

Chapter 2

The Linear Dielectric Susceptibility

Abstract In electric dipole approximation, the interaction of electromagnetic irradiation with matter results into the formation of an induced electric polarization of the medium. Then, linear optical material properties of a homogeneous and isotropic medium are completely defined by the complex linear dielectric susceptibility. Its dispersion, i.e. its frequency dependence arises from fundamental physical principles. From the susceptibility, the frequency-dependent complex dielectric function as well as the complex index of refraction may be calculated straightforwardly. The real part of the complex index of refraction (the usual refractive index) and the imaginary part (the extinction coefficient) form the pair of linear optical constants of the medium. The refractive index defines the phase velocity of the electromagnetic wave in the medium, while the extinction coefficient is responsible for the description of damping effects.

2.1 Maxwell's Equations

Any optical phenomenon is connected with the interaction of electromagnetic radiation with matter. Whenever we are looking on a landscape, a photograph, a painting and so on, our eyes receive light from the object under observation. This light may be emitted from the object itself (for example when looking into the sun, which is not recommended, or on lightnings in a thunderstorm), but it may also originate from specular as well as diffuse reflections of daylight as it is characteristic for the play of colours shown on the painting of the marble bridge on the previous page. Of course, the particular properties of this reflected irradiation are to a certain extent defined by the process of interaction of the daylight with that object. Hence, the irradiation carries some information about the properties of the object. Understanding the mechanisms of the interaction of light with matter gives us a key to disclose this information in a quantitative manner.

The light-matter interaction may be theoretically treated at different levels of difficulty. For example, one may use the purely classical description. It is on the other hand possible to build a strong quantum mechanical theory. In practice, a

large number of practically important problems may be solved working with classical models only. We will therefore start our treatment with the classical description of the radiation-with-matter interaction.

A purely classical description makes use of Maxwell's Equations for the description of the electrical and magnetic fields and classical models (for example Newton's equations of motion) for the dynamics of the charge carriers present in any terrestrial matter. On the contrary, a quantum mechanical treatment is possible within the framework of the quantization of the electromagnetic field (so-called second quantization) and a quantum theoretical treatment of matter. This description is necessary, when spontaneous optical effects have to be described (spontaneous emission, spontaneous Raman scattering, or spontaneous paramagnetic interactions in nonlinear optics) on a theoretically strong level. In applied spectroscopy, the accurate quantum mechanical description is often omitted due to the rather complicated mathematics and replaced by the so-called semiclassical treatment. Here, the properties of matter are described in terms of quantum mechanical models, while the fields are treated within the framework of Maxwell's theory. Maxwell's equations are therefore used in both classical and semiclassical approaches, and for that reason we start our discussion from these equations, which are given below:

$$\begin{aligned}
 1. \quad & \operatorname{div} \mathbf{B} = 0, \\
 & \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}), \\
 2. \quad & \operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \\
 3. \quad & \operatorname{div} \mathbf{D} = 0, \\
 4. \quad & \operatorname{curl} \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}, \\
 & \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}.
 \end{aligned} \tag{2.1}$$

Here, \mathbf{E} and \mathbf{H} represent the vectors of the electric and magnetic fields, while \mathbf{D} and \mathbf{B} stand for the electric displacement and the magnetic induction, respectively. \mathbf{P} is the polarization, and \mathbf{M} the magnetization of the medium. In (2.1), neither the free charge carrier density nor their current density is present. Keeping in mind, that optics deal with rapidly oscillating electric and magnetic fields, there is really no need to treat "free" charges separately—due to the short periods, they will only oscillate around their equilibrium position quite similar to bound charges. So in our description, the displacement vector contains information on both free and bound charges. The very few cases, where the static response of matter with free electrons becomes important in the frames of this book, cannot be treated within (2.1) and will need separate discussion.

In the following, we will assume that the media are generally non-magnetic (\mathbf{M} is a zero-vector) and isotropic. Optically anisotropic materials will be treated in

a special chapter later, but here we will assume isotropy for simplicity. When neglecting magnetization effects, from (2.1) one obtains straightforwardly:

$$\text{curl curl} \mathbf{E} = \text{grad div} \mathbf{E} - \Delta \mathbf{E} = -\mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2} \quad (2.2)$$

This is an equation with two unknown vectors. In order to proceed further, at this point, we need to establish a relationship between the vectors \mathbf{E} and \mathbf{D} , which will be done in the next subchapter.

2.2 The Linear Dielectric Susceptibility

Let us assume that a rapidly changing electric field with a completely arbitrary time-dependence interacts with a matter. One would naturally expect that the electric field tends to displace, in general, both negative and positive charges, thus creating a macroscopic dipole moment in the material system. The polarization \mathbf{P} is per definition the dipole moment per unit volume, and it will be, of course, time-dependent in a manner that is determined by the time dependence of \mathbf{E} . For the moment, we neglect the spatial dependence of \mathbf{E} and \mathbf{P} , because it is not essential for the further derivation. Generally, the polarization is thus a possibly very involved functional \mathbf{F} of the field \mathbf{E} :

$$\mathbf{P}(t) = \mathbf{F}[\mathbf{E}(t' \leq t)] \quad (2.3)$$

Of course, the polarization of the medium appears as the result of the action of the assumed electric field (here and in the following, we do *not* regard ferroelectrics!). Due to the causality principle, the polarization at a given time t can depend on the field at the same moment as well as at previous moments t' , but not on the field behaviour in the future. This is the meaning of the condition: $t' \leq t$. In order to comply with the requirement (2.3), we therefore postulate the following general relationship for the polarization as a functional of the electric field:

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^t \kappa(t, t') \mathbf{E}(t') dt' \quad (2.4)$$

Equation (2.4) postulates that the polarization at any time t may principally depend on the first power of the field at the current and all previous moments, as it follows from the integration interval that is chosen in correspondence with the mentioned causality principle. The specific way, in which the system “remembers” the field strength at previous moments, is hidden in the response function $\kappa(t, t')$, which must be specific for any material. Equation (2.4) is in fact the first (linear) term of an expansion of (2.3) into a Taylor power series of \mathbf{E} . As we keep only the

first (linear) term of the series, all optical effects that arise from (2.4) form the field of *linear optics*. Equation (2.4) thus represents a rather general writing of the material equation in linear optics.

In general, when the materials are anisotropic, $\kappa(t, t')$ is a tensor. As we restrict our attention here to optically isotropic materials, \mathbf{P} will always be parallel to \mathbf{E} , so that $\kappa(t, t')$ becomes a scalar function.

A further facilitation is possible. Due to the homogeneity of time, $\kappa(t, t')$ will in fact not depend on both individual times t and t' separately, but only on their difference $\xi \equiv t - t'$. Substituting t' by ξ , we obtain:

$$\mathbf{P}(t) = \varepsilon_0 \int_0^\infty \kappa(\xi) \mathbf{E}(t - \xi) d\xi \quad (2.5)$$

Let us now come to the utmost important case of a harmonic time dependence. Let us assume that the electric field performs rapid oscillations according to:

$$\mathbf{E}(t) = \mathbf{E}_0 e^{-i\omega t}$$

and correspondingly

$$\mathbf{E}(t - \xi) = \mathbf{E}_0 e^{-i\omega t} e^{i\omega \xi}.$$

Note, that we assume a completely monochromatic field. It is then obtained:

$$\begin{aligned} \mathbf{P}(t) &= \mathbf{E}_0 e^{-i\omega t} \varepsilon_0 \int_0^\infty \kappa(\xi) e^{i\omega \xi} d\xi \\ &= \mathbf{E}(t) \varepsilon_0 \int_0^\infty \kappa(\xi) e^{i\omega \xi} d\xi \end{aligned} \quad (2.6)$$

We define the *linear dielectric susceptibility* χ according to:

$$\chi = \int_0^\infty \kappa(\xi) e^{i\omega \xi} d\xi = \chi(\omega) \quad (2.7)$$

The thus defined susceptibility must be complex (it has both real and imaginary parts), and it depends on the frequency of the field even after having performed the integration in (2.7). Both circumstances arise mathematically from (2.5) and physically from finite inertness of any material system. Clearly, the charge carriers cannot instantaneously react on rapidly changing fields, so that their positions at a given time t depend on the history of the system, which is in fact the reason for the

complicated behaviour of the polarization with time. The information on the specific material properties is now carried by $\chi(\omega)$.

We are now able to formulate the relationship between \mathbf{E} and \mathbf{D} for monochromatic electric fields. Indeed, from (2.6) and (2.7) it follows, that

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E} \quad (2.8)$$

In combination with the definition of \mathbf{D} we have:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 [1 + \chi(\omega)] \mathbf{E} \equiv \varepsilon_0 \varepsilon(\omega) \mathbf{E} \quad (2.9)$$

where we defined the *dielectric function* $\varepsilon(\omega)$

$$\varepsilon(\omega) \equiv 1 + \chi(\omega)$$

Equation (2.9) is completely analogous to what is known from the electrostatics of dielectrics, with the only difference that ε is now complex and frequency dependent. So that we come to the conclusion, that in optics we have a similar relationship between field and displacement vectors as in electrostatics, with the difference that in optics the dielectric constant has to be replaced by the dielectric function.

Remark In the case that the incident field is not monochromatic, in (2.8) and (2.9), all of the vectors must be replaced by the amplitudes of their corresponding Fourier components. We have:

$$\mathbf{D}_\omega = \varepsilon_0 \varepsilon(\omega) \mathbf{E}_\omega \quad (2.9a)$$

And so on. Here symbols like \mathbf{D}_ω denote Fourier components of the corresponding vectors.

2.3 Linear Optical Constants

We may now turn back to (2.2). Keeping in mind that our discussion is restricted to harmonic oscillations of the fields only, the second derivative with respect to time in (2.2) may be replaced by multiplying with $-\omega^2$. Replacing moreover \mathbf{D} with (2.9), we obtain:

$$\text{curl curl} \mathbf{E} - \frac{\omega^2 \varepsilon(\omega)}{c^2} \mathbf{E} = \mathbf{0} \quad (2.10)$$

Here we used the identity:

$$\varepsilon_0 \mu_0 = c^{-2}$$

where c is the velocity of light in vacuum. For polychromatic fields, the single Fourier-components have to be treated separately in an analogous manner.

We now remember the vector identity:

$$\text{curl curl} \mathbf{E} \equiv \text{grad div} \mathbf{E} - \Delta \mathbf{E}.$$

In the case that $\varepsilon \neq 0$, from $\text{div} \mathbf{D} = 0$ it follows that $\text{div} \mathbf{E} = 0$. Thus we finally have:

$$\Delta \mathbf{E} + \frac{\omega^2 \varepsilon(\omega)}{c^2} \mathbf{E} = 0 \quad (2.11)$$

where the field vector has been replaced by a scalar field due to the assumed isotropy. A completely identical equation may be obtained for the magnetic field.

Let us remark at this point, that due to the assumed optical isotropy, we will often turn from the vectorial to the scalar mathematical description. Throughout this book, in these cases we will simply refrain from bold symbols without further notice.

Assuming that the dielectric function does not depend on the coordinates itself (homogeneous media), we are looking for a solution in the form:

$$\mathbf{E}(t, \mathbf{r}) = E_0 e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} \quad (2.12)$$

with \mathbf{k} being the wavevector. Non-trivial solutions of (2.11) exist when

$$k = \pm \frac{\omega}{c} \sqrt{\varepsilon(\omega)} \quad (2.13)$$

is fulfilled. Assuming for simplicity, that \mathbf{k} is parallel to the z -axis of a Cartesian coordinate system, (2.12) describes a planar wave travelling along the z -axis. It depends on the sign in (2.13) whether the wave is running into the positive or negative direction. We choose a wave running into the positive direction, and obtain:

$$E = E_0 e^{-i\left(\omega t - \frac{\omega}{c} \sqrt{\varepsilon(\omega)} z\right)} \quad (2.14)$$

where E_0 is the field amplitude at $z = 0$. Let us look on (2.14) in some more detail.

As we obtained in Sect. 2.2, the dielectric function may be complex; hence it may have an imaginary part. Of course, the square root will also be a complex function. We therefore have:

$$\sqrt{\varepsilon(\omega)} = \text{Re}\sqrt{\varepsilon(\omega)} + i\text{Im}\sqrt{\varepsilon(\omega)}$$

Equation (2.14) therefore describes a damped wave according to:

$$E = E_0 e^{-\frac{\omega}{c}\text{Im}\sqrt{\varepsilon(\omega)}z} e^{-i\left(\omega t - \frac{\omega}{c}\text{Re}\sqrt{\varepsilon(\omega)}z\right)} \quad (2.15)$$

with a z -dependent amplitude

$$E_{\text{ampl}} = E_0 e^{-\frac{\omega}{c}\text{Im}\sqrt{\varepsilon(\omega)}z} \quad (2.16)$$

and a phase:

$$\text{phase} = \omega t - \frac{\omega}{c}\text{Re}\sqrt{\varepsilon(\omega)}z.$$

Let us calculate the velocity dz/dt of any point at the surface of constant phase (which is a plane in our case). Regarding the phase as constant and differentiating the last equation with respect to time, we obtain the so called *phase velocity* of the wave according to:

$$\frac{dz}{dt} \equiv v_{\text{phase}} = \frac{c}{\text{Re}\sqrt{\varepsilon(\omega)}} \equiv \frac{c}{n(\omega)} \quad (2.17)$$

Here we introduced the *refractive index* $n(\omega)$ as the real part of the square root of the complex dielectric function. Naturally, the refractive index appears to be frequency dependent (so-called *dispersion* of the refractive index). In a medium with refractive index n , the phase velocity of an electromagnetic wave changes with respect to vacuum according to (2.17).

As a generalization to (2.17), one often defines the *complex index of refraction* as:

$$\hat{n}(\omega) = n(\omega) + iK(\omega) \equiv \sqrt{\varepsilon(\omega)} \quad (2.18)$$

Its real part is identical with the ordinary refractive index as defined in (2.17), while its imaginary part (the so-called *extinction coefficient*) K is responsible for the damping of a wave. Indeed, returning to (2.16), we obtain for the amplitude of the wave:

$$E_{\text{ampl}} = E_0 e^{-\frac{\omega}{c}Kz}$$

Because the intensity I of the wave is proportional to the square of the field amplitude modulus, the intensity damps inside the medium as:

$$I = I(z=0) e^{-2\frac{\omega}{c}Kz} \equiv I(z=0) e^{-\alpha z}. \quad (2.19)$$

This exponential decay of light intensity for a wave travelling in a lossy medium is well known as Beer's law of absorption with a frequency-dependent *absorption coefficient* α defined as:

$$\alpha(\omega) = 2\frac{\omega}{c}K(\omega) \quad (2.20a)$$

In terms of the identities:

$$v \equiv \frac{1}{\lambda} = \frac{\omega}{2\pi c}$$

where v is the wavenumber and λ the wavelength in vacuum, we come to a more familiar expression:

$$\alpha(v) = 4\pi v K(v) \quad (2.20b)$$

Although the refractive index n and the extinction coefficient K are dimensionless, the absorption coefficient is given in reciprocal length units, usually in reciprocal centimetres. The reciprocal value of the absorption coefficient is sometimes called *penetration depth*. The pair of n and K forms the pair of *linear optical constants* of a material.

Remark Throughout this derivation, we supposed the time dependence of the fields according to $e^{-i\omega t}$. As a consequence, we defined the complex index of refraction as $n + iK$. The same kind of theory may be built postulating a time dependence of the fields as $e^{+i\omega t}$. However, in this case the index of refraction will be $n - iK$. Both approaches are equally correct and can be found in the literature, however, they shouldn't be confused with each other.

2.4 Some General Remarks

In practice, one often has to perform calculations of different spectra with the purpose to compare them with experimentally measured ones. One of the simplest tasks is the calculation of an absorption spectrum. Although we have not yet defined what may be meant by the term "absorption spectrum", it is intuitively clear that (at least in simple cases) such an absorption spectrum should resemble the wavelength dependence of the absorption coefficient of the material that is investigated. From the theoretical stuff described so far, we find however that the calculation of any

absorption spectrum will contain at least two different parts: First of all, one has to find a suitable model for the dielectric function that contains the information about the *material*. After that, the optical constants may be calculated. Secondly, having this model in hands, one has to solve the wave equation (2.11) to account for the particular *geometry* valid for the (given or assumed) experiment. Having solved the wave equation with realistic boundary conditions, we obtain electric and/or magnetic fields that may be converted into light intensities, which in turn may be compared with experimental data. Changing the systems geometry will change the intensities obtained at the output, although the material might be the same. For example, in Sect. 2.3 we have solved the wave equation, assuming however that the dielectric function is the same at any point. In other words, we assumed there a completely homogeneous medium, particularly without any interfaces. That resulted in Beer's law (2.19), but the latter cannot be applied in other geometries, for example in thin film spectroscopy (although it is often done!). So that *both* material and geometry specifics must be considered in any spectra calculation.

There is a further complication in real live. What we have described so far is the philosophy of the forward search: We start from a model, calculate the optical constants, solve the wave equation, and finally calculate the intensities. In terms of the terminology introduced in Fig. 1.1 that means, that we calculate the system output, assuming the input as well as the system properties as known. In practice, one is much more often confronted with reverse search tasks: The absorption (or any other) spectrum (the output) has been measured, and the optical constants have to be calculated. In this case, the task is rather to gain information on the systems properties, when input and output are assumed to be known. In several geometries (and particularly in thin film spectroscopy), the reverse search procedures are much more complicated than the forward search. The next section will exemplify a part of a *forward* search, namely the calculation of the dielectric function of a material consisting of permanent microscopic dipoles.

2.5 Example: Orientation Polarization and Debye's Equations

Let us assume a material that is built from permanent microscopic electric dipoles. The dipoles are allowed to rotate freely with some damping. This is the typical situation in a liquid built from polar molecules (for example water). When no external electric field is applied, the stochastic thermally activated movement of the dipoles will not be able to create a macroscopic polarization. However, in an external electric field, the dipoles will more or less align with the field, creating a resulting macroscopic polarization. We shall find the frequency dependence of the dielectric function (and consequently of the optical constants) of such a material.

We will solve this task by a direct application of (2.7). Because we still do not know the response function $\kappa(\xi)$, we start from the following thought experiment:

Let us assume, that a static electric field has been applied to the system for a sufficiently long time, so that a static polarization of the liquid has been well established. Let us further assume that the field is switched off at the moment $t = 0$. We model this situation by means of the electric field:

$$E(t) = E_0[1 - \theta(t)]$$

where $\theta(t)$ is a step function that has the value one for $t \geq 0$ and zero elsewhere. It makes no sense to assume that the polarization will vanish instantaneously with a vanishing external field. On the contrary, we shall assume, that due to the thermal movement of the particles, the macroscopic polarization decreases smoothly and asymptotically approaches the value of zero. This situation may be described by an exponentially descending behaviour with a time constant τ according to:

$$P(t) = P_0 e^{-t/\tau}; t > 0$$

Furthermore, from (2.5) we have:

$$P(t) = P_0 e^{-t/\tau} = \varepsilon_0 \int_0^\infty \kappa(\xi) E_0 [1 - \theta(t - \xi)] d\xi$$

The only action of the step function is to reduce the integration interval:

$$P_0 e^{-t/\tau} = -\varepsilon_0 E_0 \int_\infty^t \kappa(\xi) d\xi$$

We differentiate with respect to time and make use of the identity:

$$f(x) = \frac{d}{dx} \left[\int_a^x f(\xi) d\xi \right]$$

That leads us to the following expression for the response function $\kappa(t)$:

$$\kappa(t) = \frac{P_0}{\varepsilon_0 E_0 \tau} e^{-t/\tau} \equiv \kappa_0 e^{-t/\tau} \quad (2.21)$$

Having found the response function, the further treatment is straightforward. Equations (2.7) and (2.9) yield the dielectric function:

$$\begin{aligned}
 \varepsilon(\omega) &= 1 + \chi(\omega) = 1 + \int_0^{\infty} \kappa(\xi) e^{i\omega\xi} d\xi \\
 &= 1 + \kappa_0 \int_0^{\infty} e^{(i\omega - \frac{1}{\tau})\xi} d\xi = 1 + \frac{\kappa_0 \tau}{1 - i\omega\tau}
 \end{aligned}$$

or

$$\varepsilon(\omega) = 1 + \frac{\chi_{stat}}{1 - i\omega\tau} \quad (2.22)$$

where χ_{stat} is the static ($\omega = 0$) value of the susceptibility. The real and imaginary parts of the dielectric function may be written as follows:

$$\begin{aligned}
 \text{Re } \varepsilon \equiv \varepsilon' &= 1 + \frac{\chi_{stat}}{1 + \omega^2 \tau^2} \\
 \text{Im } \varepsilon \equiv \varepsilon'' &= \frac{\chi_{stat} \omega \tau}{1 + \omega^2 \tau^2}
 \end{aligned} \quad (2.23)$$

The thus obtained dielectric function represents a simplified version of Debye's equations valid for the dielectric function in polar viscous media. In Fig. 2.1, the spectral shapes of real and imaginary parts of this particular dielectric function are presented. Figure 2.2 shows the corresponding optical constants. In these Figures, a static susceptibility of $\chi_{stat} = 80$ has been assumed, similar to what is valid in ordinary water. Obviously, the presence of permanent dipoles in the medium results in a high static dielectric constant, while for higher frequencies, the real part of the dielectric function may be essentially lower. Thus, in the visible spectral range, water has a dielectric function with a real part of approximately 1.77 and a refractive index of 1.33. This behaviour is consistent with the predictions from Debye's equations, where the refractive index is expected to steadily decrease with increasing frequency.

Fig. 2.1 Real and imaginary parts of the dielectric function according to (2.23)

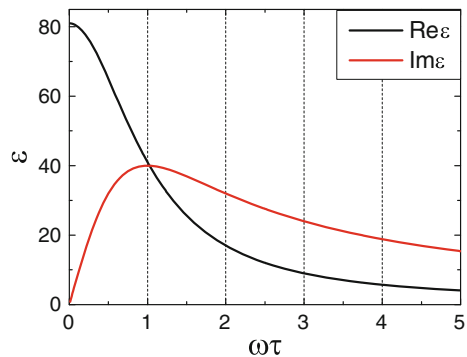
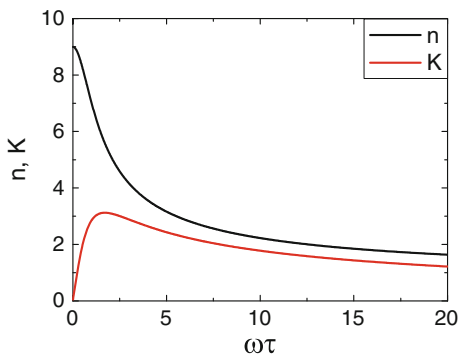


Fig. 2.2 Optical constants n and K for the dielectric function presented in Fig. 2.1, but in a broader spectral region



There is another interesting fact that becomes obvious from Fig. 2.1. The imaginary part of the dielectric function has its maximum value exactly at the angular frequency $\omega = \tau^{-1}$. Consequently, the result of a *spectral* measurement (determining the peak position of $\text{Im } \varepsilon$) reveals information about the *dynamic* behaviour of the system (the decay time of polarization). This is one example for the validity of a more general fundamental principle, that in optics the spectral ($\chi(\omega)$) and time domain ($\kappa(t)$) representations embody the same information and may be transferred into each other. Indeed, (2.7) is in fact a Fourier transformation of the response function, performed however only over a semi-infinite interval for reasons of causality. One may formally multiply the response function with a step-function:

$$\tilde{\kappa}(\xi) = \kappa(\xi)\theta(\xi) \quad (2.24)$$

The thus obtained modified response function may be integrated over the full time interval, so that we have:

$$\chi(\omega) = \int_{-\infty}^{\infty} \tilde{\kappa}(\xi) e^{i\omega\xi} d\xi \quad (2.25)$$

In (2.25), the susceptibility appears to be the Fourier-transform of the modified response function.

Remark The derivation presented in Sect. 2.5 is in fact an example for the tight interplay of forward and reverse search procedures, as it is often observed in practice. Clearly, the calculation of the susceptibility from a material model (rotating permanent dipoles) should be interpreted as a forward search task. Nevertheless, during this derivation we also had to perform a reverse search, namely the calculation of the response function from the time dependence of the polarization (the systems output), caused by a step-function like electric field (the input).

2.6 Energy Dissipation

A high extinction coefficient (high damping) is not necessarily connected with a high imaginary part of the dielectric function. For example, a real but negative dielectric function will result in a purely imaginary refractive index, i.e. a possibly high extinction coefficient. This seemingly exotic situation is in fact a characteristic model assumption in metal optics and will be treated in the section dedicated to total internal reflection. Here the penetrating wave is indeed damped, but the light is rather reflected than absorbed. Therefore, the generally accepted terminus “absorption coefficient” may be misleading in special cases. In fact, for light absorption (energy dissipation) it is essential that $\text{Im } \varepsilon \neq 0$.

Let us illustrate this fact. Indeed, the volume density of the power dissipated from the electromagnetic field can be written as:

$$\frac{\partial W_{diss}}{V \partial t} = \mathbf{j} \mathbf{E}$$

This notation corresponds to real fields \mathbf{E} and current densities \mathbf{j} . In order to express the real functions by their complex counterparts, we simply make use of (1.5) and obtain:

$$\frac{\partial W_{diss}}{V \partial t} = 4 \text{Re} \mathbf{j} \text{Re} \mathbf{E}$$

Making further use of:

$$\frac{\partial \mathbf{P}}{\partial t} = \frac{1}{V} \sum_l q_l \dot{\mathbf{r}}_l = \mathbf{j} \quad (2.26)$$

and considering a harmonic time dependence of the oscillating electric field in the wave, we can write:

$$\mathbf{j} = -i \varepsilon_0 \omega (\varepsilon - 1) \mathbf{E}$$

Therefore we have:

$$\begin{aligned} \text{Re} \mathbf{j} &= \varepsilon_0 \omega [(\text{Re } \varepsilon - 1) \text{Im } \mathbf{E} + \text{Im } \varepsilon \text{Re } \mathbf{E}] \\ \Rightarrow \text{Re} \mathbf{j} \text{Re } \mathbf{E} &= \varepsilon_0 \omega \left[(\text{Re } \varepsilon - 1) \text{Im } \mathbf{E} \text{Re } \mathbf{E} + \text{Im } \varepsilon (\text{Re } \mathbf{E})^2 \right] \end{aligned}$$

Let us finally use expression (2.15) for the electric field. Once the real part of the electric field oscillates according to a cosine function, and the imaginary according to a sine function, the time average over a period results in:

$$\left\langle \frac{\partial W_{diss}}{V \partial t} \right\rangle = 2\epsilon_0 \omega \text{Im } \epsilon |E_0|^2 e^{-\alpha z} \quad (2.27)$$

Thus, light absorption (or energy dissipation) is observed when $\text{Im } \epsilon \neq 0$ is fulfilled.

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