

Chapter 2

Coefficient of Restitution in Badminton Racket

Abstract This chapter explains in greater detail on the development of generating the finite element model for badminton racket using ABAQUS (Explicit). Among the details involved material selections, measurement and important boundary conditions of the finite element models were presented. Several relevant theories behind the analysis of the badminton racket's performance, such as the equation of coefficient of restitution, COR in badminton racket were explained. Other than that, the important information about the experimental procedures and apparatus setup were explained in detail. The experimental analysis is crucial since the results will be used to validate the finite element model.

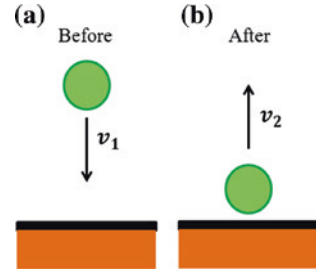
Keywords Coefficient of Restitution • Sweet Spot • Finite Element Analysis

2.1 Coefficient of Restitution (COR)

The theory of coefficient of restitution (COR) was commonly used to analyse the performance of racket based sports such as badminton, tennis, squash and table tennis. In theory, the coefficient of restitution (COR) was a measurement of the energy loss between collisions of two bodies, whereas COR was ranged at $0 \leq \text{COR} \leq 1$ [1–3]. In ideal cases, $\text{COR} = 0$ was referred to as perfectly inelastic collision while $\text{COR} = 1$ was for perfectly elastic collision. In the collision between ball and racket, COR was defined as the ratio of ball rebound velocity, v_2 to the ball incident velocity, v_1 . The COR can also be determined by using the equation

$$\text{COR} = \frac{v_2}{v_1} = \sqrt{\frac{h_r}{h_d}} \quad (2.1)$$

Fig. 2.1 Illustration of collision between a ball and a surface complying with COR theory, **a** incident velocity, v_1 , **b** rebound velocity, v_2



where,

v_1 is the incident velocity,

v_2 is the rebound velocity,

h_r is the ball rebound height, and

h_d is the ball drop height.

The common procedure in obtaining COR was by dropping a ball onto the racket string-bed with the racket handle or racket head clamped [1, 3, 4]. Normally, rackets that produce higher COR value is categorised as a good racket. Thus, the value of COR is used to measure the potential performance for specific racket. Figure 2.1 shows the illustration collision between a ball and a surface complying with the COR theory.

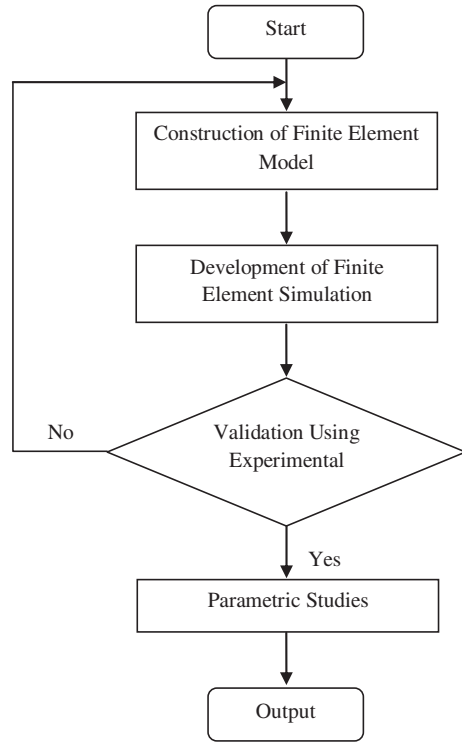
2.2 Finite Element Model

In this study, finite element simulation was developed as a tool to analyse the racket design parameter towards racket performance. Basically, the finite element model for this simulation consists of three major components; the racket frame model, the string-bed model and the rubber ball model. Each 3D model was created in SolidWorks® 2012 and then imported to ABAQUS Explicit for the employment of finite element analysis. Then, results from this simulation were compared with experimental results for validation processes. The complete methodology for the development of finite element analysis on the badminton racket is summarized in Fig. 2.2.

2.2.1 Racket Frame Model

The main finite element model of the badminton racket frame was developed based on an isometric head shaped racket design. The overall dimensions of the isometric head frame are 197 mm in width, w and 249 mm in length, l . This racket frame model has the total overall length, l_o of 516 mm. This model also consists of

Fig. 2.2 Research framework of finite element simulation of badminton racket



the shaft cross-sectional diameter, d_o of 7 mm and shaft cross-sectional thickness, t of 1 mm. Figure 2.3 shows the schematic diagram of the badminton racket frame model.

The finite element model of the badminton racket was assigned with linear elastic material to enable the deformation of the racket frame during the simulation [5]. The material used for the racket frame model was carbon fiber material which had Young's modulus, E of 25,000 MPa, density, ρ of 1750 kg/m³, and Poisson's ratio, ν of 0.3.

2.2.2 String-Bed Model

The string-bed model was inter-woven to replicate the actual string-bed design. Figure 2.4 shows the schematic diagram of the isometric string-bed model. The isometric string-bed model had the overall dimensions of 239 mm length, l and 187 mm width, w with 22 main (longitudinal axis) and 22 cross strings.

The diameter of cross-section area of the string, ϕ was set at 0.66 mm. The material used for the string was made of nylon having a Young's modulus, E of 7200 MPa, density, ρ of 1100 kg/m³ and the Poisson's ratio, ν of 0.3 [6]. The

Fig. 2.3 Schematic diagram of badminton racket frame model

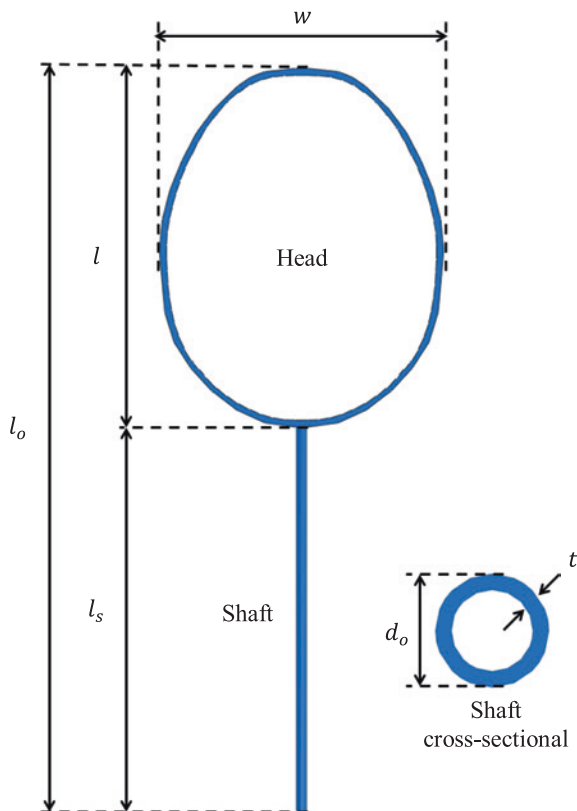
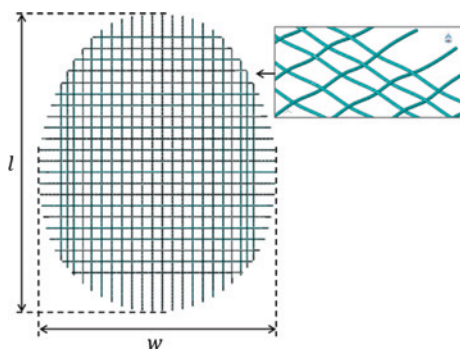


Fig. 2.4 Schematic diagram of inter-woven string-bed model of a badminton racket



interaction between string in the string—bed area was defined as surface-to-surface contact (explicit) with a friction coefficient of 0.4, while the interaction of rubber ball and the string-bed was defined as surface-to-surface contact with a friction coefficient of 0.1 [6]. Since there were limited studies done on the badminton racket, the friction coefficient of 0.4 and 0.1 for the string and string-bed and also rubber ball and the string-bed respectively, were selected as the

reference obtained from previous studies on tennis rackets. These coefficients were used merely to set a benchmark in designing the finite element analysis of the badminton racket. The latter may be expanded and further investigated once the fundamentals of the finite element model for the badminton racket have been clearly established. The interaction of surface-to-surface contact was set based on ABAQUS Explicit's requirement to prevent any unintended penetration between string-to-string and string-to-ball. To connect the string-bed with the racket head, the end of each string was applied using tie constraint with outer surface of the racket head frame.

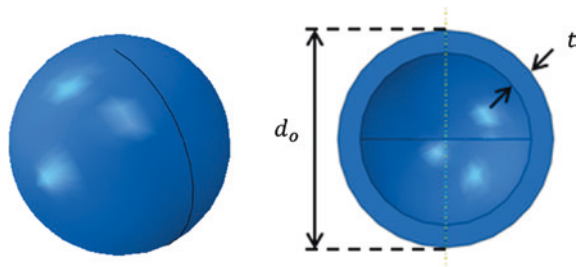
The tension of the string is another important boundary condition that should be considered in order to simulate the actual string-bed condition. In this finite element study, the most critical boundary condition is to ensure the string-bed behaves such in a constant initial tension condition. Theoretically, if the string was applied with tensional force at the end of the string, internal normal stress would be present. Therefore, in order to replicate the behaviour of string tension, another alternative method was applied using normal stresses on the string-bed. This can be done by applying stress in INITIAL CONDITION using Edit Keywords function in ABAQUS Explicit [5]. By applying this technique, the string-bed would be constantly in tension condition.

2.2.3 Rubber Ball Model

In this study, the rubber ball has been chosen to replace the shuttlecock for the simulation impact on the badminton racket's string-bed. The shuttlecock has several limitations that would greatly affect the COR result in the finite element simulation. Basically, the cork section is heavier compared to the feather section, thus, the difference of the weight distribution of the shuttlecock makes it less stable and more inclined to flip during the collision. As a result, inconsistent values of rebound velocity will be obtained and affect the COR performance. Due to the geometry issue of the shuttlecock, the rubber ball has been utilised in order to obtain precise and consistent results of COR.

The dimension of the simulated rubber ball model is based on the real-life rubber ball dimension used in the experiment whereby the outer diameter, d_o of this model was set at 43.5 mm and thickness, t of 4.5 mm (Fig. 2.5). The rubber ball

Fig. 2.5 Schematic diagram of rubber ball model



model consists of Mooney-Rivlin hyper elastic material model with constants $C_{10} = 0.69$ MPa, $C_{01} = 0.173$ MPa and $D_1 = 0.0145$ MPa⁻¹ [5]. The density, ρ of the rubber ball was set to be 1068 kg/m³.

2.3 Finite Element Simulation

This finite element simulation mimics the collision between a rubber ball and a badminton racket. The simulation was performed using an isometric head shaped racket model with the total overall racket length, l_o of 516 mm, shaft cross-sectional diameter, d_o of 7 mm and shaft cross-sectional thickness, t of 1 mm. The head of this racket model was anchored as a boundary condition for this simulation as shown in Fig. 2.6 to prevent any uncalled head movement during collision.

The simulation was carried out with varying string tension from 14, 20, 24, 28 and 34 lbs. The rubber ball was dropped with initial height, h_d of 10 mm under the gravitational acceleration, g of 9.81 m/s² onto the COM of the racket string-bed. Then, the maximum rebound height, h_r of the ball was analysed to obtain the values of COR. Later, the results from this simulation were compared with experimental results for validation analysis.

2.4 Experimental Procedures

The main purpose of conducting this experiment was to validate the finite element simulation of the badminton racket. The experiment was performed by colliding the rubber ball with the badminton racket. The racket head was clamped in order to preserve the integrity of the shaft's structural stiffness and thus, preventing any involuntary effect as a result from the collision on the COR performance. Therefore, the performance of COR was only affected by string parameters. The racket head frame was assumed to behave as a rigid body condition where there was no deformation of the said frame during the impact.

Then, the rubber ball was freely dropped from a 10 mm height, h_d onto the centre of mass (COM) of the string-bed [7]. The COM of the string-bed is the softest

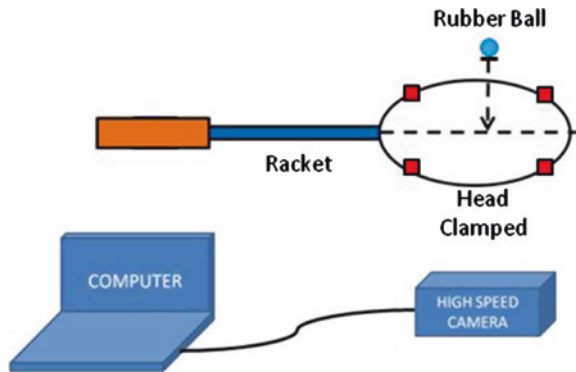
Fig. 2.6 Badminton racket frame model in anchored boundary condition



Fig. 2.7 Phantom V12.1 high speed camera



Fig. 2.8 Illustration of the experimental apparatus setup



area on the racket string-bed whereby it produces the maximum value of COR [4]. The location of COM of the string-bed was obtained based on the measurement from a 3D model of an isometric racket in SolidWorks[®]. This experimental procedure was repeated 10 times to get the average values of ball rebound height, h_r . The rebound heights, h_r of each trial were recorded using a Phantom V12.1 high speed camera (HSC) with frame rates of 1000 fps (Fig. 2.7) [6, 8]. Figure 2.8 shows illustration of the apparatus setup for this experiment while Fig. 2.9 shows the picture of the experiment setup. Later, the data obtained were analysed using Phantom Camera Control Version: 9.3.692.0-C PhCon:692 software in order to determine the maximum values of ball rebound height, h_r .

2.5 Validation

The validation of finite element model is very essential and necessary in order to ensure the reliability and accuracy of the finite element simulation results. Theoretically, the finite element model is reliable if the simulation results have

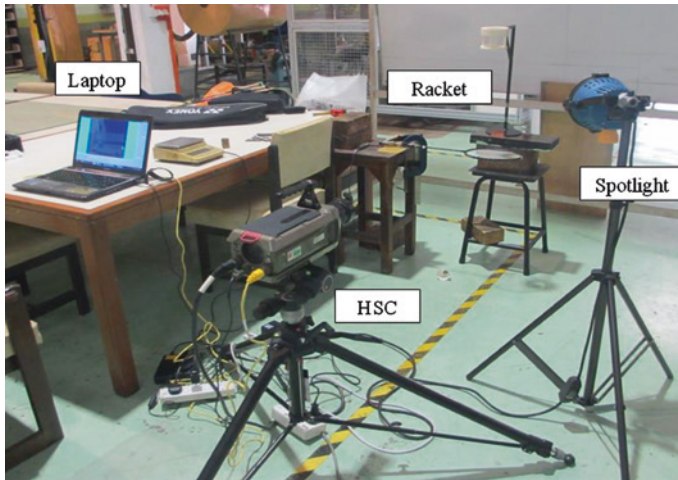


Fig. 2.9 Picture of the experimental apparatus setup

good correlation with the experimental results. Since the accuracy and reliability of the simulation results are important, therefore, mesh convergent study was conducted to determine the sufficient number of elements for this simulation.

2.5.1 Mesh Convergence Study

The convergence study was done to determine the appropriate number of elements that could compromise between the accuracy of the results and the time consumed in finite element analysis. Figure 2.10 shows the COR results of the collision between the rubber ball and the centre of the racket string-bed in various numbers of elements. It was observed that the COR was converged on the increasing number of elements from 7985 elements until 217210 elements. Referring to Fig. 2.10, the suitable number of elements that could be used in this simulation was in the range of 7985–217210 elements, where the results observed between these elements were slightly constant.

One of the criteria that should be considered in convergence study is the simulation time. The computer utilised in this simulation study was equipped with Intel® Xeon® 2.30 GHz processor and an installed memory, RAM of 16 GB. With this computer capacity performance, the simulation using 217210 elements takes about 136 h to complete and it was the longest time taken to finish the simulation. Meanwhile, the simulation using 7985 elements took about 65 h to complete. Basically, the higher number of elements took a longer duration to complete the simulation compared to the lower number of elements.

Fig. 2.10 Mesh convergence results of finite element simulation

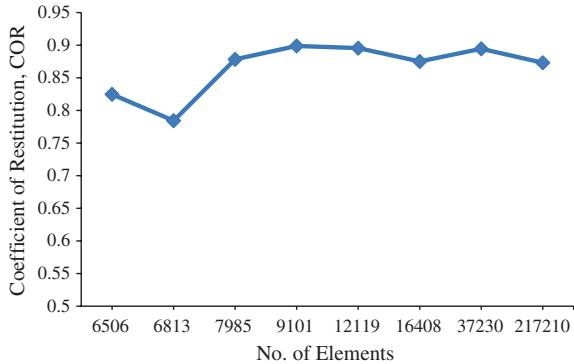


Table 2.1 Results accuracy and time consumption of the finite element simulation

No. of elements	Accuracy (%)	Time (h)
6506	5.5	54
6813	10.2	59
7985	0.6	65
9101	2.3	69
12119	2.6	75
16408	0.2	82
37230	2.4	88
217210	0	136

Other than that, the accuracy of the data must also be considered in choosing the appropriate number of elements. Table 2.1 shows the accuracy, % and the simulation time consumed by each number of elements used in this finite element analysis. In order to obtain the accuracy, each COR was compared with the COR obtained from 217210 elements. Based on the results, the accuracy of data obtained using 7985 elements had a difference of only 0.6 % compared to the data using 217210 elements. Therefore, due to time constraint and computer performance limitations, the elements number of 7985 was considered as the most appropriate element to be selected in this simulation study.

2.5.2 Validation of Finite Element Simulation

Figure 2.11 shows the comparison between COR obtained from the experiment and simulation against various string tensions. Referring to Fig. 2.11, both results from experiment and simulation showed that the values of COR slightly decreased as the string tension increased.

In comparison between the results obtained from the experiment and simulation, the recorded overall percentage error was less than 7 % whereas the

Fig. 2.11 The COR from the experiment and the simulation against various string tensions

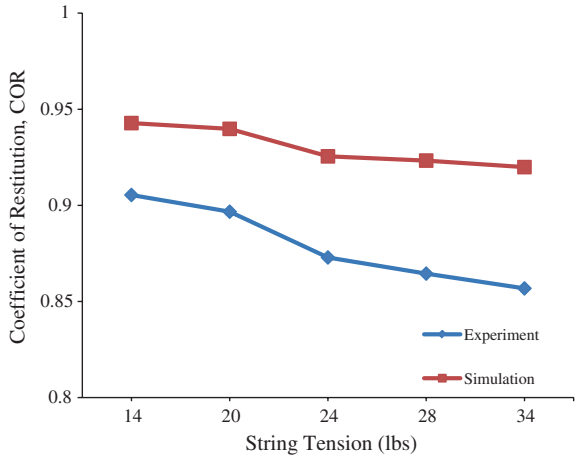


Table 2.2 The COR percentage error of the simulation

String tension (lbs)	Experiment	Simulation	Error (%)
14	0.9054	0.9428	3.97
20	0.8967	0.9398	4.59
24	0.8729	0.9255	5.68
28	0.8645	0.9233	6.37
34	0.8568	0.9199	6.86

maximum error was 6.8 % at string tension of 34 lbs while the minimum error recorded was 3.9 % at string tension of 14 lbs. Table 2.2 enlists the percentage error of the simulation of various string tensions. Therefore, there was an agreement on COR between the experiment and simulation, since the error falls within the acceptable limit.

The sequence from the finite element model for an impact at the COM of the string-bed is shown in Fig. 2.12. During the pre-impact phase, the ball started to fall onto the COM of the string-bed at a height of 10 mm under gravitational acceleration, g of 9.81 m/s^2 . Then, during the impact phase, the string-bed absorbs the energy from the ball and causes the deformation of the string. Finally during the post-impact phase, ball has left the string-bed and stops until reaches the maximum rebound height. The maximum rebound heights were recorded to calculate the COR.

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Finite Element Analysis on Badminton Racket Design
Parameters

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