Numbers

You do not have to read this chapter! But … if you get in trouble with numbers later, you can come back and get what you need out of it at any time. I put it toward the front so it would be easy for you to find.

You don't need to be embarrassed if numbers have always seemed difficult to you. That's true for almost everybody. And yet, numbers are easier than you think! Big ones, small ones—I'll show you.

Recently, I read about a case in which a hamburger company spent a lot of money advertising 1/3 pound burgers for only slightly more money than the well-known source of 1/4-pounders was charging, only to find in a focus group that people thought they were getting cheated with 1/3 pound. After all, three is less than four, right?!

Does that seem funny to you?

You know that there are three thirds of a pound in a whole pound (a bit over 5 ounces each piece), and four quarters of a pound (4 ounces each), so I hope you know that 1/4 is less than 1/3. Still … equations and big numbers freeze your mind, right? I hope we'll fix that!

I promise never to subject you to equations other than Einstein's famous $E = mc^2$ in this book. But I want you to understand what that equation means, and be able to do calculations with it, from the mass (m) of something and the speed of light (c). We'll talk about big and small numbers, like femtoseconds and Petaflops, and I need you to be comfortable with those, because today's world works with numbers. You can't afford not to know them, and using them is fun. This chapter is about that.

If all of this seems pretty elementary to you, just go on to the next chapter.

Pretty early, we learn about basic numbers: 1, 2, 3, … 10, 100 and, later, 0.1, 0.01, and so on. Did you know that *zero* wasn't even a number until a bit more than 1,000 years ago? Indian mathematicians first thought of zero as a placeholder (for writing numbers like 1000) and Islamic mathematicians like al-Kwarizmi (see the chapter on *Islamic Science*) went on to develop the whole number system we and the Chinese and everyone else use today. This is during the period Europeans call the Dark Ages. We call them Arabic numerals.

C. Phipps, *No Wonder You Wonder!*, DOI 10.1007/978-3-319-21680-5_2

Zero means "nothing," which is why people had a hard time understanding it a thousand years ago. How can anything that really exists not exist? Well, it's not really a mind twisting koan after all, just an example of how people twist their own minds. Truth be told, Indian mathematicians had a lot to do with developing those numbers, so really they are Hindu-Arabic, even though you might not recognize them, the way they looked in the 800s AD. Things go back.

Numbers like −1 and −9 are harder to comprehend, because they're less than nothing, but we use them all the time when we want to subtract in our checkbooks to find our account balance.

Also, you know that there are other kinds of numbers that aren't whole, fractions like the weight of those hamburger patties, 1/3 and 1/4. And, when you write them in base 10 (I'll explain later), 1/3 goes on forever: 0.33333333 …, but not ¼. One quarter is just 0.25 and that's it.

The "square root" of two, 1.414212356 … is a fractional number that also goes on without end, just like 1/3, but the digits are all random. Funny as it sounds, the "square root" of a number is the answer to the question, "what number taken times itself is equal to a number?" Taking something times itself is squaring it, so this is called the square root, because roots are the answers to queries like that.

What about π? Pi is just the distance around any circle divided by its diameter, whether the circle is an atomic orbit, a crop circle, or the Earth's Equator. It's also the area of a sphere divided by its diameter squared. Think about that! Why should it be so simple, that one number can describe both things?

Party knowledge: did you know that Pi times ten million is about the number of seconds in a year, to within 1/3 of a percent? "Pi" is equal to 3.14159265 …, another of those endless numbers. Some people with good memories and a need to impress can recite it out to 100 places. Computer nuts have calculated it out to *ten trillion digits* now! It is not a whole number of anything, no matter how far you look, or what base you count in. But it's very useful, and a part of nature! Not to say a "natural number," which is a word people use to talk about the whole numbers like 1, 2, and 3.

Did I say "base" in that last paragraph? The one you know about is base 10, where the digits go from 0 to 9. We use that unless we use base 2. You hackers out there are already comfortable with base 2, where the digits are either 0 or 1. That's because electronic things today are (mostly, still!) either on or off, 0 or 1, and we developed a counting system to match them.

For the rest of us: how does base 2 work? Take a look at Table 1.

If you've never seen "binary" before, can you figure it out? It isn't hard! It's a game, a secret code and, fundamentally, just another bookkeeping system. "Base" is just the thing you're taking powers of to write down big numbers in an efficient way. In binary, that base is 2. Eight is 2^3, so there's a 1 followed by three zeros, just like a thousand in base ten is 10^3, and you write *that* as a 1 followed by three zeros. That is all there is to it. There are little games you use to add and multiply in both systems. You know how to do that in base 10, but I won't get into it here for base 2.

Table 1 Counting in two systems

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000

Woops—I'm getting ahead of myself again. What did I mean with that superscript 3? 10^3 just means $10 \times 10 \times 10$, ten times itself three times. The superscript counts the zeros. Just another bookkeeping system. It's easier to write it that way, no?

By writing "c" for the speed of light, we're just using a shorthand code to avoid writing 299,792.458 kilometers/second each time we talk about the speed of light. So "c" is about 300 million meters per second, or 3×10^8. In this chapter, forget about being precise for a while.

Aha! So by c^2, we mean the number which is c times c. That is about nine times 10^{16}. Is that hard to understand? No. Big numbers exist. Instead of writing 10,000,000,000,000,000, we write 10^{16}. Then, you don't have to count 'em. Easier, no? The action happens in that superscript, called an exponent.

Those of you that use Excel know that 10^{16} can also be written 1E16, and writing it that way is better for two reasons: it's even easier when the important number is easier to see for people that use glasses. The "E" in this case is not energy, but just a placeholder for "10 to the …" My friends who are theoreticians always seem to make graphs with numbers along the axis in 9-point type, so the exponent is 6-point, and anyone who is not sitting in the front row at a conference can't read *the only important part*. That's why I always write 10^{16} as 1E16 in 14-point bold type when I present an Excel chart.

So $E = mc^2$ just means that energy E (in this case) is m (mass) times c times c. If m is 1 kg, the energy E is 9E16 J. A trillion is 1E12, so 9E16 joules is 90 thousand trillion joules! Scientists at Los Alamos decided that the energy of a ton of exploding TNT was 4.18E9 J, or 4.18 GJ. *That's no worse to think about than 4 GHz in*

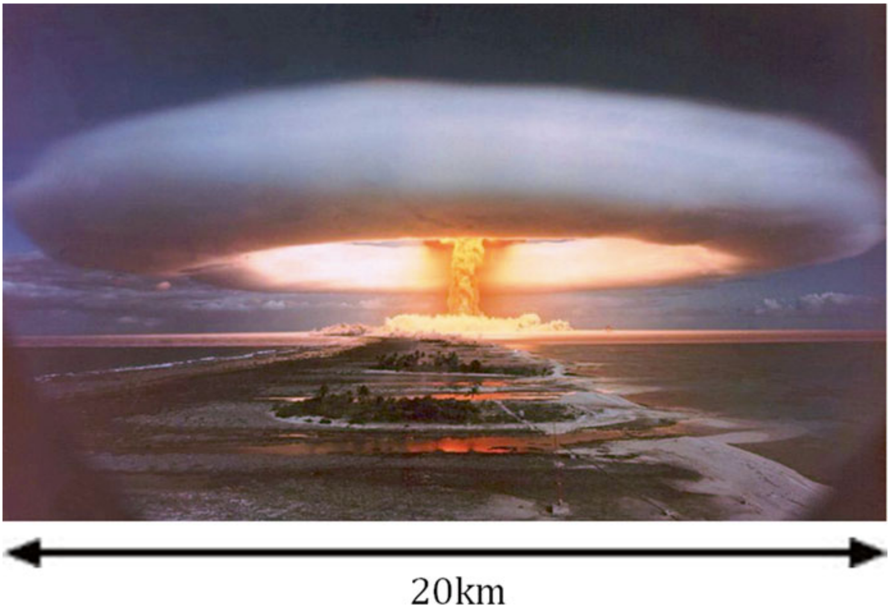

Fig. 1 Twenty megatons from your coffee cup (DoE public domain)

your computer, right? Big, important numbers. Just 4E9. And Petaflops, which you hear about all the time in the news. To conclude this bit, the energy put out by converting a kilogram of mass into pure energy, 9E16 J, is a bit more than 21 million tons of TNT, 21 MT (Fig. 1). *Now*, you can get a sense for that amount of energy!

To multiply big numbers, just add the exponents! 100 times 100 is 1E2 × 1E2, and that's 1E4 or ten thousand. You know that. A billion (1E9) is a thousand million (1E3 × 1E6). In the sixth grade, I wasted whole afternoons trying to multiply big numbers using arithmetic, on paper (we did that back then!). You can do it, but it takes a long time to get the answer to the area of the Earth (Pi times d^2 is 3.14159265 times 12756.328 kilometers times 12756.328 kilometers), and that's a lot more accuracy then you need. Instead, you just multiply the first few numbers out in front and add the exponents. The Earth's diameter is 6.38E3 km within 1 % accuracy, which is good enough for most purposes. By the way, each time you multiply two numbers that are inaccurate, the resulting inaccuracy is a little larger, so the area of the Earth is $3.14 \times (1.27)^2 \times$ 1E8, or 5.11E8 km^2 to within about 2 %. That's 500 million square kilometers. Or, 5.11E14 square meters, because there are a million of those in each km^2. By the way, there are ten thousand (1E4) square meters in a hectare, so the Earth has a bit over 50 billion hectares on it, right? And, only a fraction of those hectares can be plowed.

Now, look at Table 2. We also have names for little numbers as well as big ones. Not so hard after all, huh? *Just another bookkeeping system*. Don't ask *me* why the popular names for these go the way they do. They seem to count groups of three

Table 2 Abbreviations for powers of ten

Prefix	Prefix	Power of ten (± number of zeros)	Short-hand	Popular number names
-----	---	33	1E33	Decillion
Watta	W	30	1E30	Nondecillion
Xenna	X	27	1E27	Octillion
Yotta	Y	24	1E24	Septillion
Zetta	Z	21	1E21	Sextillion
Exa	E	18	1E18	Quintillion
Peta	P	15	1E15	Quadrillion
Tera	T	12	1E12	Trillion
Giga	G	9	1E9	Billion
Mega	M	6	1E6	Million
Kilo	k	3	1E3	Thousand
Centi	c	−2	1E-2	A hundredth
Milli	m	−3	1E-3	A thousandth
Micro	μ	−6	1E-6	A millionth
Nano	n	−9	1E-9	A billionth
Pico	p	−12	1E-12	A trillionth
Femto	f	−15	1E-15	A millionth of a billionth
Atto	a	−18	1E-18	A billionth of a billionth
Zepto	z	−21	1E-21	A billionth of a trillionth
Yocto	Y	−24	1E-24	A trillionth of a trillionth
Hella	h	−27	1E-27	A trillionth of a quadrillionth

Plus power means 1 followed by that many zeroes. Minus power means a fraction with 1 on top and 1 followed with that many zeroes on the bottom. The convention is that plus powers are capitalized and negative powers not

zeros, but missing one, right? I mean "bi"llion ought to be 1 followed by two groups of 000's, "tri"llion, 1 with three 000's, "qunt"illion, 1 with five 000's, etc. right? They all seem to be off by one group of 000's. Go figure.

Of course, a good scientific calculator can do all this for you, but I want you to *understand* these things without a calculator, really understand what the answer you get means, and be able to estimate things with just a pencil and the "back of an envelope," or even your mental blackboard which I hope you can develop and exercise while reading this chapter.

People haven't *named* anything smaller or bigger than what's in Table 2, yet. It already covers 60 powers of ten! Of course, there are bigger and smaller numbers. Why would anybody want to measure something as small as 1E-15? Femtosecond lasers put out a pulse that is that brief. During 1 fs, light travels 1E-15 × 3E8 = 3E-7 m, or 0.3 μm (0.3E-6), a wavelength of ultraviolet light. Attosecond lasers are being worked on. Can you imagine that? A hydrogen atom electron takes 150 attoseconds to go around its nucleus, so you can see that, with a few-attosecond pulse, you can take a flash photograph of a chemical reaction happening, freezing the electrons with their pants down, so to speak (electrons do the reacting).

Why worry about an Exasecond? The age of the Universe is 0.44 Es. The mass of the Earth is about 6 Xennagrams, or Xg. What could possibly be interesting about a hellagram (hg)? An electron weighs 0.9 hg. The diameter of our Milky Way Galaxy is about 0.9 Zettameters (Zm). The nearest star is about 4 ly (light years) away, and our Galaxy is about 1E5 ly across and 2,000 ly thick. If every star were spaced just like the nearest one is to us, there would be about $\pi/4*(1E5)^2 \times 1E3/4^3 = 2.4E11$ (240 billion) stars in our Galaxy.

Light year? That's just the *distance* (not a time) light can go in a year, about 9.46 Petameters (Pm) (Fig. 2). Sounds funny, right? You can do this stuff! It isn't hard to do even astronomical calculations!

Fig. 2 Petameter (F. Wicke)

Mental Blackboard

Does this sound hard? It isn't, and it's very useful.

Here's a meditation! Go inside somewhere it's not too bright and close your eyes. Imagine your hand out there with a big bright, fluorescent marker and write "12." Write another "12" under that and add them. Don't open your eyes! Can you see 'em? Can you get "24" in your mind's eye below the line? Now erase this and write "12" with a "3" under it and multiply. Can you get "36"? Now, something more complicated: Add "1234" and "1111." Can you do that? Meditate until it comes into focus. Ah, there! Keep at it until it's easy in odd moments this week. People will just think you're meditating or spaced out. In 2 weeks, I want you to be able to multiply 123 by 3.14 and compute $(123)^2$! You can do it. When you're expert at that, you can figure the area of a circle in your head!

My message to you: Don't be afraid of numbers, even if they're very big or very small!! They're your friends and they'll help you a lot. It's far too popular these days to laugh at geeks and think people who are good at mathematics are strange. Here's a whole new country for you to explore, and it's much easier than you've been led to think. All you need to be able to do is multiply a few small numbers, and add.

Read this chapter again whenever you're having trouble with numbers. Rather than dragging you through a whole bunch of boring exercises, I'd just like you to understand everything in it!

http://www.springer.com/978-3-319-21680-5

No Wonder You Wonder!
Great Inventions and Scientific Mysteries
Phipps, C.
2016, XVIII, 331 p. 289 illus., 233 illus. in color.,
ISBN: 978-3-319-21680-5