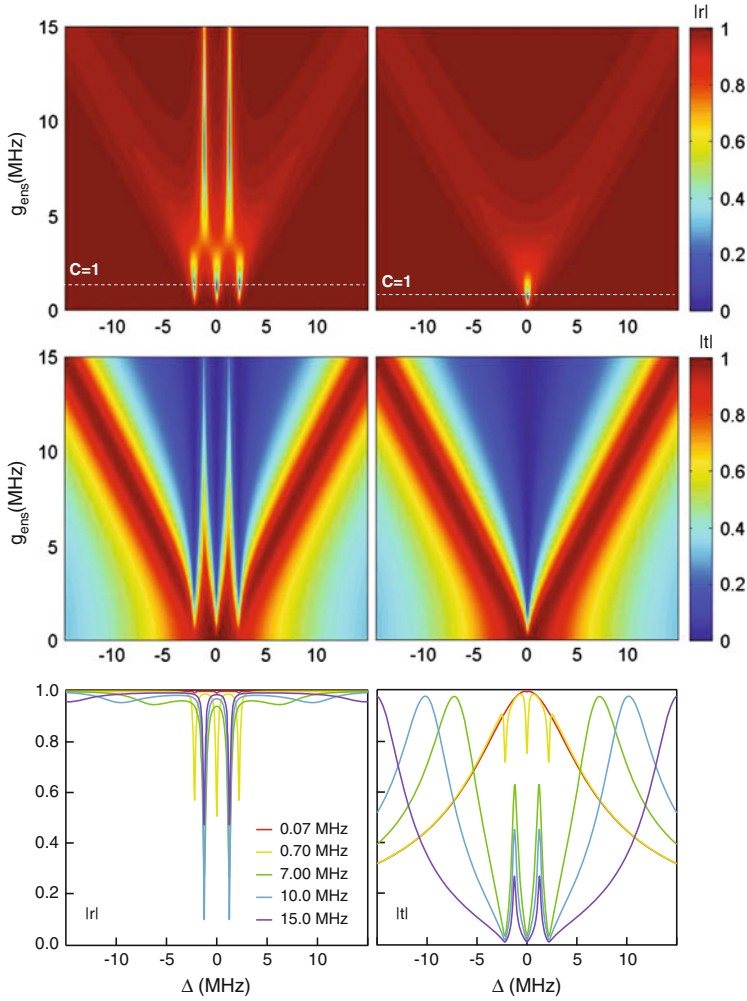


Fig. 2.41 Reflection and transmission of a resonator resonantly coupled to a distribution of NV center spins. Case $\Gamma/(2\pi) = 0.1$ MHz and $\kappa/(2\pi) = 10$ MHz. The same condition for the single Lorentzian distribution is shown on the *right* for comparison. In the *bottom panel*, the graphs for $g_{\text{ens}}/2\pi = 0.07$ MHz (red), 0.7 MHz (yellow), 7 MHz (green), 10 MHz (light blue) and 15 MHz

changes to the cavity frequency and quality factor. The depth of these absorption dips increases with g_{ens} , until it reaches perfect absorption on the reflection coefficient ($r(\omega_s) = 0$, or $t(\omega_s) = 1/2$ in transmission), which corresponds to the situation $C = 1$ as already explained. For larger values of g_{ens} , the two polaritonic peaks, separated by $2g_{\text{ens}}$, become visible, marking the strong coupling regime.

Figure 2.41 shows the reflection and transmission spectra for NV centers with $\kappa/(2\pi) = 10$ MHz and $\Gamma/(2\pi) = 0.1$ MHz. The same condition for a single-

Lorentzian distribution is plotted on the right for comparison. In the weak coupling regime, the hyperfine structure is directly visible as 3 distinct absorption peaks. Note that the condition $C = 1$, corresponding to $r(\omega_s) = 0$, is reached at a $\sqrt{3}$ larger g_{ens} for the NV center case than in the case of a single Lorentzian, since the number of spins in each peak is divided by 3. At the other extreme, for $g_{\text{ens}} \gg \Delta_{\text{hf}}$, only two polaritonic peaks are visible, exactly as in the single-Lorentzian case, which implies that the details of the spin distribution function are “washed out” in the strong coupling regime. In the intermediate regime where $g_{\text{ens}} \approx \Delta_{\text{hf}}$, an interesting phenomenon occurs, with the appearance of two narrow peaks in the spectrum, which are reminiscent of dark modes [46].

DYNAMICS

As will be clear in Chaps. 4 and 5, we also need to compute the dynamics of an excitation initially stored in the system, in the absence of external drive. We follow the main lines of the derivation done in [46]. We consider the spin-resonator system, described by Hamiltonian Eq. 2.103 without drive, and initialized at $t = 0$ in state $|1, G\rangle$, i.e. with all spins in their ground state and 1 photon in the resonator. The goal is to determine the probability at $t > 0$ that the excitation is still in the resonator $p(t) = |\alpha(t)|^2$ with $\alpha(t) = \langle 0 | a(t) a^\dagger(0) | 0 \rangle$. This quantity can be calculated by considering an effective non-Hermitian Hamiltonian

$$H_{\text{eff}}/\hbar = \begin{pmatrix} \tilde{\omega}_0 & i g_1 & i g_2 & \dots \\ -i g_1 & \tilde{\omega}_1 & & \\ -i g_2 & & \tilde{\omega}_2 & \\ \vdots & & & \ddots \end{pmatrix}. \quad (2.120)$$

with complex angular frequencies $\tilde{\omega}_r = \omega_r - i\kappa/2$ and $\tilde{\omega}_k = \omega_k - i\gamma/2$. Introducing the vector $X(t)$ of coordinates $\left[\langle a(t) a^\dagger(0) \rangle, \dots, \langle \hat{b}_j(t) a^\dagger(0) \rangle, \dots \right]$ it can be shown that $dX/dt = -(i/\hbar) H_{\text{eff}} X$. The formal solution to this equation is

$$X(t) = \mathcal{L}^{-1}[(s + i H_{\text{eff}}/\hbar)^{-1} X(0)] \quad (2.121)$$

with $X(0) = x_G$ and $x_G \equiv (1, 0, \dots, 0)$. This implies that $\alpha(t) = x_G^\dagger \cdot X(t) = \mathcal{L}^{-1}[t_1(s)]$ with $t_1(s) = x_G^\dagger \cdot (s + i H_{\text{eff}})^{-1} \cdot x_G$ and $\mathcal{L}[f(s)] = \int e^{-st} f(t) dt$, s being a complex number. Since $t_1(s)$ is not singular on its imaginary axis, we only need the transmission coefficient t_1 given by Eq. 2.22 in Sect. 2.3 for pure imaginary argument $s = -i\omega$ to perform the Laplace transform inversion. The computation of the intracavity field $\alpha(t)$ follows.

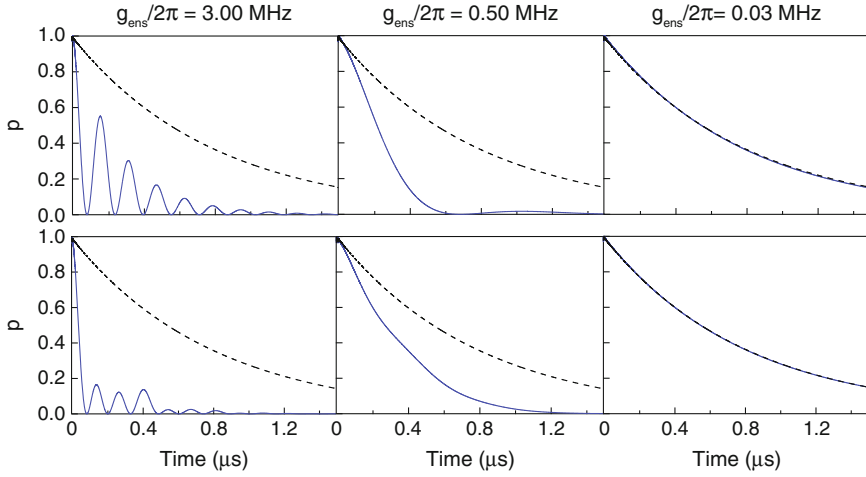


Fig. 2.42 Dynamics of a photon in a resonator resonantly coupled to a distribution of spins. Probability $p(t)$ for a single Lorentzian (*top*) and sum of three Lorentzian (*bottom*) spin distributions. The exchange of the photon is only possible in the strong coupling regime $g_{\text{ens}} \gg \kappa, \Gamma$. The case $g/2\pi = 0$ MHz that reveals the damping in the cavity alone is plotted in *dashed line*

We show in Fig. 2.42 the computed probability $p(t)$ for different coupling strengths g_{ens} in the case of the single Lorentzian and sum of three Lorentzian spin distributions, assuming a width $w/2\pi = 1$ MHz for each Lorentzian peak, and taking $\gamma = 0$. The resonator is resonant with the spins ($\omega_r = \omega_s$) and has a damping rate $\kappa = 1.2 \times 10^6 \text{ s}^{-1}$. When the coupling constant is low so that the two systems are in the weak coupling regime (right panel), p is exponentially damped with a time constant slightly shorter than in the absence of spins, which shows that the absorption of the spin takes place on a longer time scale and is therefore not able to efficiently absorb the photon before it leaks out of the cavity. Increasing the coupling constant makes the absorption faster, up to the point that coherent effects are visible (middle then left panel, strong coupling case). There, oscillations are seen in $p(t)$ with period π/g_{ens} , revealing the coherent exchange of a single excitation between the two systems. The damping of the oscillations is due mainly this time to the transfer of the energy from the bright spin mode into the spin dark modes (with a time constant $T_2^* = 2/w = 300 \text{ ns}$), as evidenced from the fact that it takes place faster than cavity damping. In the case of the Lorentzian triplet distribution, the conclusions are unchanged, except that the oscillations in the strong coupling regime are non-periodic and non-exponentially damped. Corresponding experimental results are reported in Chap. 4.

2.3.4 The Resonator-Spins System Under Strong Drive Powers

The Holstein-Primakoff approximation, in which the spin operators are linearized, makes it possible to compute the quantum dynamics of the resonator-spin ensemble system, as explained in the previous section and required for the experiments described in Chap. 4 where single-photon fields are stored into the ensemble. But in order to retrieve this quantum field after writing into the spin-ensemble memory, a π pulse should be applied to all the spins (see Introduction), which requires strong classical microwave pulses and clearly implies a breakdown of the Holstein-Primakoff approximation. A quantitative estimate of the fidelity of the quantum memory requires not only a calculation of the mean values of the spin and field operators, but also of their quantum statistics, which then becomes a difficult theoretical problem in particular because of inhomogeneous broadening, as explained above. In this section we qualitatively outline the steps taken by our collaborator B. Julsgaard, from Aarhus University, to address this problem numerically as explained in more details in [50, 51]. The simulations developed at this occasion will be used extensively in Chap. 5 to compute the response of the spins to microwave pulses beyond the HP approximation, including Hahn-echo pulse sequences.

2.3.4.1 The Model

The cavity—spin ensemble system is modelled by the following Hamiltonian, which takes into account the inhomogeneity of spin frequency and of coupling constant, and which includes the effect of a drive through port 1:

$$\hat{H}_2/\hbar = \omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \sum_{j=1}^N \omega_j \hat{\sigma}_z^{(j)} + \sum_{j=1}^N \left(g_j \hat{\sigma}_+^{(j)} \hat{a} + g_j^* \hat{\sigma}_-^{(j)} \hat{a}^\dagger \right) + i\sqrt{\kappa_1} (\beta \hat{a}^\dagger - \beta^* \hat{a}). \quad (2.122)$$

Damping is treated as in the previous section by a master equation approach using the Lindblad formalism. This implies as before that the baths are Markovian, which constitutes a crude approximation when applied to the spins. The cavity damping operator is kept unchanged. Spin decoherence is described by the operator $\hat{c}_{2,j}^\dagger = \sqrt{\gamma_2/2} \hat{\sigma}_-$ which damps the phase of a state superposition at the Hahn-echo damping rate γ_2 .

2.3.4.2 Integration

To treat the problem, the approach developed in [50] consists in dividing the spin ensemble into M sub-ensembles, each of them containing a smaller number of spins N_m with homogeneous coupling strength g_m and frequency ω_m . This subdivision

is done according to the distributions of coupling constant and frequency $\rho(g)$ and $\rho(\omega)$ which are input to the problem. As explained in Sect. 2.3.2.1 about the Tavis-Cummings model, this allows to re-write the Hamiltonian using only the collective spin operators $\hat{S}_{x,y,z}^{(m)}$, which considerably reduces the number of variables. In the end, there are $3M + 2$ operators (3 for each spin subdivision, and 2 for the two field quadratures $\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ and $\hat{P} = -i(\hat{a} - \hat{a}^\dagger)/\sqrt{2}$) whose evolution needs to be computed. The time-dependence of their mean values is thus obtained by solving a set of $3M + 2$ coupled differential equations, which can be achieved numerically provided M is not too large. This is the approach used for the simulations presented in Chap. 5, where we compute the response of the spins to strong resonant microwave pulses and to Hahn-echo sequences.

Knowing the mean value evolution is not sufficient to assess the fidelity of a quantum memory. Indeed, a quantum state of the field is characterized also by its fluctuations. Imagine that an input field in a coherent state $|\alpha\rangle$ is stored in the memory, with mean values $\langle\hat{X}\rangle = \alpha$ and $\langle\hat{P}\rangle = 0$. The variance of these quadratures $\hat{X}$ and $\hat{P}$ then verifies $\langle\delta X^2\rangle = \langle\delta P^2\rangle = 1/2$. These values should be preserved at the end of the protocol: the final state of the field should still be in a state of minimal uncertainty, otherwise the fidelity will be low even though the mean values of the quadrature are faithfully restored. Fortunately, the approach outlined above not only makes it possible to integrate the time evolution of the operators mean value, but also of their co-variance matrix. Indeed this matrix has a dimension of $(3M + 2)^2$; its coefficients are governed by intricate equations which were derived in [50]. Again, for M not too large, this set of coupled $(3M + 2)^2$ differential equations can be integrated numerically.

References

1. B. Yurke, J.S. Denker, Quantum network theory. *Phys. Rev. A* **29**, 1419–1437 (1984)
2. M.H. Devoret, *Quantum Fluctuations in Electrical Circuits* (Elsevier, 1997), p. 351
3. M.J. Collett, C.W. Gardiner, Squeezing of intracavity and traveling-wave light fields produced in parametric amplification. *Phys. Rev. A* **30**, 1386 (1984)
4. A. Blais, R.-S. Huang, A. Wallraff, S.M. Girvin, R.J. Schoelkopf, Cavity quantum electrodynamics for superconducting electrical circuits: an architecture for quantum computation. *Phys. Rev. A* **69**, 062320 (2004)
5. A. Palacios-Laloy, Superconducting qubit in a resonator: test of the Leggett-Garg inequality and single-shot readout (2010)
6. R.N. Simons, *Coplanar Waveguide Circuits, Components, and Systems* (Wiley, 2001)
7. B. Josephson, Possible new effects in superconductive tunneling. *Phys. Lett.* **1**, 251–253 (1962)
8. J. Clarke. *The SQUID Handbook: Fundamentals and Technology of SQUIDs and SQUID Systems* (Wiley, 2005)
9. I. Siddiqi, R. Vijay, F. Pierre, C.M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, M.H. Devoret, RF-driven Josephson bifurcation amplifier for quantum measurement. *Phys. Rev. Lett.* **93**, 207002 (2004)
10. R. Vijay, M.H. Devoret, I. Siddiqi, Invited review article: the Josephson bifurcation amplifier. *Rev. Sci. Instrum.* **80**(11), 111101–1111017 (2009)

11. F. Mallet, F.R. Ong, A. Palacios-Laloy, F. Nguyen, P. Bertet, D. Vion, D. Esteve, Single-shot qubit readout in circuit quantum electrodynamics. *Nat. Phys.* **5**, 791 (2009)
12. A. Palacios-Laloy, F. Nguyen, F. Mallet, P. Bertet, D. Vion, D. Esteve, Tunable resonators for quantum circuits. *J. Low Temp. Phys.* **151**, 1034 (2008)
13. J. Koch, T.M. Yu, J. Gambetta, A.A. Houck, D.I. Schuster, J. Majer, A. Blais, M.H. Devoret, S.M. Girvin, R.J. Schoelkopf, Charge-insensitive qubit design derived from the cooper pair box. *Phys. Rev. A* **76**, 042319 (2007)
14. V. Bouchiat, D. Vion, P. Joyez, D. Esteve, M.H. Devoret, Quantum coherence with a single cooper pair. *Physica Scripta* **T76**, 165 (1998)
15. A. Cottet, Implementation of a quantum bit in a superconducting circuit. Ph.D. thesis, Université Paris VI, Paris, 2002
16. Y. Nakamura, Y.A. Pashkin, J.-S. Tsai, Coherent control of macroscopic quantum states in a single-cooper-pair box. *Nature* **398**, 786 (1999)
17. J.E. Mooij, T.P. Orlando, L. Levitov, L. Tian, C.H. van der Wal, S. Lloyd, Josephson persistent-current qubit. *Science* **285**(5430), 1036–1039 (1999)
18. J.M. Martinis, S. Nam, J. Aumentado, C. Urbina, Rabi oscillations in a large Josephson-junction qubit. *Phys. Rev. Lett.* **89**, 117901 (2002)
19. D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, M.H. Devoret, Manipulating the quantum state of an electrical circuit. *Science* **296**(5569), 886–889 (2002)
20. I. Chiorescu, P. Bertet, K. Semba, Y. Nakamura, C.J.P.M. Harmans, J.E. Mooij, Coherent dynamics of a flux qubit coupled to a harmonic oscillator. *Nature* **431**(7005), 159–162 (2004)
21. H. Paik, D.I. Schuster, L.S. Bishop, G. Kirchmair, G. Catelani, A.P. Sears, B.R. Johnson, M.J. Reagor, L. Frunzio, L.I. Glazman, S.M. Girvin, M.H. Devoret, R.J. Schoelkopf, Observation of high coherence in Josephson junction qubits measured in a three-dimensional circuit QED architecture. *Phys. Rev. Lett.* **107**, 240501 (2011)
22. R. Barends, J. Kelly, A. Megrant, D. Sank, E. Jeffrey, Y. Chen, Y. Yin, B. Chiaro, J. Mutus, C. Neill, P. O'Malley, P. Roushan, J. Wenner, T.C. White, A.N. Cleland, J.M. Martinis, Coherent Josephson qubit suitable for scalable quantum integrated circuits. *Phys. Rev. Lett.* **111**, 080502 (2013)
23. F. Mallet, F.R. Ong, A. Palacios-Laloy, F. Nguyen, P. Bertet, D. Vion, D. Esteve, Single-shot qubit readout in circuit quantum electrodynamics. *Nat. Phys.* **5**(11), 791–795 (2009)
24. J. Loubser, J.A. van Wyk, Electron spin resonance in the study of diamond. *Rep. Prog. Phys.* **41**, 1201 (1978)
25. N.R.S. Reddy, N.B. Manson, E.R. Krausz, Two-laser spectral hole burning in a colour centre in diamond. *J. Lumin.* **38**, 46–47 (1987)
26. G. Balasubramanian, P. Neumann, D. Twitchen, M. Markham, R. Koslov, N. Mizuochi, J. Isoya, J. Achard, J. Beck, J. Tessler, V. Jacques, P.R. Hemmer, F. Jelezko, J. Wrachtrup, Ultralong spin coherence time in isotopically engineered diamond. *Nat. Mater.* **8**, 383 (2009)
27. N. Bar-Gill, L. Pham, A. Jarmola, D. Budker, R. Walsworth, Solid-state electronic spin coherence time approaching one second. *Nat. Commun.* **4**, 1743 (2013)
28. N.B. Manson, J.P. Harrison, M.J. Sellars, Nitrogen-vacancy center in diamond: model of the electronic structure and associated dynamics. *Phys. Rev. B* **74**, 104303 (2006)
29. A. Gruber, A. Dräbenstedt, C. Tietz, L. Fleury, J. Wrachtrup, C. Von Borczyskowski, Scanning confocal optical microscopy and magnetic resonance on single defect centers. *Science* **276**(5321), 2012–2014 (1997)
30. S. Felton, A.M. Edmonds, M.E. Newton, P.M. Martineau, D. Fisher, D.J. Twitchen, Electron paramagnetic resonance studies of the neutral nitrogen vacancy in diamond. *Phys. Rev. B* **77**, 081201 (2008)
31. N. Aslam, G. Waldherr, P. Neumann, F. Jelezko, J. Wrachtrup, Photo-induced ionization dynamics of the nitrogen vacancy defect in diamond investigated by single-shot charge state detection. *New J. Phys.* **15**(1), 013064 (2013)
32. J. Harrison, M.J. Sellars, N.B. Manson, Optical spin polarisation of the nv centre in diamond. *J. Lumin.* **107**(1), 245–248 (2004)

33. J. Harrison, M.J. Sellars, N.B. Manson, Measurement of the optically induced spin polarisation of NV centres in diamond. *Diam. Relat. Mater.* **15**(4), 586–588 (2006)
34. A. Batalov, C. Zierl, T. Gaebel, P. Neumann, I.-Y. Chan, G. Balasubramanian, P.R. Hemmer, F. Jelezko, J. Wrachtrup, Temporal coherence of photons emitted by single nitrogen-vacancy defect centers in diamond using optical Rabi-oscillations. *Phys. Rev. Lett.* **100**(7), 077401 (2008)
35. L. Robledo, L. Childress, H. Bernien, B. Hensen, P.F.A. Alkemade, R. Hanson, High-fidelity projective read-out of a solid-state spin quantum register. *Nature* **477**, 574 (2011)
36. M.V. Gurudev Dutt, L. Childress, L. Jiang, E. Togan, J. Maze, F. Jelezko, A.S. Zibrov, P.R. Hemmer, M.D. Lukin, Quantum register based on individual electronic and nuclear spin qubits in diamond. *Science* **316**(5829), 1312–1316 (2007)
37. R. Hanson, O. Gywat, D.D. Awschalom, Room-temperature manipulation and decoherence of a single spin in diamond. *Phys. Rev. B* **74**, 161203 (2006)
38. V.V. Dobrovitski, A.E. Feiguin, D.D. Awschalom, R. Hanson, Decoherence dynamics of a single spin versus spin ensemble. *Phys. Rev. B* **77**, 245212 (2008)
39. J.R. Maze, J.M. Taylor, M.D. Lukin, Electron spin decoherence of single nitrogen-vacancy defects in diamond. *Phys. Rev. B* **78**, 094303 (2008)
40. N. Zhao, S.-W. Ho, R.-B. Liu, Decoherence and dynamical decoupling control of nitrogen vacancy center electron spins in nuclear spin baths. *Phys. Rev. B* **85**, 115303 (2012)
41. J.A. van Wyk, E.C. Reynhardt, G.L. High, I. Kiflawi, The dependences of esr line widths and spin-spin relaxation times of single nitrogen defects on the concentration of nitrogen defects in diamond. *J. Phys. D Appl. Phys.* **30**, 1790–1793 (1997)
42. M. Tavis, F.W. Cummings, Exact solution for an n -molecule-radiation-field hamiltonian. *Phys. Rev.* **170**, 379 (1968)
43. R.H. Dicke, Coherence in spontaneous radiation processes. *Phys. Rev. Lett.* **93**, 99 (1954)
44. T. Holstein, H. Primakoff, Field dependence of the intrinsic domain magnetization of a ferromagnet. *Phys. Rev.* **58**, 1098 (1940)
45. Z. Kurucz, J.H. Wesenberg, K. Mølmer, Spectroscopic properties of inhomogeneously broadened spin ensembles in a cavity. *Phys. Rev. A* **83**, 053852 (2011)
46. I. Diniz, S. Portolan, R. Ferreira, J.M. Gérard, P. Bertet, A. Auffèves, Strongly coupling a cavity to inhomogeneous ensembles of emitters: potential for long-lived solid-state quantum memories. *Phys. Rev. A* **84**, 063810 (2011)
47. S. Haroche, J.-M. Raimond, *Exploring the Quantum* (Oxford University Press, 2006)
48. B. Julsgaard, K. Mølmer, Reflectivity and transmissivity of a cavity coupled to two-level systems: coherence properties and the influence of phase decay. *Phys. Rev. A*, **85**, 013844 (2012)
49. A. Abragam, *Principles of Nuclear Magnetism* (Oxford Science Publications, Clarendon Press, 1986)
50. B. Julsgaard, C. Grezes, P. Bertet, K. Mølmer, Quantum memory for microwave photons in an inhomogeneously broadened spin ensemble. *Phys. Rev. Lett.* **110**, 250503 (2013)
51. C. Grezes, B. Julsgaard, Y. Kubo, M. Stern, T. Umeda, J. Isoya, H. Sumiya, S. Abe, S. Onoda, T. Ohshima, V. Jacques, J. Esteve, D. Vion, D. Esteve, K. Moelmer, P. Bertet, Multimode storage and retrieval of microwave fields in a spin ensemble. *Phys. Rev. X* **4**, 021049 (2014)

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