Chapter 2
Mathematics and Temperature

Mathematical analysis has therefore necessary relations with sensible phenomena; its object is not created by human intelligence; it is a pre-existent element of the universal order, and is not in any way contingent or fortuitous.

—Joseph Fourier

2.1 Introduction

In the previous chapter, we looked at the preliminary discourse of Fourier's *Analytical Theory of Heat*. Herein, he provided an overview of the types of (previously insoluble) problems his theory would address, and how his theory would be formulated: in terms of differential equations. These differential equations govern the flow of heat—and hence the temperature distribution—throughout bodies subjected to sources of heat. In order to arrive at a solution to these differential equations for a given body, one must have knowledge of certain specific qualities of the body. In particular, one must know the body's (i) *specific heat* (its power to contain heat), (ii) *surface conductivity* (its power to receive or transmit heat across its surface), and (iii) *thermal conductivity* (its power to conduct heat through the interior of its mass).Once these qualities are known—along with the thermal conditions existing at the surface of the body—then finding the temperature distribution within the body is reduced to the mathematical process of solving a differential equation given certain boundary conditions. This is not to say it is easy: Fourier would have to develop a new method—the series solution—to solve many such problems. But the technique which Fourier discovered provided a new way of addressing the problem of heat. In the reading selection that follows, Fourier begins to flesh out the mathematical methods outlined in the preliminary discourse. He begins by way of example—describing the distribution of temperature within bodies subjected to various sources of heat.

K. Kuehn, *A Student's Guide Through the Great Physics Texts*,
Undergraduate Lecture Notes in Physics, DOI 10.1007/978-3-319-21828-1_2

2.2 Reading: Fourier, *The Analytical Theory of Heat*

Fourier, J., *The Analytical Theory of Heat*, Cambridge University Press, London, 1878.

2.2.1 Statement of the Object of the Work

1. The effects of heat are subject to constant laws which cannot be discovered without the aid of mathematical analysis. The object of the theory which we are about to explain is to demonstrate these laws; it reduces all physical researches on the propagation of heat, to problems of the integral calculus whose elements are given by experiment. No subject has more extensive relations with the progress of industry and the natural sciences; for the action of heat is always present, it penetrates all bodies and spaces, it influences the processes of the arts, and occurs in all the phenomena of the universe. When heat is unequally distributed among the different parts of a solid mass, it tends to attain equilibrium, and passes slowly from the parts which are more heated to those which are less; and at the same time it is dissipated at the surface, and lost in the medium or in the void. The tendency to uniform distribution and the spontaneous emission which acts at the surface of bodies, change continually the temperature at their different points. The problem of the propagation of heat consists in determining what is the temperature at each point of a body at a given instant, supposing that the initial temperatures are known. The following examples will more clearly make known the nature of these problems.
2. If we expose to the continued and uniform action of a source of heat, the same part of a metallic ring, whose diameter is large, the molecules nearest to the source will be first heated, and, after a certain time, every point of the solid will have acquired very nearly the highest temperature which it can attain. This limit or greatest temperature is not the same at different points; it becomes less and less according as they become more distant from that point at which the source of heat is directly applied.
   When the temperatures have become permanent, the source of heat supplies, at each instant, a quantity of heat which exactly compensates for that which is dissipated at all the points of the external surface of the ring.
   If now the source be suppressed, heat will continue to be propagated in the interior of the solid, but that which is lost in the medium or the void, will no longer be compensated as formerly by the supply from the source, so that all the temperatures will vary and diminish incessantly until they have become equal to the temperatures of the surrounding medium.
3. Whilst the temperatures are permanent and the source remains, if at every point of the mean circumference of the ring an ordinate be raised perpendicular to the plane of the ring, whose length is proportional to the fixed temperature at

that point, the curved line which passes through the ends of these ordinates will represent the permanent state of the temperatures, and it is very easy to determine by analysis the nature of this line. It is to be remarked that the thickness of the ring is supposed to be sufficiently small for the temperature to be sensibly equal at all points of the same section perpendicular to the mean circumference. When the source is removed, the line which bounds the ordinates proportional to the temperatures at the different points will change its form continually. The problem consists in expressing, by one equation, the variable form of this curve, and in thus including in a single formula all the successive states of the solid.

4. Let z be the constant temperature at a point m of the mean circumference, x the distance of this point from the source, that is to say the length of the arc of the mean circumference, included between the point m and the point o which corresponds to the position of the source; z is the highest temperature which the point m can attain by virtue of the constant action of the source, and this permanent temperature z is a function $f(x)$ of the distance x. The first part of the problem consists in determining the function $f(x)$ which represents the permanent state of the solid.
   Consider next the variable state which succeeds to the former state as soon as the source has been removed; denote by t the time which has passed since the suppression of the source, and by v the value of the temperature at the point m after the time t. The quantity v will be a certain function $F(x, t)$ of the distance x and the time t; the object of the problem is to discover this function $F(x, t)$, of which we only know as yet that the initial value is $f(x)$, so that we ought to. have the equation $f(x) = F(x, 0)$.
5. If we place a solid homogeneous mass, having the form of a sphere or cube, in a medium maintained at a constant temperature, and if it remains immersed for a very long time, it will acquire at all its points a temperature differing very little from that of the fluid. Suppose the mass to be withdrawn in order to transfer it to a cooler medium, heat will begin to be dissipated at its surface; the temperatures at different points of the mass will not be sensibly the same, and if we suppose it divided into an infinity of layers by surfaces parallel to its external surface, each of those layers will transmit, at each instant, a certain quantity of heat to the layer which surrounds it. If it be imagined that each molecule carries a separate thermometer, which indicates its temperature at every instant, the state of the solid will from time to time be represented by the variable system of all these thermometric heights. It is required to express the successive states by analytical formulæ, so that we may know at any given instant the temperatures indicated by each thermometer, and compare the quantities of heat which flow during the same instant, between two adjacent layers, or into the surrounding medium.
6. If the mass is spherical, and we denote by x the distance of a point of this mass from the centre of the sphere, by t the time which has elapsed since the commencement of the cooling, and by v the variable temperature of the point m, it is easy to see that all points situated at the same distance x from the centre

of the sphere have the same temperature v. This quantity v is a certain function $F(x, t)$ of the radius x and of the time t; it must be such that it becomes constant whatever be the value of x, when we suppose t to be nothing; for by hypothesis, the temperature at all points is the same at the moment of emersion. The problem consists in determining that function of x and t which expresses the value of v.

7. In the next place it is to be remarked, that during the cooling, a certain quantity of heat escapes, at each instant, through the external surface, and passes into the medium. The value of this quantity is not constant; it is greatest at the beginning of the cooling. If however we consider the variable state of the internal spherical surface whose radius is x, we easily see that there must be at each instant a certain quantity of heat which traverses that surface, and passes through that part of the mass which is more distant from the centre. This continuous flow of heat is variable like that through the external surface, and both are quantities comparable with each other; their ratios are numbers whose varying values are functions of the distance x, and of the time t which has elapsed. It is required to determine these functions.
8. If the mass, which has been heated by a long immersion in a medium, and whose rate of cooling we wish to calculate, is of cubical form, and if we determine the position of each point m by three rectangular co-ordinates x, y, z, taking for origin the centre of the cube, and for axes lines perpendicular to the faces, we see that the temperature v of the point m after the time t, is a function of the four variables x, y, z, and t. The quantities of heat which flow out at each instant through the whole external surface of the solid, are variable and comparable with each other; their ratios are analytical functions depending on the time t, the expression of which must be assigned.
9. Let us examine also the case in which a rectangular prism of sufficiently great thickness and of infinite length, being submitted at its extremity to a constant temperature, whilst the air which surrounds it is maintained at a less temperature, has at last arrived at a fixed state which it is required to determine. All the points of the extreme section at the base of the prism have, by hypothesis, a common and permanent temperature. It is not the same with a section distant from the source of heat; each of the points of this rectangular surface parallel to the base has acquired a fixed temperature, but this is not the same at different points of the same section, and must be less at points nearer to the surface exposed to the air. We see also that, at each instant, there flows across a given section a certain quantity of heat, which always remains the same, since the state of the solid has become constant. The problem consists in determining the permanent temperature at any given point of the solid, and the whole quantity of heat which, in a definite time, flows across a section whose position is given.
10. Take as origin of co-ordinates x, y, z, the centre of the base of the prism, and as rectangular axes, the axis of the prism itself, and the two perpendiculars on the sides: the permanent temperature v of the point m, whose co-ordinates are x, y, z, is a function of three variables $F(x, y, z)$: it has by hypothesis a constant value, when we suppose x nothing, whatever be the values of y and

z. Suppose we take for the unit of heat that quantity which in the unit of time would emerge from an area equal to a unit of surface, if the heated mass which that area bounds, and which is formed of the same substance as the prism, were continually maintained at the temperature of boiling water, and immersed in atmospheric air maintained at the temperature of melting ice.

We see that the quantity of heat which, in the permanent state of the rectangular prism, flows, during a unit of time, across a certain section perpendicular to the axis, has a determinate ratio to the quantity of heat taken as unit. This ratio is not the same for all sections: it is a function $\phi(x)$ of the distance x, at which the section is situated. It is required to find an analytical expression of the function $\phi(x)$.

11. The foregoing examples suffice to give an exact idea of the different problems which we have discussed.

The solution of these problems has made us understand that the effects of the propagation of heat depend in the case of every solid substance, on three elementary qualities, which are, its capacity for heat, its own conductibility, and the exterior conductibility.

It has been observed that if two bodies of the same volume and of different nature have equal temperatures, and if the same quantity of heat be added to them, the increments of temperature are not the same; the ratio of these increments is the inverse ratio of their capacities for heat. In this manner, the first of the three specific elements which regulate the action of heat is exactly defined, and physicists have for a long time known several methods of determining its value. It is not the same with the two others; their effects have often been observed, but there is but one exact theory which can fairly distinguish, define, and measure them with precision.

The proper or interior conductibility of a body expresses the facility with which heat is propagated in passing from one internal molecule to another. The external or relative conductibility of a solid body depends on the facility with which heat penetrates the surface, and passes from this body into a given medium, or passes from the medium into the solid. The last property is modified by the more or less polished state of the surface; it varies also according to the medium in which the body is immersed; but the interior conductibility can change only with the nature of the solid.

These three elementary qualities are represented in our formulæ by constant numbers, and the theory itself indicates experiments suitable for measuring their values. As soon as they are determined, all the problems relating to the propagation of heat depend only on numerical analysis. The knowledge of these specific properties may be directly useful in several applications of the physical sciences; it is besides an element in the study and description of different substances. It is a very imperfect knowledge of bodies which ignores the relations which they have with one of the chief agents of nature. In general, there is no mathematical theory which has a closer relation than this with public economy, since it serves to give clearness and perfection to the practice of the numerous arts which are founded on the employment of heat.

12. The problem of the terrestrial temperatures presents one of the most beautiful applications of the theory of heat; the general idea to be formed of it is this. Different parts of the surface of the globe are unequally exposed to the influence of the solar rays; the intensity of their action depends on the latitude of the place; it changes also in the course of the day and in the course of the year, and is subject to other less perceptible inequalities. It is evident that, between the variable state of the surface and that of the internal temperatures a necessary relation exists, which may be derived from theory. We know that, at a certain depth below the surface of the earth, the temperature at a given place experiences no annual variation: this permanent underground temperature becomes less and less according as the place is more and more distant from the equator. We may then leave out of consideration the exterior envelope, the thickness of which is incomparably small with respect to the earth's radius, and regard our planet as a nearly spherical mass, whose surface is subject to a temperature which remains constant at all points on a given parallel, but is not the same on another parallel. It follows from this that every internal molecule has also a fixed temperature determined by its position. The mathematical problem consists in discovering the fixed temperature at any given point, and the law which the solar heat follows whilst penetrating the interior of the earth.

    This diversity of temperature interests us still more, if we consider the changes which succeed each other in the envelope itself on the surface of which we dwell. Those alternations of heat and cold which are reproduced every day and in the course of every year, have been up to the present time the object of repeated observations. These we can now submit to calculation, and from a common theory derive all the particular facts which experience has taught us. The problem is reducible to the hypothesis that every point of a vast sphere is affected by periodic temperatures; analysis then tells us according to what law the intensity of these variations decreases according as the depth increases, what is the amount of the annual or diurnal changes at a given depth, the epoch of the changes, and how the fixed value of the underground temperature is deduced from the variable temperatures observed at the surface.
13. The general equations of the propagation of heat are partial differential equations, and though their form is very simple the known methods do not furnish any general mode of integrating them; we could not therefore deduce from them the values of the temperatures after a definite time. The numerical interpretation of the results of analysis is however necessary, and it is a degree of perfection which it would be very important to give to every application of analysis to the natural sciences. So long as it is not obtained, the solutions may be said to remain incomplete and useless, and the truth which it is proposed to discover is no less hidden in the formulæ of analysis than it was in the physical problem itself. We have applied ourselves with much care to this purpose, and we have been able to overcome the difficulty in all the problems of which we have treated, and which contain the chief elements of the theory of heat. There is not one of the problems whose solution does not provide convenient and exact means for discovering the numerical values of the temperatures acquired, or

those of the quantities of heat which have flowed through, when the values of the time and of the variable coordinates are known. Thus will be given not only the differential equations which the functions that express the values of the temperatures must satisfy; but the functions themselves will be given under a form which facilitates the numerical applications.

14. In order that these solutions might be general, and have an extent equal to that of the problem, it was requisite that they should accord with the initial state of the temperatures, which is arbitrary. The examination of this condition shews that we may develop in convergent series, or express by definite integrals, functions which are not subject to a constant law, and which represent the ordinates or irregular or discontinuous lines. This property throws a new light on the theory of partial differential equations, and extends the employment of arbitrary functions by submitting them to the ordinary processes of analysis.
15. It still remained to compare the facts with theory. With this view, varied and exact experiments were undertaken, whose results were in conformity with those of analysis, and gave them an authority which one would have been disposed to refuse to them in a new matter which seemed subject to so much uncertainty. These experiments confirm the principle from which we started, and which is adopted by all physicists in spite of the diversity of their hypotheses on the nature of heat.
16. Equilibrium of temperature is effected not only by way of contact, it is established also between bodies separated from each other, which are situated for a long time in the same region. This effect is independent of contact with a medium; we have observed it in spaces wholly void of air. To complete our theory it was necessary to examine the laws which radiant heat follows, on leaving the surface of a body. It results from the observations of many physicists and from our own experiments, that the intensities of the different rays, which escape in all directions from any point in the surface of a heated body, depend on the angles which their directions make with the surface at the same point. We have proved that the intensity of a ray diminishes as the ray makes a smaller angle with the element of surface, and that it is proportional to the sine of that angle.[1] This general law of emission of heat which different observations had already indicated, is a necessary consequence of the principle of the equilibrium of temperature and of the laws of propagation of heat in solid bodies.

    Such are the chief problems which have been discussed in this work; they are all directed to one object only, that is to establish clearly the mathematical principles of the theory of heat, and to keep up in this way with the progress of the useful arts, and of the study of nature.
17. From what precedes it is evident that a very extensive class of phenomena exists, not produced by mechanical forces, but resulting simply from the presence and accumulation of heat. This part of natural philosophy cannot be

[1] 1 *Mem. Acad. d. Sc.* Tome V. Paris, 1826, pp. 179–213. [A. F.].

connected with dynamical theories, it has principles peculiar to itself, and is founded on a method similar to that of other exact sciences. The solar heat, for example, which penetrates the interior of the globe, distributes itself therein according to a regular law which does not depend on the laws of motion, and cannot be determined by the principles of mechanics. The dilatations which the repulsive force of heat produces, observation of which serves to measure temperatures, are in truth dynamical effects; but it is not these dilatations which we calculate, when we investigate the laws of the propagation of heat.

18. There are other more complex natural effects, which depend at the same time on the influence of heat, and of attractive forces: thus, the variations of temperatures which the movements of the sun occasion in the atmosphere and in the ocean, change continually the density of the different parts of the air and the waters. The effect of the forces which these masses obey is modified at every instant by a new distribution of heat, and it cannot be doubted that this cause produces the regular winds, and the chief currents of the sea; the solar and lunar attractions occasioning in the atmosphere effects but slightly sensible, and not general displacements. It was therefore necessary, in order to submit these grand phenomena to calculation, to discover the mathematical laws of the propagation of heat in the interior of masses.
19. It will be perceived, on reading this work, that heat attains in bodies a regular disposition independent of the original distribution, which may be regarded as arbitrary.
    In whatever manner the heat was at first distributed, the system of temperatures altering more and more, tends to coincide sensibly with a definite state which depends only on the form of the solid. In the ultimate state the temperatures of all the points are lowered in the same time, but preserve amongst each other the same ratios: in order to express this property the analytical formulæ contain terms composed of exponentials and of quantities analogous to trigonometric functions.
    Several problems of mechanics present analogous results, such as the isochronism of oscillations, the multiple resonance of sonorous bodies. Common experiments had made these results remarked, and analysis afterwards demonstrated their true cause. As to those results which depend on changes of temperature, they could not have been recognised except by very exact experiments; but mathematical analysis has outrun observation, it has supplemented our senses, and has made us in a manner witnesses of regular and harmonic vibrations in the interior of bodies.
20. These considerations present a singular example of the relations which exist between the abstract science of numbers and natural causes.
    When a metal bar is exposed at one end to the constant action of a source of heat, and every point of it has attained its highest temperature, the system of fixed temperatures corresponds exactly to a table of logarithms; the numbers are the elevations of thermometers placed at the different points, and the logarithms are the distances of these points from the source. In general, heat distributes itself in the interior of solids according to a simple law expressed by a partial differential

equation common to physical problems of different order. The irradiation of heat has an evident relation to the tables of sines, for the rays which depart from the same point of a heated surface, differ very much from each other, and their intensity is rigorously proportional to the sine of the angle which the direction of each ray makes with the element of surface.

If we could observe the changes of temperature for every instant at every point of a solid homogeneous mass, we should discover in these series of observations the properties of recurring series, as of sines and logarithms; they would be noticed for example in the diurnal or annual variations of temperature of different points of the earth near its surface.

We should recognise again the same results and all the chief elements of general analysis in the vibrations of elastic media, in the properties of lines or of curved surfaces, in the movements of the stars, and those of light or of fluids. Thus the functions obtained by successive differentiations, which are employed in the development of infinite series and in the solution of numerical equations, correspond also to physical properties. The first of these functions, or the fluxion properly so called, expresses in geometry the inclination of the tangent of a curved line, and in dynamics the velocity of a moving body when the motion varies; in the theory of heat it measures the quantity of heat which flows at each point of a body across a given surface. Mathematical analysis has therefore necessary relations with sensible phenomena; its object is not created by human intelligence; it is a pre-existent element of the universal order, and is not in any way contingent or fortuitous; it is imprinted throughout all nature.

21. Observations more exact and more varied will presently ascertain whether the effects of heat are modified by causes which have not yet been perceived, and the theory will acquire fresh perfection by the continued comparison of its results with the results of experiment; it will explain some important phenomena which we have not yet been able to submit to calculation; it will shew how to determine all the thermometric effects of the solar rays, the fixed or variable temperature which would be observed at different distances from the equator, whether in the interior of the earth or beyond the limits of the atmosphere, whether in the ocean or in different regions of the air. From it will be derived the mathematical knowledge of the great movements which result from the influence of heat combined with that of gravity. The same principles will serve to measure the conductivities, proper or relative, of different bodies, and their specific capacities, to distinguish all the causes which modify the emission of heat at the surface of solids, and to perfect thermometric instruments.

The theory of heat will always attract the attention of mathematicians, by the rigorous exactness of its elements and the analytical difficulties peculiar to it, and above all by the extent and usefulness of its applications; for all its consequences concern at the same time general physics, the operations of the arts, domestic uses and civil economy.

2.3 Study Questions

QUES. 2.1. How does Fourier's theory address the problem of heat?

QUES. 2.2. What is the temperature distribution within a metallic ring held above a source of heat, such as a candle?

a) Does the ring reach a uniform temperature? Why or why not? How may the temperature distribution be expressed mathematically? What assumption does Fourier make?
b) What happens to the temperature of the ring when the heat source is suddenly extinguished? What, then, is the ultimate goal of Fourier's theory?

QUES. 2.3. What is the temperature of a solid sphere alternately immersed and removed from a warm fluid?

a) Does the sphere attain a uniform temperature immediately upon immersion? Does it ever attain a uniform temperature? If so, what is its value?
b) When the sphere is removed from the warm fluid, is its rate of cooling uniform? Is its temperature uniform during the cooling process? Do any two points on its surface differ in temperature?
c) More generally, how may the sphere's temperature distribution be expressed mathematically? Does the temperature distribution have any symmetries?
d) Is the behavior of a cube different than that of a sphere? Does the temperature distribution have any symmetries?

QUES. 2.4. What is the temperature of a long rectangular prism one end of which is held at a high temperature?

a) Does the prism attain a uniform temperature immediately upon immersion? Does it ever attain a uniform temperature?
b) Is the temperature constant along the length of the prism? What about within any cross-section of the prism?
c) Does the temperature distribution have any symmetries? Is the temperature distribution time-dependent?
d) What does Fourier select as the unit of heat? Why might he have chosen this unit?
e) In its final state, is the heat flowing through any cross-section of the prism time-dependent, constant, or perhaps even zero? Is it the same for every cross-sectional area along the length of the prism?

QUES. 2.5. Upon what elementary quantities doe the propagation of heat through a body depend? How are each of these quantities defined? What can a knowledge of these quantities provide?

QUES. 2.6. What are some significant problems that Fourier's theory of heat allows one to solve? What does a solution to such problems entail? Are all such problems solvable analytically?

QUES. 2.7. Must bodies be in physical contact in order to achieve equilibrium? Must there be anything at all between them? What law governs the intensity of radiant heat from a surface? In particular, how does its intensity depend on its emission angle?

QUES. 2.8. Is Fourier's theory of heat a dynamical theory of heat? That is: does his theory of heat follow from Newton's three laws of motion? Is there any relationship at all between Newton's three laws of motion and, for instance, atmospheric or oceanic currents?

QUES. 2.9. Does the final state of a body depend on its initial distribution of temperatures? Upon what does the final state depend? Are any other phenomena in nature independent of initial conditions?

QUES. 2.10. In what way does mathematical analysis, and derivatives in particular, relate to sensible phenomena?

a) In geometry, what quantity does the first derivative of a line express?
b) In dynamics, what quantity does the first derivative of the position of a moving body express?
c) In the theory of heat, what quantity does the first derivative of the temperature distribution inside a body express?

2.4 Exercises

EX. 2.1 (NATURE AND MATHEMATICS ESSAY). Consider the Fourier quote at the outset of this chapter. What does he mean when he asserts that mathematics "is a pre-existent element of the universal order" and that it "is not created by human intelligence"? Do you agree with these assertions, or do you have a different view of the origin of mathematics? Can you defend your view?

EX. 2.2 (TEMPERATURE AND SYMMETRY). Make sketches which clearly indicate the temperature distribution within objects under the following conditions: (i) a thin ring of metal heated for a long time at a single point along its circumference, (ii) a solid homogeneous sphere immersed for a long time in fluid at 50°C, (iii) the same homogeneous sphere shortly after it has been removed from the fluid and placed in the air at 20°C, and (iv) a long, thin rectangular prism in 20°C air, one end of which is heated to 50°C for a long time.

EX. 2.3 (HEAT CAPACITY AND THE METHOD OF MIXTURES). According to the *caloric theory*, heat is an invisible and imponderable (weightless) fluid which, upon entering a substance raises its temperature, and upon leaving a substance lowers its temperature. Different substances have different abilities to store heat—different *heat capacities*. The mathematical relationship between the amount of heat added to a substance, ΔQ, the heat capacity of the substance, C, and the change in temperature of the substance, ΔT, is given by

$$\Delta Q = C \Delta T \tag{2.1}$$

The *method of mixtures* is an experimental technique used to measure the relative heat capacities of two substances, one of which is a fluid. Suppose, for instance, that a 1 pound sheet of lead foil is loosely rolled up and then suspended in the steam rising from a tea kettle until it finally reaches $212°F$. It is then plunged into a 1-pound water bath initially at $57°F$. After the lead foil and water achieve thermal equilibrium, the water is measured to be $62°F$. If 1 unit of heat is now defined as the amount of heat required to raise 1 pound of water by $1°F$, then what is the heat capacity of 1 pound of lead? (ANSWER: $\frac{1}{30}$)

EX. 2.4 (LATENT HEAT OF ICE MELTING). According to the caloric theory, when heat enters a substance it may cause it to undergo a phase transformation *instead of raising its temperature*. After such a phase transformation, the added heat or *caloric* is hidden within the substance in the form of *latent heat*. For example, when a certain quantity of heat is added to ice at 0°C, it melts and becomes water at 0°C. This latent heat becomes manifest—it reappears and must be removed—when water is refrozen into ice. In order to determine the latent heat associated with the melting of ice, consider the following two laboratory experiments.

Experiment 1: 100 grains of ice at 0°C are added to a 5000-grain bath of water at 55°C. After equilibration, the final temperature is 52.3°C.

Experiment 2: 1000 grains of water at 5°C are poured into a 5000-grain bath of water at 56°C. After equilibration, the final temperature is 47.5°C.

According to convention, a grain is a unit of weight approximately equivalent to 65 mg,[2] and a unit of heat is the amount required to raise one grain of water by 1°C.

a) In experiment 1, what was the change in temperature of the ice? of the (initially) warm water bath?
b) In experiment 2, what was the change in temperature of the (initially) cool water? of the (initially) warm water bath? How many units of heat did the (initially) cool water absorb? And how many units of heat left the (initially) warm water bath? (ANSWER: 42,500 units)
c) In experiment 1, how many units of heat left the (initially) hot water bath? How much of this heat went into raising the temperature of the melted ice? And how much went into melting the ice? What, then, is the latent heat of one grain of ice? (ANSWER: 82.3)

[2] The grain is part of the traditional English weight system; it is still used today by apothecaries, dentists, and ammunition manufacturers.

2.5 Vocabulary

1. Propagation
2. Equilibrium
3. Dissipate
4. Incessant
5. Ordinate
6. Commencement
7. Emersion
8. Prism
9. Conductibility
10. Diurnal
11. Facilitate
12. Requisite
13. Shew
14. Conform
15. Dynamical
16. Exponential
17. Logarithm
18. Differentiation
19. Tangent
20. Contingent
21. Fortuitous
22. Thermometric

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