Chapter 2
Problem Definition, Dynamical Model Formulation

Abstract This chapter presents the problem definition and dynamical model formulation of an oscillatory-base manipulator considering an illustrative example, which we will work on throughout the monograph. We will consider three types of control problems, specifically attitude control in local coordinates (base-fixed coordinates) which is associated with on-board operations such as cargo handling, attitude control in global coordinates (earth-fixed coordinates), e.g., radar gimbal systems, and position control in global coordinates, e.g., heave-motion-compensated cranes. Further, as patterns of base oscillation, we consider three patterns, single-frequency sinusoidal oscillation, double-frequency sinusoidal one, and ocean-wave imitated oscillation based on the Bretschneider spectrum. Using combinations of those cases, we will demonstrate control system design and analysis, control simulations and experiments.

2.1 Introduction

First, this chapter defines motion control problems of an OBM which we will work on throughout this monograph, by choosing an illustrative model and setting some important assumptions. Then, the problems are categorized into two classes, local-coordinate problems and global-coordinate ones, depending on the coordinate frame to be referred by the control system. We consider three types of control problems. Subsequently, being based on the problem definition, the corresponding dynamical model is derived. Further, we introduce three patterns of base oscillation which we will use for various demonstrations together with the control problems.

2.2 Problem Definition

As an illustrative model of OBMs, we consider a two-DOF manipulator with a one-DOF oscillatory base and a payload as depicted in Fig. 2.1, and motions of which are restricted on the vertical plane. The reason that this simple model has been

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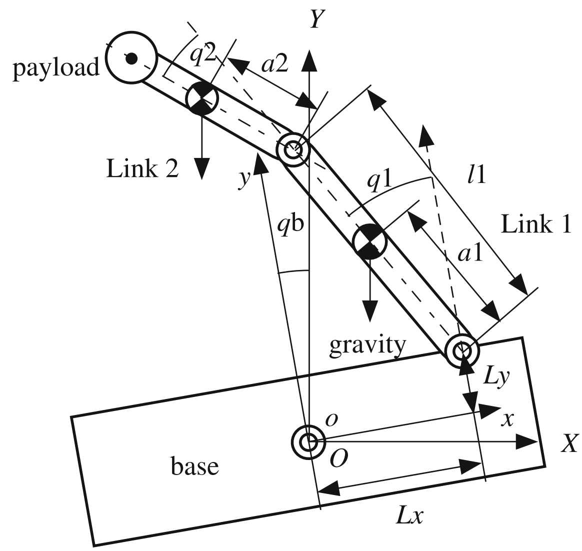

Fig. 2.1 Schematic diagram of the illustrative model of OBMs (a two-DOF manipulator with a one-DOF oscillatory base and a payload)

employed is because, in practical cases of offshore mechanical systems, roll and heave (vertical) oscillations among six-DOF ones are manifest and critical. It should be noted that the roll motion of the model with respect to the origin is one-DOF rotating one, however, with respect to another point, e.g., the root of the manipulator, involves heave and sway motions as well as roll ones. Further, from the viewpoint of control, forces parallel and torques orthogonal to the rotating axes of manipulator joints do not affect the manipulator dynamics. Therefore, this illustrative model is simple for exposition, but is realistic enough. Further, notice that a payload is attached to the tip of the second link in Fig. 2.1, which is assumed to have uncertainty in its physical parameters, e.g., mass and inertia moment, in order to analyze robust control problems.

Here the following important assumptions are made in addressing motion control problems of OBMs;

A1 the frequency range of the base motion is known in advance;
A2 the forces and torques exerted on the base by the manipulator are negligible to the base motion;
A3 the actuator dynamics is negligible;
A4 the payload has uncertainty in its physical parameters, e.g., mass and inertia moment;
A5 except the payload, the physical parameters of the manipulator are known;
A6 the joint angles and velocities of the manipulator can be measured; and
A7 the base oscillation angle can be measured.

A1 is natural and is not restrictive considering the ocean environment and, further, is very important to make our proposed $\mathcal{H}_\infty$-control-based technique highly effective.

In fact it has been reported that the frequencies of ocean surface waves are within the range 1/30–1 Hz [9]. A2, which supports A1 together with the facts in oceanography and also makes the control problem simple, shall be discussed later in detail and be clarified not to be restrictive, but to be reasonable. A3 is only for simplicity. A4 is intended to analyze robust control problems, and is often the case for offshore mechanical systems such as a crane handling mineral and fishery resources. A5 and A6 are necessary for control systems we will demonstrate and standard conditions in terms of motion control. These assumptions are taken into account in Chaps. 2–6, and Chap. 7 which presents an estimation method of the base oscillation aims at implementing A7.

Taking into account all the above conditions, now the motion control problem of OBMs to be dealt with can be stated as "*under the assumptions A1–A7, synthesize an controller that achieves successful motion control of the manipulator in the presence of disturbance due to the base oscillation and model uncertainties in the payload physical parameters*."

2.3 Local-Coordinate and Global-Coordinate Problems

See that in Fig. 2.1 two different coordinate frames are set; OXY is an inertia frame whose Y–axis is parallel to the direction of gravity and oxy is a frame attached to the base whose origin o is fixed at O. Motion control problems of OBMs can be typically categorized into two classes depending on the coordinate frame to be referred by the control system. One class is of the case where the control system performs being based on a base-fixed coordinate frame, i.e., a local-coordinate frame (as oxy in Fig. 2.1), which is called the class of *local-coordinate problems*. The other class contains *global-coordinate problems* where an earth-fixed coordinate frame, i.e., an inertia frame (as OXY in Fig. 2.1), is referred. The local-coordinate problems are associated with practical applications with base-fixed task spaces, such as a ship-mounted crane performing load/unload operations of cargo on the ship. On the other hand, an on-board radar gimbals and a ship-mounted crane performing on land fall into the class of global-coordinate problems.

The common feature between both the problems in global and local coordinates is the disturbances due to the base oscillation. On the contrary, the difference between them is in that in global coordinates the desired trajectory of motion must be generated according to the base motion, while it is not the case in local coordinates where the reference signals do not contain information on the base oscillation and further there is no need to measure the base motion as shown later.

Moreover, regardless of coordinate frames and in general, motion control problems are divided into *position control* ones and *attitude control* ones. In the case of OBMs, "position control" implies that the position of the payload is to be controlled, while "attitude control" does controlling the attitudes of the links. Hence, by combining the coordinate frame classes with these motion control types, four patterns of motion control problems of OBMs can be considered as depicted Figs. 2.2, 2.3, 2.4

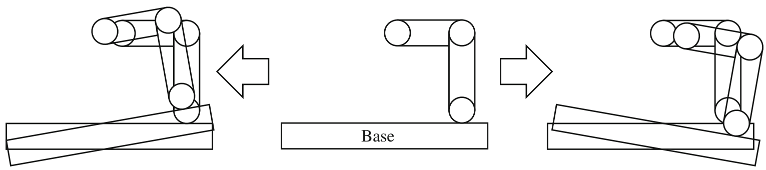

Fig. 2.2 Position control in a base-fixed coordinate system (local case)

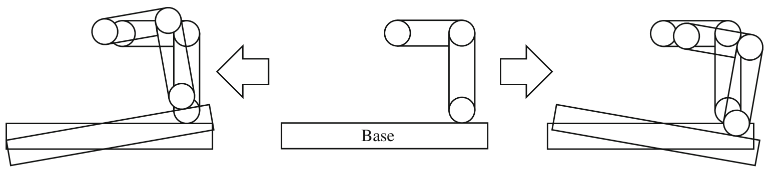

Fig. 2.3 Attitude control in a base-fixed coordinate system (local case)

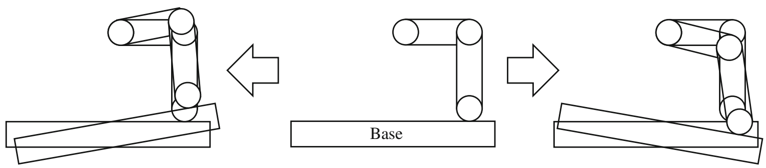

Fig. 2.4 Position control in a global-coordinate system

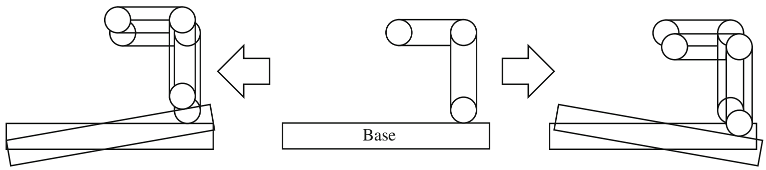

Fig. 2.5 Attitude control in a global-coordinate system

and 2.5. As seen from the figures, notice that, in local-coordinate problems, position control, and attitude control are equivalent as long as the redundancy and/or multiplicity of solution of the joint space mapped to the given task space can be ignored, whereas it is not the case for global-coordinate problems.

2.4 Dynamical Model Formulation

2.4.1 Dynamical Model of an OBM

The dynamics of n-rigid link manipulators with revolutionary joints subject to m-DOF base oscillation can be described by the standard formulation for manipulator dynamics plus the disturbance due to the base oscillation as follows:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + D\dot{\boldsymbol{q}} + G(\boldsymbol{q}, \boldsymbol{q}_\mathrm{b}) + H(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_\mathrm{b}, \ddot{\boldsymbol{q}}_\mathrm{b}) = \boldsymbol{\tau}, \tag{2.1}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ and $\boldsymbol{q}_\mathrm{b} \in \mathbb{R}^m$ denote the position vectors of the manipulator links and the base, respectively, and $\boldsymbol{\tau}$ is the input torque vector; $M(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the inertia matrix of the manipulator, which is a symmetric positive definite matrix; $C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in \mathbb{R}^n$ represents the centripetal and Coriolis torque depending only on the states of the manipulator; $D \in \mathbb{R}^{n\times n}$ is the damping coefficient matrix of the manipulator joints, which is a positive definite constant diagonal matrix; $G(\boldsymbol{q}, \boldsymbol{q}_\mathrm{b}) \in \mathbb{R}^n$ is the gravitational torque; $H(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_\mathrm{b}, \ddot{\boldsymbol{q}}_\mathrm{b}) \in \mathbb{R}^n$ represents the inertia torque and the centripetal and Coriolis torque due to the base oscillation. $G(\boldsymbol{q}, \boldsymbol{q}_\mathrm{b})$ and $H(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_\mathrm{b}, \ddot{\boldsymbol{q}}_\mathrm{b})$ form the disturbance prescribed, which are nonlinearly coupled with both the states of the manipulator and the base.

Now, we derive the specific formulation of (2.1) for the illustrative model in Fig. 2.1 by applying Lagrangian mechanics. The notations used here are as follows.

q_b, q_1, q_2	Position angles of the base and the links as defined in Fig. 2.1
J_1, J_2	Inertia moment of each link with respect to the centroid
m_1, m_2	Mass of each link
D_1, D_2	Damping coefficient of each joint
L_x, L_y, l_1, a_1, a_2	Geometric parameters as defined in Fig. 2.1
τ_1, τ_2	Control torque applied to each joint
g	Gravitational acceleration
t	Time variable

Then, each term in the model (2.1) can be explicitly represented by (2.2)–(2.6) and the base oscillation is described by (2.7); $(\cdot)^T$ denotes the transpose.

$$\begin{aligned}
M(\boldsymbol{q}) &= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \\
M_{11} &= m_1 a_1^2 + m_2(a_2^2 + l_1^2 + 2a_2 l_1 \cos(q_2)) + J_1 + J_2 \\
M_{12} &= M_{21} = m_2(a_2^2 + a_2 l_1 \cos(q_2)) + J_2 \\
M_{22} &= m_2 a_2^2 + J_2,
\end{aligned} \tag{2.2}$$

$$C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = \begin{bmatrix} -m_2 a_2 l_1 \sin(q_2)\dot{q}_2(2\dot{q}_1 + \dot{q}_2) \\ m_2 a_2 l_1 \sin(q_2)\dot{q}_1^2 \end{bmatrix}, \tag{2.3}$$

$$D = \mathrm{diag}[D_1,\ D_2], \tag{2.4}$$

$$\begin{aligned}
G(\boldsymbol{q}, \boldsymbol{q}_\mathrm{b}) &= [G_1, G_2]^T \\
G_1 &= -\{m_1 a_1 \sin(q_b + q_1) + m_2(l_1 \sin(q_b + q_1) \\
&\quad + a_2 \sin(q_\mathrm{b} + q_1 + q_2))\}g \\
G_2 &= -m_2 a_2 \sin(q_\mathrm{b} + q_1 + q_2)g,
\end{aligned} \tag{2.5}$$

$$
\begin{aligned}
H(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_{\mathrm{b}}, \ddot{\boldsymbol{q}}_{\mathrm{b}}) &= [H_1, H_2]^T \\
H_1 &= \{m_1(a_1{}^2 - a_1 L_x \sin(q_1) + a_1 L_y \cos(q_1)) \\
&\quad + m_2(a_2^2 + l_1{}^2 - l_1 L_x \sin(q_1) + l_1 L_y \cos(q_1) \\
&\quad - a_2 L_x \sin(q_1 + q_2) + a_2 L_y \cos(q_1 + q_2) \\
&\quad + 2 l_1 a_2 \cos(q_2)) + J_1 + J_2\}\ddot{q}_{\mathrm{b}} \\
&\quad + \{m_1(a_1 L_x \cos(q_1) + a_1 L_y \sin(q_1)) \\
&\quad + m_2(l_1 L_x \cos q_1 + l_1 L_y \sin q_1 \\
&\quad + a_2 L_x \cos(q_1 + q_2) + a_2 L_y \sin(q_1 + q_2))\}\dot{q}_{\mathrm{b}}^2 \\
&\quad - 2 m_2 a_2 l_1 \sin(q_2)\dot{q}_{\mathrm{b}}\dot{q}_2 \\
H_2 &= \{m_2(a_2^2 + a_2 l_1 \cos(q_2) - a_2 L_x \sin(q_1 + q_2) \\
&\quad + a_2 L_y \cos(q_1 + q_2)) + J_2\}\ddot{q}_{\mathrm{b}} \\
&\quad + m_2(a_2 l_1 \sin(q_2) + a_2 L_x \cos(q_1 + q_2) \\
&\quad + a_2 L_y \sin(q_1 + q_2))\dot{q}_{\mathrm{b}}^2 \\
&\quad + 2 m_2 a_2 l_1 \sin(q_2)\dot{q}_{\mathrm{b}}\dot{q}_1 . \qquad (2.6)\\
q_{\mathrm{b}} &= \Sigma_{i=1}^{n_\omega} A_{\omega i} \sin(\omega_i t + \phi_i) \qquad (2.7)
\end{aligned}
$$

As in (2.7), the base oscillation is modeled as a linear combination of multiple sinusoidal motions. Equation (2.6) indicates that the manipulator is strongly influenced by the base oscillation and that the larger amplitude and angular frequency of the base will induce the larger $H(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_{\mathrm{b}}, \ddot{\boldsymbol{q}}_{\mathrm{b}})$. Hence, to achieve desirable motion control, how to overcome $H(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_{\mathrm{b}}, \ddot{\boldsymbol{q}}_{\mathrm{b}})$ is the central issue.

2.4.2 Base Oscillation Model

In this monograph, we have chosen three types of base oscillations in (2.7) for demonstrations of control design and control system evaluation. The first one is a single-frequency one with $\omega_1 = 2\pi$ (rad/s), $A_{\omega 1} = 10^{(\circ)}$, and $\phi_1 = 0$ (rad), and the second one is a double-frequency one with $\omega_1 = \pi$, $\omega_2 = 2\pi$ (rad/s), $A_{\omega 1} = A_{\omega 2} = 5^{(\circ)}$, and $\phi_1 = \phi_2 = 0$ (rad). Furthermore, for more realistic demonstrations, continuously distributed frequencies imitating an ocean-wave spectrum are also considered for the third oscillation model. As in [15], we have employed *the two-parameter Bretschneider spectrum* [22], and by approximating the spectrum each component of base motion in (2.7) is generated as in the following.

$$
\omega_i = \omega_{min} + \frac{\omega_{max} - \omega_{min}}{n_\omega - 1}(i - 1) \quad (i = 1, 2, \cdots, n_\omega) \qquad (2.8)
$$

$$
S_{\omega i} = \frac{1.25}{4}\frac{\omega_0^4}{\omega_i^5} A_s^2 e^{-1.25(\omega_0/\omega_i)^4} \qquad (2.9)
$$

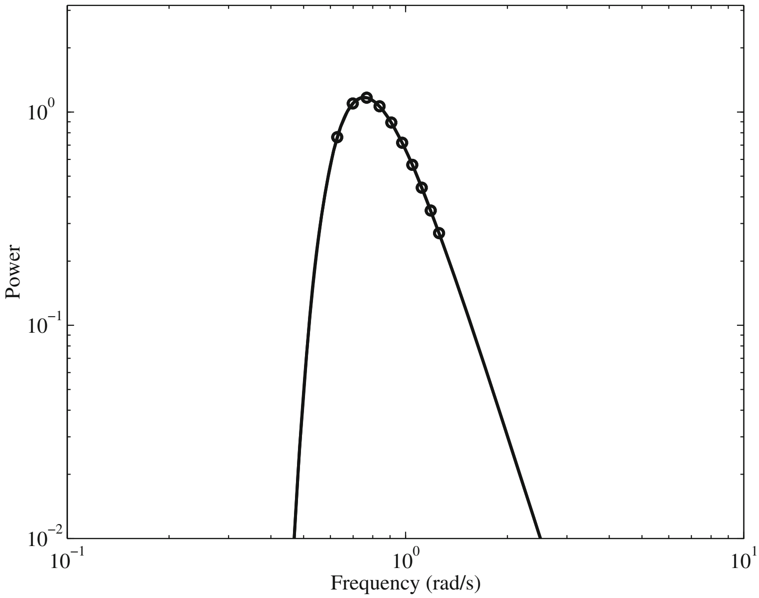

Fig. 2.6 Power spectrum of the Bretschneider model

$$A_{\omega i} = \frac{\omega_i^2}{9.8}\sqrt{2S_{\omega i}\frac{\omega_{max} - \omega_{min}}{n_\omega - 1}} \tag{2.10}$$

where ω_0, ω_{min}, and ω_{max} are the modal, minimum, and maximum frequencies, respectively, and A_s is the significant roll amplitude. Given these parameters, the spectral density $S_{\omega i}$ and amplitude A_i can be obtained. The phase ϕ_i is given as a random number between 0 and 2π with the uniform distribution.

To generate a base rolling motion, those parameters are set as $\omega_0 = 0.24\pi$, $\omega_{min} = 0.2\pi$, $\omega_{max} = 0.4\pi$ (rad/s), $n_\omega = 10$, $A_s = \pi$ (rad). With those parameters, the power spectrum is displayed in Fig. 2.6, where the circles represent the employed frequencies. Moreover, since our experimental apparatus is a small-scale model, taking into account the scale effect, (2.7) is modified as

$$q_{\mathrm{b}}(t) = \Sigma_{i=1}^{n_\omega} A_{\omega i} \sin(5\omega_i t + \phi_i), \tag{2.11}$$

i.e., the frequencies are magnified by 5 times so as to make the disturbance affect enough. We refer to the oscillation as *the Bretschneider oscillation* hereafter.

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