Chapter 1
Introduction

Abstract In this chapter the problem of mass variation is discussed. Some phenomena in celestial mechanics which are caused by mass variation are presented. Machines and mechanisms with variable mass are described. In this book special attention is given to transportation and measuring devices, lifting systems, cranes, sieves, but also to rotating machines like pumps, centrifuges, fans, mills and rotors of machines in textile, paper, cable, carpet industry etc. In this chapter effect of mass and moment of inertia variation on dynamics of systems is underlined.

Problem of motion of mass variable systems is evident since the 17th century. Galileo discovered the anomaly in the Moon motion which he believed is the function of the system mass variation. Lately, Laplace theoretically explained the phenomena of the secular acceleration of the Moon. Dufour (1866) explained that the mass of the earth varies continuously due to the falling shooting-stars and also due to combustion or spending in the atmosphere. He found that the dust of shooting-stars which fall on the surface of France in one year can cover a volume of 0.1 m^3. Oppalzer (1884) was the first to analyze the reason for secular acceleration of the Moon as the result of Earth and Moon mass increase. Namely, during a hundred year a 2.8 mm dust layer is formed on the Earth. Gylden (1884) extended the previous investigations in celestial mechanics by analyzing of the relative motion of two variable mass systems under influence of the Newton force. Meshchersky (1893) continued the investigation and found that the body with variable mass would move along a spiral, tending toward zero, or it would increase the distance to the other mass variable system.

Cayley in his works (Cayley 1857, 1858) was the first to consider the influence of the continual mass variation on the motion of the body. Class of the dynamic problem he studied was the 'continuous reactive problem', i.e., the problem when continuously the infinitesimal small mass is added to a system which causes the velocity of the system continuously to be changed for a definite value. Two most widely discussed examples were (see Cayley 1859): one, the chain is on the table and is dropping vertically down from the table (Fig. 1.1), and the second, the chain is moving straightforward on a horizontal plane without friction while its length is changing due to the fact that the other end of the chain is rolling up on a drum.

At the end of the nineteenth century and at the beginning of the 20th century Meshchersky (1897) laid the foundations of the modern dynamics of a particle with

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Fig. 1.1 Chain dropping vertically from the table

variable mass. He was the first to worn out the differential equation of motion of a particle with variable mass. After that publication numerous investigations have been done and the dynamics of variable mass systems is developed. In the 'variable mass systems' particles are added and/or separated during motion.

Two kinds of systems are identified:

- 'continuously' particle-ejecting systems, where mass variation is a continual function of time, position (see Grudtsyn 1972) or velocity of the particle, and
- 'discretely' particle-ejecting systems, where mass variation is a discontinual function (for example, automatic weapons that fire rounds, one at a time).

For the case of discontinual mass variation Meshchersky (1952) calculated velocity of the particle after mass variation. Finite discontinual mass variation in a very short time was not of special interest for a long time and was not intensively discussed. Meshchersky was the first to consider the velocity change of a translatory moving body during step-like mass variation. Mass which is separated or added has a finite amount and mass variation of the body is discontinual. Theory is mostly applied for solving of the Keplerian two-body problem (see Luk'yanov 2005), and also the three and four-body problems (Cveticanin and Kovacic 2007).

Motion of continuously mass variable systems is much more investigated due to its application in rocket theory (Meirovitch 1970; Cornelisse 1979; Tran and Eke 2005), astronomy (Kayuk and Denisenko 2004), for charged particle motion in a magnetic field with decreasing mass and charge (Howard 2007), in robotics (McPhee and Dubey 1991; Djerassi 1998) in machinery (Cveticanin 1984, 1988, 1989, 1991, 1993a, b, 1995, 1998a, 2001) etc. Motion is described with differential equations with variable parameters (Kayuk and Tilavov 1987; Wang and Eke 1995; Eke and Mao 2002; Pesce 2003; Irschik and Holl 2004). Effect of expulsion and/or capture of particles on the motion of the continuously mass variable system is evident as changes in the integration variables of the governing dynamic equations.

In spite of the fact that mass variable systems are nonconservative, for certain conditions first integrals may exist (Leubnert and Krumm 1990; Cveticanin 1994). Nonholonomic properties of mass variable systems are also treated (Ge and Cheng 1982; Ge 1984). For the case when mass is continually varying in time, the influence of the reactive force on the motion (see Apykhtin and Jakovlev 1980; Azizov 1986; Cveticanin 1992, 1993c, 2004) and also on the stability (Ignat'yev 1991; Cveticanin 1996a, b) were investigated. Reactive force is mathematically described as a product of mass variation function and relative velocity of mass separated or added to the particle. Usually, two special cases were considered: first, relative velocity of mass variation is zero and second, absolute velocity of separated or added mass is zero. If the relative velocity is zero, i.e., the absolute velocity of the separated or added particle is equal to the velocity of the basic particle, then the reactive force is also zero. Levi-Civita (1928), investigated the motion of the particle for the case when the absolute velocity of the separated or added mass is zero and the reactive force exists. In the papers of Cveticanin (2012, 2013) an overview on dynamics of bodies with variable mass is given, while the most comprehensive consideration of the dynamics is given in the books of Bessonov (1967), Cornelisse (1979), Cveticanin (1998b, 2014).

In this chapter various types of machines and mechanisms with mass variable elements are described. Expression "variable mass element" or "body with variable mass" as used in the context of this book, refers to mechanical systems that lose and/or gain mass while in motion.

Examples of such devices abound in contemporary engineering literature. They include complex systems such as aircraft, rockets, automobiles, moving robots picking up or lifting objects, as well as simpler systems such as transportation machines, mining machines, excavators, vibrating machines used as conveyors, separators, machines for segregation, rolling mills, metallurgical machines, casting machines, agricultural machines, centrifuges, measuring mechanisms, etc. In these mechanisms, mass of elements and position of their centre of mass and moment of inertia vary during addition or removal of the material. In this chapter various types of machines and mechanisms with mass variable elements are described. Their constructive properties and working procedures are explained.

Pouring machine, shown in Fig. 1.2, represents a mechanism with variable mass. Hook of the crane grips the vessel at point B, and rotates it around A, pouring out the molten metal for continuous casting. Basic requirement for the equipment is that pouring of metal from the vessel has to be uniform, and it is regulated by the velocity of the hook. To estimate the process of lifting of the vessel and to give good control it is necessary to take into consideration the mass variation of the vessel, because it is the basic cause of uniform flow of metal.

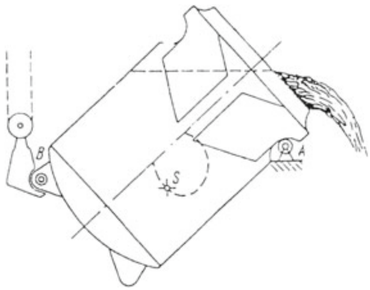


Fig. 1.2 Mechanism for tipping a vessel

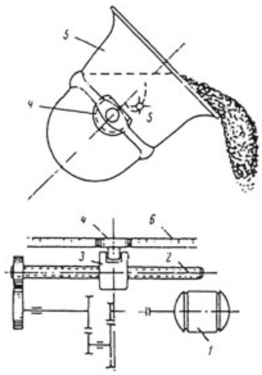


Fig. 1.3 Mechanism for rotating a vessel

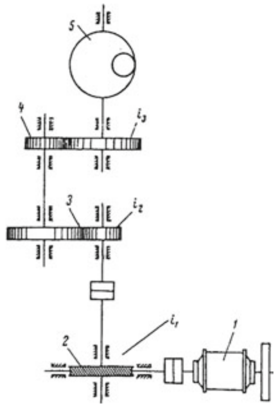


Fig. 1.4 Mechanism for converter turning

Position of the centre of mass of the vessel and metal varies. As simplification this system can be considered as a rotating bar AB with variable mass and variable position of the centre of mass inside the element. Let us consider some other mechanisms. Pouring of concrete from the vessel in Fig. 1.3 is achieved by a mechanism which contains an electric motor 1, which by a system of gears 4 and 6 and screws 2 and 3 transmits the motion to 4. As 4 is fixed to vessel 5 its motion about 6 turns the vessel. To analyze the elements and the process of vessel turning, variable mass of the vessel has to be taken into account. Only in this way the correct position of the turning axle and of the torque for vessel rotating can be obtained.

Converters are used for steel production. For melting the steel, the converter-vessel system is usually lifted and turned during the introduction some additives and pouring out the finished steel.

An electric motor 1 with a system of gear-wheels 2, 3 and 4 rotates the converter 5 (Fig. 1.4). Exact dynamic analysis, taking into account the variation of mass of metal in the converter, is necessary for automatic control of the process. A system of a rotor type of wagon turner is shown in Fig. 1.5. Wagon with material 1 is in the rotor 2. Rotor rotates on cylinders 3 and is driven with a mechanism 4 and rope 5.

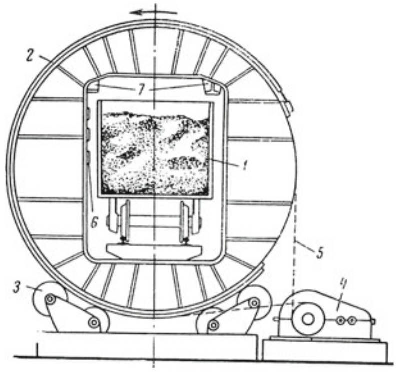


Fig. 1.5 Mechanism for wagon turning

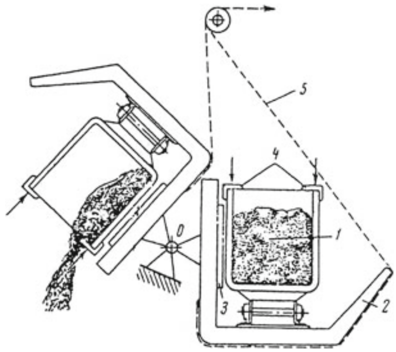


Fig. 1.6 Mechanism for discharging a wagon

Wagon is positioned by 6 and 7. By turning the wagon by an angle smaller than 180 degrees discharging is completed. The efficiency of this mechanism is very high.

Mechanism for discharging wagon, which is used in the metallurgical industry for minerals, is of the rotating type (Fig. 1.6). Wagon 1 is on the carrier 2, and is supported on the wall 3 and elements 4. The rope 5 is turning on the wheel and tips the carrier, discharging the material from the wagon. Torques are balanced with a system of counterweights. Mechanism productivity is higher for higher velocity of discharging. For dynamic analysis, mass variation has to be taken into consideration.

Wagon 2, which moves laterally, is shown in Fig. 1.7. It is driven by a motor 1. Another type of laterally moved wagon is shown in Fig. 1.8. Movement in one direction increases the mass of the wagon, while in other direction it decreases. Mechanism is a kind of link mechanisms whose motion is reversible.

A rolling mill is shown in Fig. 1.9. Tin rolls down off the drum 1, travels through the rollers 2 and rolls up on the drum 3. System is fixed by supporting rollers 4. Cold rolling is with velocity 10 to 15 m/sec and more. During rolling up or rolling down of the tin, radius, mass and moment of inertia of the drum varies. One of the most important requirements is constant velocity of rolling up, and special automatic regulation and controls have to be introduced. To increase the efficiency of these machines, dynamics of motion of this system with variable mass have to be analyzed.

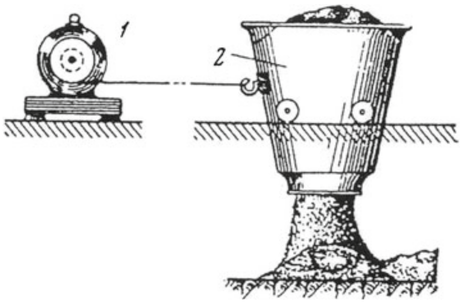


Fig. 1.7 Wagon with variable mass

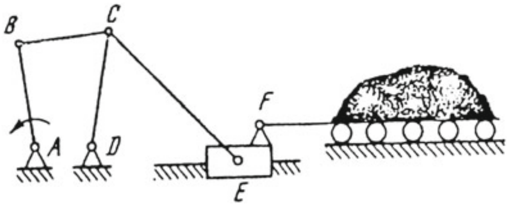


Fig. 1.8 Transportation mechanism

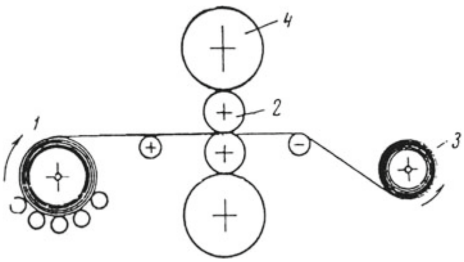


Fig. 1.9 Rolling machine scheme

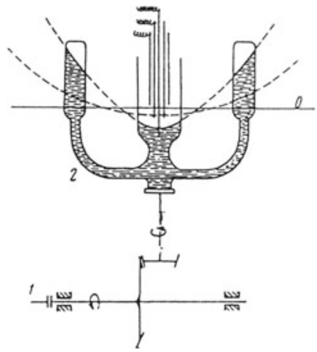


Fig. 1.10 Centrifugal regulator

Another mechanism with variable moment of inertia is shown in Fig. 1.10. It is a centrifugal lifting basket regulator of motion. It contains shaft 1, system of gears, and a system of mercury filled tubes. The velocity of the shaft and of tubes regulates the velocity of the basket. If the velocity of tubes is zero, the level of mercury in all tubes is the same. If the velocity increases, the mercury level attains a parabolic shape and

Fig. 1.11 Excavator model

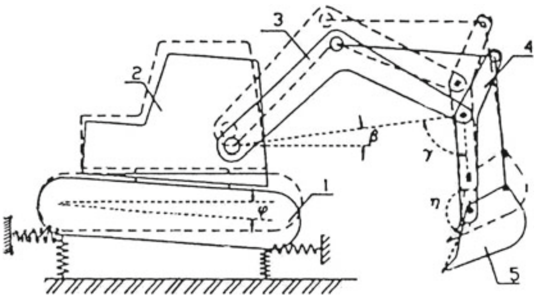
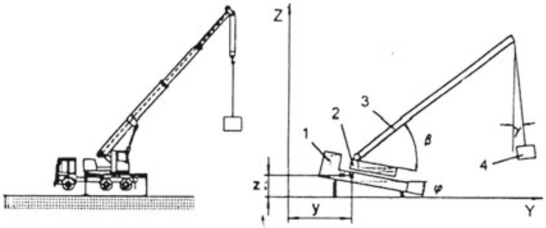


Fig. 1.12 Sectional model of crane mechanism



the level in the middle tube decreases. If it gets to a very small level, the connection with the motor is broken and the shaft 1 slows down. If the velocity of basket motion is smaller than permitted, the regulator sets up the motor and transfers the motion to the shaft 1. As position of the mercury depends on velocity, the moment of inertia varies, as a function of angular velocity. To give the correct instructions for regulator dynamic behavior it is necessary to know the change of moment of inertia of the mechanism.

A simple planar model of an excavator is shown in Fig. 1.11. It consists of five rigid bodies: body 1 which is in contact with the ground, body 2 mounted on 1, body 3 connected to 2 by means of pivot, body 4 which is an excavator arm connected with 3 by means of a joint, and body 5 which is the excavator scoop and can rotate around the body 4. Excavator scoop has a varying mass. The operating movements can be realized by means of the jib 3, arm 4 or the scoop 5 movements. A crane represents a mechanism for load transportation.

A typical sectional model of a crane (Fig. 1.12) consists of four rigid bodies: body 1 which is in elastic contact with the ground, body 2 mounted on 1, body 3 connected with 2 by means of joint and a component 4 connected with 3 by a flexible cord. Component 4 has a time variable mass, because it includes the load. Variation of mass and its distribution along the elements of a mechanism have a significant influence on belt type automatic dosing devices.

The aim of the mechanism shown in Fig. 1.13 is to achieve constant mass flow of material. Mechanism contains: basket 1, belt conveyor 2 and bar mechanism 3. Bar is a sensitive element which directs the opening and shutting of the dosing device. Process of separating material from the conveyor may result in large vibrations around O. These vibrations are undesirable and they are due to mass variation.

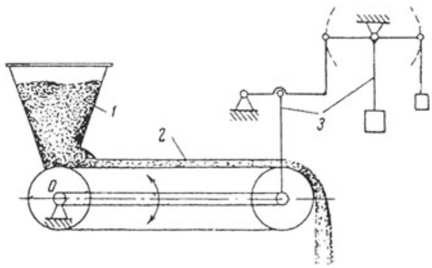


Fig. 1.13 Belt type automatic dosing device

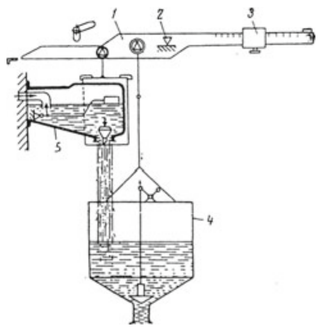


Fig. 1.14 Mechanism for automatic measurement of liquid

A mechanism for automatic measurement of liquid is shown in Fig. 1.14. It contains bar 1, which is pivoted at 2 and is under the influence of weight 3, and vessel 4 which is to be filled with liquid. Quantity of liquid is regulated by a special dosing device 5 which automatically regulates the liquid in the vessel by stopping the liquid getting in. It is connected to bar 1. Recently, such a model is considered by Otto et al. (2013).

The higher the velocity of the lorry, the higher the influence of mass variation on the dynamics. A special group of mechanisms are the rotors with variable mass. Rotors with variable mass are the fundamental working elements of many machines in process, cable, textile and paper industry, as well as in transportation, etc. The term "rotor with variable mass" will refer to all parts which are mounted on the rotating shaft and whose mass is varying. The shaft which is a fundamental element of the rotor is designed to support machine parts rotating with it and to transmit bending moments and torques and also longitudinal forces.

As it can be seen, mechanisms with variable mass are widely used in various industrial fields: process, transportation, civil engineering, regulators etc. Most of them can be reduced to a simple dynamic model with mass variation which is a function of time (for example, vibration mechanisms, dosing devices, etc.). For correct dynamic analysis of the mechanical system it is necessary to take this property of the system into consideration.

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