

Chapter 2

The Standard Model and Its Supersymmetric Extensions

Abstract This chapter starts with a theoretical introduction to the Standard Model (SM) of particle physics. We outline some shortcomings of the SM, before we turn to the discussion of supersymmetric (SUSY) models. Supersymmetry is motivated and introduced, followed by a detailed description of the particle sectors of the Minimal Supersymmetric Standard Model (MSSM). Then we go to Next-to-minimal Supersymmetric Standard Model (NMSSM) and show how the Higgs and neutralino sectors are modified compared to the MSSM.

2.1 The Standard Model

The Higgs boson was for a long time the last missing piece predicted by the Standard Model of particle physics. This gap was filled by the spectacular discovery of a particle at the LHC in July 2012 with properties compatible with the SM Higgs boson.

The Standard Model of particle physics [1–3] is a theory formulated (in its current version) in the 1970s, which describes all fundamental particles which make up for the visible matter in the universe and the interactions between them, apart from gravity. It is a quantum field theory (QFT) that exhibits translation invariance and Lorentz invariance, two global symmetries following from special relativity. These global symmetries define the Poincaré group. Further the SM is locally gauge invariant under the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.¹ The SM gauge group is split into two parts: Quantum Chromo Dynamics (QCD) and the quantum theory of electroweak interactions. QCD is the theory of strong interactions, described by the $SU(3)_C$ gauge group of colour. The electroweak theory is based on $SU(2)_L \otimes U(1)_Y$. The existence of massive fields implies that the electroweak gauge group must be broken. The breaking is described by the BEH mechanism, which entails the existence of a Higgs boson.

¹The subscripts refer to colour, left chirality and weak hypercharge.

In this section we will outline the symmetries of the SM, the concept of electroweak symmetry breaking and the particle sectors of the SM. The last part of this section discusses the shortcomings of the SM, motivating the study of ‘new physics’ models.

2.1.1 Symmetries

The kinetic terms of the SM fields are fully determined by the global symmetry assuring translation invariance and invariance under Lorentz transformations. The SM fields are classified into fermionic (spin 1/2) fields, bosonic (spin 1) fields and a scalar boson (spin 0) field. Fermions account for the (visible) matter of the universe, spin 1 bosons carry the forces between them. The special role of the scalar will be discussed in Sect. 2.1.2. The possible kinetic terms for Dirac fermions ψ and vector bosons A_μ^a are

$$\begin{aligned}\mathcal{L}_{\text{kin}} &= \mathcal{L}_{\text{kin}}^{\text{fermion}} + \mathcal{L}_{\text{kin}}^{\text{vector}} \\ &= \bar{\psi} i \not{\partial} \psi - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a}\end{aligned}\quad (2.1)$$

with $\bar{\psi} = \psi^\dagger \gamma^0$ and $\not{\partial} = \gamma^\mu \partial_\mu$. The field strength tensor is

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c. \quad (2.2)$$

Here g denotes the gauge coupling of a gauge group with generators T^a , where $[T^a, T^b] = i f^{abc} T^c$ defines the structure constants f^{abc} .

The interactions of the SM fields are given by the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Demanding local gauge invariance, the derivatives in the kinetic terms must be replaced by the covariant derivatives, leading to a coupling of the vector fields to fermions and scalars. For a general gauge theory the covariant derivative is $D_\mu = \partial_\mu - i g T^a A_\mu^a$. For the SM gauge group the derivatives in Eq. (2.1) are replaced by²

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - i g_2 I^a W_\mu^a - i g_1 \frac{Y}{2} B_\mu - i g_s \frac{\lambda^a}{2} g_\mu^a. \quad (2.3)$$

Here g_2 , g_1 and g_s are the coupling constants of $SU(2)_L$, $U(1)_Y$ and $SU(3)_C$. We define $\alpha_s = g_s^2/4\pi$ for the strong $SU(3)_C$ interactions and

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \quad \alpha = \frac{e^2}{4\pi} \quad (2.4)$$

²We adopt the sign conventions for the $SU(2)_L$ covariant derivative used in the code `FeynArts` [4–9], where (for historical reasons) the $SU(2)_L$ covariant derivative in the SM is defined by $\partial_\mu - i g_2 I^a W_\mu^a$ (as in Eq. (2.3)), while it is defined by $\partial_\mu + i g_2 I^a W_\mu^a$ in the (N)MSSM, as we will discuss later.

for the electroweak $SU(2)_L \otimes U(1)_Y$ interactions. The generators of $SU(2)_L$ are $I^a = \sigma^a/2$ (where σ^a are the Pauli matrices), defining the weak isospin I^3 of a field. The generator of $U(1)_Y$ is $Y/2$ defining the hypercharge, and the generators of $SU(3)_C$ are $\lambda^a/2$ (λ^a are the Gell-Mann matrices) defining the colour charge. The gauge bosons of $SU(2)_L$, $U(1)_Y$ and $SU(3)_C$ are W^a ($a = 1, 2, 3$), B and g^a ($a = 1 \dots 8$). The gauge bosons g^a of QCD are called gluons.

2.1.2 Electroweak Theory and the BEH Mechanism in the SM

It is impossible to write down gauge-invariant explicit mass terms for vector boson fields. However among the electroweak gauge bosons only the photon is massless,³ while the other electroweak gauge bosons are massive, so the $SU(2)_L \otimes U(1)_Y$ gauge symmetry must be broken down to $U(1)_{\text{em}}$. The breaking is accomplished via the BEH mechanism, which, furthermore, is also responsible for the generation of fermion masses. In this framework, gauge boson masses are obtained by adding additional terms

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger (D^\mu \phi) - V_H^{\text{SM}} \quad (2.5)$$

to the Lagrangian of the electroweak SM. The scalar Higgs field ϕ is a $SU(2)_L$ doublet with hypercharge $Y = 1$.

Requiring gauge invariance and renormalizability,⁴ the potential can be written as

$$V_H^{\text{SM}} = -\mu^2 |\phi|^2 + \frac{\lambda}{4} |\phi|^4, \quad (2.6)$$

where λ must be positive, so that the potential is bounded from below. One chooses $\mu^2 > 0$, such that the potential is minimised at $|\langle \phi \rangle|^2 = 2\mu^2/\lambda \equiv v^2/2$, where v is the (non-zero) vacuum expectation value (vev).⁵ One specific minimum is conventionally chosen as

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (2.7)$$

This ground state does not reflect the symmetry of the potential anymore. This feature is termed spontaneous symmetry breaking. Expanding around the minimum, the full Higgs field can be written as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + H + i G^0 \end{pmatrix}. \quad (2.8)$$

³In the strong sector the gluons of $SU(3)_C$ are also massless.

⁴The concept of renormalization is explained in Sect. 3.2.

⁵ Note that the vev v of the SM Higgs field differs from the value v which we will define in the MSSM (in Eq. (2.58)) using a different convention. The numerical value here is $v \sim 246 \text{ GeV}$.

From the four degrees of freedom, the three unphysical fields, $G^\pm = G_1^\pm \pm i G_2^\pm$ and G , (called Goldstone bosons) can be absorbed in a suitable gauge transformation. The gauge in which the Goldstone bosons are absent is called unitary gauge.

Expanding the kinetic term $(D_\mu \phi)^\dagger (D^\mu \phi)$ of Eq.(2.5) around the minimum of the Higgs doublet, masses are generated for the fields

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2), \quad (2.9)$$

called (charged) W bosons and for the neutral Z boson

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}, \quad (2.10)$$

while the photon A remains massless. Here s_W and c_W are the sine and cosine of the weak mixing angle, which at tree level are given by

$$s_W \equiv \sin \Theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad c_W \equiv \cos \Theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}. \quad (2.11)$$

The generated masses are

$$M_W = c_W M_Z = \frac{1}{2} g_2 v, \quad M_A = 0. \quad (2.12)$$

The photon is the mediator of the electromagnetic interaction and remains massless. All charged particles are subject to the electromagnetic interaction. The weak interaction is carried by the charged gauge bosons W^\pm and the neutral gauge boson Z . The neutral interaction involves all (left- and right-handed) fermions, while W^\pm couples only to left-handed fermions.

The remaining real degree of freedom in Eq.(2.8) is the only physical scalar field– the Higgs boson, H . The mass of the Higgs boson can be written as $M_H^2 = 2\mu^2$. However since μ^2 is arbitrary, M_H is a free parameter in the SM that must be determined by experiment. Later we shall call the SM Higgs H^{SM} to avoid confusion when we simultaneously talk about the SM and extensions with several Higgs bosons.

2.1.3 Fermion Sector

The fermions in the SM consist of leptons and quarks. Leptons are not charged under $SU(3)_C$ while quarks carry colour. While leptons exist as free particles, quarks are always bound inside hadrons, such as protons and neutrons. The fermions can be ordered into three ($i = 1, 2, 3$) generations or families, which are identical with respect to the quantum numbers of their contents and differ only by the mass of the particles. The fermions building $SU(2)_L$ doublets ($l_{i,L}$ for leptons and $q_{i,L}$ for

quarks) are called left-handed, while the fermions building $SU(2)_L$ singlets ($e_{i,R}$ for leptons, $u_{i,R}$ for up-type quarks, $d_{i,R}$ for down-type quarks) are right-handed.

An explicit Dirac-type fermion mass term in the Lagrangian would not preserve gauge invariance. Fermion masses are generated by so called Yukawa couplings of the Higgs field to the fermion fields which can be written as

$$\mathcal{L}_{\text{Yukawa}} = - \left(\bar{q}_L \mathbf{y}_u \phi^C u_R + \bar{q}_L \mathbf{y}_d \phi d_R + \bar{l}_L \mathbf{y}_l \phi e_R + h.c. \right) \quad (2.13)$$

where $\phi^C = i\sigma_2 \phi^*$ is the charge conjugated Higgs field (note that the same Higgs doublet is used to give mass to up-type and down-type fermions), q_L, l_L, u_R, d_R, e_R are 3-component vectors in family space, and $\mathbf{y}_u, \mathbf{y}_d$ and \mathbf{y}_l are the 3×3 Yukawa coupling matrices.

There are no right-handed neutrinos in the SM (in its established form) and the neutrinos remain massless in the SM.⁶

Replacing the Higgs field by its vacuum expectation value, one finds the lepton mass matrix (which can be diagonalised)

$$\mathbf{m}_l = \frac{v}{\sqrt{2}} \mathbf{y}_l. \quad (2.14)$$

The mass eigenstates of the quarks are obtained by unitary transformations on the quark fields; the diagonalised mass matrices for up- and down type fermions read

$$\mathbf{m}_u = \frac{v}{\sqrt{2}} (U_L^U)^\dagger \mathbf{y}_u U_R^U, \quad \mathbf{m}_d = \frac{v}{\sqrt{2}} (U_L^D)^\dagger \mathbf{y}_d U_R^D. \quad (2.15)$$

The product

$$V_{\text{CKM}} = (U_L^U)^\dagger U_L^D \quad (2.16)$$

is referred to as the Cabibbo-Kobayashi-Maskawa (CKM) matrix. A complex phase in the quark mixing matrix provides the only source of \mathcal{CP} -violation in the SM.

2.1.4 Gauge Fixing, Ghost Sector

The quantisation of the SM, requires the insertion of additional, gauge fixing, terms in the Lagrangian. Using a renormalizable 't Hooft gauge the gauge fixing term is

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2} \left[(F^A)^2 + (F^Z)^2 + 2F^{W^+} F^{W^-} \right], \quad (2.17)$$

⁶ The evidence of neutrino oscillation (see e.g. [10]) implies that neutrinos are (against the original assumption) massive. Introducing right handed neutrinos, Dirac mass terms can easily be added. Another possibility is to write down Majorana mass terms. In this thesis neutrinos can be assumed to be massless.

with

$$\begin{aligned}
 F^{W^\pm} &= (\xi_1^W)^{-\frac{1}{2}} \partial^\mu W_\mu^\pm \mp i M_W (\xi_2^W)^{\frac{1}{2}} G^\pm \\
 F^Z &= (\xi_1^Z)^{-\frac{1}{2}} \partial^\mu Z_\mu - M_Z (\xi_2^Z)^{\frac{1}{2}} G^0 \\
 F^A &= (\xi_1^A)^{-\frac{1}{2}} \partial^\mu A_\mu.
 \end{aligned} \tag{2.18}$$

Here $\xi_1^W, \xi_2^W, \xi_1^Z, \xi_2^Z$ and ξ_1^A are five gauge parameters. The parameters ξ_i^α can be chosen freely, since in the end the physical observables must be independent of the gauge fixing. In most parts of this work (if not stated otherwise) the particularly simple Feynman-'t Hooft gauge is chosen, where all ξ_i^α are set equal to 1.

In this formulation non-physical contributions appear, which must be canceled. This cancellation is achieved by introducing the so called Faddeev-Popov ghost $u^\alpha(x)$ and antighost $\bar{u}^\alpha(x)$ fields ($\alpha = W^\pm, A, Z$). Ghosts are unphysical mathematical entities, which do not correspond to 'real' external particles and only appear as virtual particles within loops. The additional Faddeev-Popov term in the Lagrangian is

$$\mathcal{L}_{\text{ghost}} = \sum_{\alpha, \beta=W^\pm, A, Z} \bar{u}^\alpha(x) \frac{\delta F^\alpha}{\delta \theta^\beta(x)} u^\beta(x). \tag{2.19}$$

where the θ^α denote infinitesimal gauge transformations and $\delta F^\alpha / \delta \theta^\beta$ are variations of the gauge fixing operators F^α ($\alpha = W^\pm, A, Z$) under θ^α . In the Feynman-'t Hooft gauge each ghost field acquires the same mass parameter as its associated gauge boson field. Also the Goldstone bosons acquire a non-zero mass parameter in this gauge and must be included in our calculations.

2.1.5 Full SM Lagrangian

The full Lagrangian density of the SM is

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}} \tag{2.20}$$

where in the term \mathcal{L}_{kin} the kinetic term is written down for $\psi = l_{i,L}, e_{i,R}, q_{i,L}, u_{i,R}, d_{i,R}$ ($i = 1, 2, 3$) and $A_\mu^a = W^a$ ($a = 1, 2, 3$), B, g^a ($a = 1 \dots 8$). The derivative in Eq. (2.1) is replaced by the covariant derivative of Eq. (2.3).

2.1.6 Shortcomings of the SM

The Standard Model of particle physics is a very successful theory describing most experimental measurements with high precision. However there are some observations and theoretical shortcomings indicating that the SM cannot be the

complete description of elementary particle physics but needs to be embedded in a more complete theory. Some shortcomings of the SM are outlined in the following.

As a start the gravitational force which has profound implications for our everyday lives cannot be described within the SM. Quantum gravitational effects are expected to become relevant only at very high scales ($M_{\text{Planck}} = 10^{19} \text{ GeV}$) and therefore are expected to have hardly any impact on particle physics phenomenology. Nevertheless the failure of the SM to include a description of gravity clearly indicates that the SM cannot be an exhaustive theory of nature: It is known to fail (at the latest) at the Planck scale where quantum gravitational effects become important, which implies that the SM must be an effective theory which can be valid only up to a cutoff scale Λ , at which new physics appears.

This has drastic implications for the stability of the Higgs mass. In the SM the Higgs mass is a free parameter, while one might expect that, in a more fundamental theory, the Higgs mass value can be predicted. Quantum level effects affect the Higgs mass and must be included in the calculation of the Higgs mass value giving a correction term ΔM_H^2 , thus

$$M_H^2 = M_{H,0}^2 + \Delta M_H^2. \quad (2.21)$$

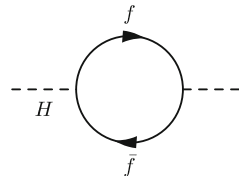
Diagrams such as the one depicted in Fig. 2.1 (showing the one-loop correction from a fermion loop) contribute to ΔM_H^2 . Calculating this diagram and cutting off the integral at the new-physics scale Λ yields a correction term to the Higgs mass

$$\Delta M_H^2 = -\frac{y_f^2}{8\pi^2} \Lambda^2 + \dots \quad (2.22)$$

The ellipsis denote terms that grow at most logarithmically with Λ . This means that for $\Lambda = M_{\text{Planck}}$ the corrections to the Higgs mass are of the size $\Delta M_H^2 \approx 10^{38} \text{ GeV}^2$. On the other hand we have observed a Higgs boson at 126 GeV . To get $M_H \sim 126 \text{ GeV}$, an immense cancellation between the ΔM_H^2 correction and $M_{H,0}^2$ is necessary (extreme ‘fine-tuning’). This seems very unnatural and is known as the Hierarchy Problem.

There is another theoretical unaesthetic feature of the SM: The SM gauge group is not simple, so that the cancellation of gauge anomalies is accidental and the existence of electric charges in fractional amounts is not explained. Another shortcoming (mentioned earlier) is the observation of non-zero neutrino masses, which are not

Fig. 2.1 Fermion loop diagram which leads to quadratic divergent corrections to the Higgs mass



described in the SM in its current form. Further the hierarchy of the fermion masses ($5 \times 10^{-4} \rightarrow 10^2 \text{ GeV}$) remains unexplained in the SM ('Flavour imbalance').

Many astrophysical observations have shown evidence that more gravitationally interacting matter (so called dark matter) than the visible baryonic matter must exist in the universe. Recent results reveal that the largest part ($\sim 68.3\%$) of the total energy in the universe consists of so called dark energy, while dark matter accounts for $\sim 26.8\%$ [11, 12]. Neither dark energy nor dark matter can be explained within the SM. If dark matter consists of elementary particles, these can at most interact weakly with other particles and they have to be stable over cosmological timescales. Another observation is the baryon asymmetry in the universe [11, 13]. This discrepancy cannot be explained by just the \mathcal{CP} -violation from the CKM phase in the SM alone and indicates that further sources of \mathcal{CP} -violation beyond the SM must exist.

2.2 The Minimal Supersymmetric Standard Model

Supersymmetry (SUSY) is an attractive and popular guideline to extend the SM. Some of the shortcomings of the SM mentioned in the previous section are addressed in supersymmetric models. It is a natural extension of the space-time symmetry of the SM which relates fermionic and bosonic degrees of freedom. In the Minimal Supersymmetric Standard Model (MSSM) the global symmetries of the SM are minimally extended while the local gauge symmetries remain unchanged. In this section we will introduce and motivate weak scale supersymmetry and its minimal realisation, the MSSM. I will discuss how the shortcomings of the SM are addressed in the MSSM. Further I will introduce the particle content of the MSSM and set the notation for later chapters.

2.2.1 Concepts of Supersymmetric Models

Possible extensions of the Poincaré group are highly restricted by the Haag-Łopuszański-Sohnius theorem [14] stating that (in 4-dimensional QFT) the Poincaré group can only (non trivially) be extended by (N) fermionic operators. The generator of ($N = 1$) supersymmetry is a fermionic operator Q which converts a bosonic state into a fermionic state and vice versa: $Q |\text{boson}\rangle = |\text{fermion}\rangle$ and $Q |\text{fermion}\rangle = |\text{boson}\rangle$ and has to fulfil the SUSY algebra

$$\begin{aligned} \{Q_\alpha, Q_\alpha^\dagger\} &= (\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu \\ \{Q_\alpha, Q_\beta\} &= 0, \quad \{Q_\alpha^\dagger, Q_\beta^\dagger\} = 0 \\ [Q_\alpha, P_\mu] &= 0, \quad [Q_\alpha^\dagger, P_\mu] = 0 \end{aligned} \tag{2.23}$$

where P_μ is the four-momentum, α, β and $\dot{\alpha}, \dot{\beta}$ are spinor indices.

In supersymmetric extensions of the Standard Model the SM fermions and gauge bosons get superpartners, with identical quantum numbers except for the spin. This implies that, in an unbroken supersymmetric model, particles and superparticles have degenerate masses.

This already suggests a solution of the hierarchy problem of the SM. In SUSY all fermions have superpartners, \tilde{f} , which give additional corrections to the Higgs mass. Diagrams as the one depicted in Fig. 2.2 lead to a contribution

$$\Delta M_H^2 = \frac{\tilde{y}_f}{8\pi^2} \Lambda^2 + \dots \quad (2.24)$$

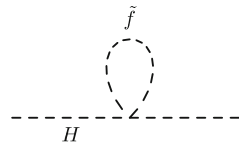
Here again the ellipsis denote terms that grow at most logarithmically with the cut off scale. Adding the contributions to the Higgs mass from fermions and their superpartners, we see that (independent on the masses if the superpartners) the quadratic divergent terms cancel if $\tilde{y}_f = y_f^2$. In an unbroken supersymmetric model the fermions and their superpartners have the same mass and the Higgs mass corrections cancel completely. But as we will argue in Sect. 2.2.4, supersymmetry must be broken, which implies that the masses of the superpartners differ from the masses of the SM particles. This will not spoil the cancellation of the quadratic divergencies, as long as we require the SUSY breaking to maintain the relation $\tilde{y}_f = y_f^2$. Naively one would think that the remaining corrections (after the cancellation of the quadratically divergent parts) are proportional to squared mass difference $m_{\tilde{f}}^2 - m_f^2$, however calculating the corrections in the MSSM, one finds that in the Higgs mass is only logarithmically sensitive to the mass difference between fermions and their superpartners

$$\Delta M_H^2 \sim \log \left(\frac{m_{\tilde{f}}^2}{m_f^2} \right). \quad (2.25)$$

Therefore the remaining corrections to the Higgs mass stay relatively small and the Higgs mass in the MSSM is protected from large loop corrections. However in the discussion of radiative electroweak symmetry breaking (see below) we will see that also in the context of the MSSM, large corrections sensitive to $m_{\tilde{f}}^2$ appear, indicating that splitting between the masses of the SM and the SUSY particles should not be not too large.

The SM particles and their superpartners are combined within supermultiplets. Quarks and leptons receive scalar superpartners, squarks and sleptons. Supersymmetric extensions of the SM contain several scalar Higgs bosons (as we will discuss

Fig. 2.2 Sfermion loop diagram which leads to quadratic divergent corrections to the Higgs mass



below), which get fermionic superpartners, the higgsinos. All these particles are described by so called chiral supermultiplets, each containing a two-component Weyl⁷ fermion ψ , a complex scalar field ϕ and an auxiliary field F . Also the electroweak gauge bosons of the SM and the gluons get fermionic superpartners. These are described by vector multiplets, containing a spin-1 vector boson A_μ^a , a spin-1/2 Majorana fermion λ^a and a scalar auxiliary field D^a , where the index runs over the adjoint representation of the gauge group: $a = 1, \dots, 8$ for $SU(3)_C$, $a = 1, \dots, 3$ for $SU(2)_L$ and $a = 1$ for $U(1)_Y$.

The part of a supersymmetric Lagrangian describing the n free chiral supermultiplets ($i = 1, \dots, n$) is

$$\mathcal{L}_{\text{free}} = \partial_\mu \phi_i^\dagger \partial^\mu \phi_i + \psi_i^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_i + F_i^\dagger F_i. \quad (2.26)$$

The interaction term of the chiral multiplets can be written as (we define W_i and W_{ij} below)

$$\mathcal{L}_{\text{int}} = \left(-\frac{1}{2} W_{ij} \psi_i \psi_j + W_i F_i \right) + c.c. \quad (2.27)$$

Using the Euler-Lagrange equation of motion for the auxiliary fields F_i and F_i^\dagger one finds $F_i = -W_i^\dagger$, $F_i^\dagger = -W_i$. Here

$$W_i = \frac{\partial W}{\partial \phi_i} \text{ and } W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}. \quad (2.28)$$

W is a complex analytic (or holomorphic) function, which determines the allowed interaction terms for chiral multiplets and is called superpotential. The part of the Lagrangian describing chiral multiplets is $\mathcal{L}_{\text{chiral}} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$.

The Lagrangian describing the vector multiplets is

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + i \lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a, \quad (2.29)$$

One proceeds as in the SM and promotes the derivatives in Eq. (2.26) to covariant derivatives, in order to obtain a gauge invariant Lagrangian.⁸ This couples the gauge boson in the vectormultiplets to the fermions and scalars of the chiral multiplets. Additional terms must be added to respect supersymmetry.

Including all possible gauge invariant, renormalizable interaction terms, the supersymmetric Lagrangian is

⁷In Sect. 2.1 we used Dirac notation to describe the SM fermions. However it turns out to be more convenient to use the two-component Weyl spinor notation for the fermions in the supermultiplets. For the definition of Weyl fermions see Ref. [15].

⁸As mentioned earlier, we define the $SU(2)_L$ covariant derivative in the SUSY models with opposite sign than in the SM, following the FeynArts [4–9] conventions.

$$\mathcal{L}_{\text{susy}} = \mathcal{L}_{\text{chiral}}(\text{with } \partial_\mu \rightarrow D_\mu) + \mathcal{L}_{\text{gauge}} - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) + g(\phi^* T^a \phi)D^a. \quad (2.30)$$

As introduced earlier T^a are the generators of the gauge group. For the auxiliary fields D^a one finds the equation of motion $D^a = -g(\phi^* T^a \phi)$. The scalar potential part of $\mathcal{L}_{\text{susy}}$ is given by the term $-V(\phi, \phi^*)$ with

$$V(\phi, \phi^*) = F_i^\dagger F_i + \frac{1}{2}D^a D^a = W_i^\dagger W_i + \frac{1}{2}g^2(\phi^\dagger T^a \phi)^2. \quad (2.31)$$

The first term (‘F-term’) comes from $\mathcal{L}_{\text{chiral}}$, the second term (‘D-term’) combines the last term of Eq. (2.30) and the last term in Eq. (2.29). It is a peculiarity of supersymmetric models that the scalar potential is given by the Yukawa (F-term) and gauge (D-term) interactions.

2.2.2 The MSSM Superpotential

The chiral multiplets in the MSSM are given in Table 2.1. One can see from the table that the MSSM has two Higgs doublets, H_1 and H_2 .⁹ We will explain below why two Higgs doublets are needed. We follow the convention to define chiral multiplets in terms of left-handed Weyl spinors. That means we regard the right-handed fermions and their superpartners as conjugates of the left-handed fields. The vectormultiplets of the MSSM are listed in Table 2.2.

The superpotential for the MSSM with conserved R-parity (see Sect. 2.2.3) is given by

$$W^{\text{MSSM}} = \bar{u}_Y Q H_2 - \bar{d}_Y Q H_1 - \bar{e}_Y L H_1 + \mu H_2 H_1 \quad (2.32)$$

Table 2.1 Chiral supermultiplets in the MSSM. Family and colour indices are suppressed

	Label	Spin 0	Spin 1/2	$(SU(3)_C, SU(2)_L, U(1)_Y)$
Squarks, quarks	Q	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	\bar{u}	$\tilde{\bar{u}} = \tilde{u}_L^\dagger$	\bar{u}_L	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	\bar{d}	$\tilde{\bar{d}} = \tilde{d}_L^\dagger$	\bar{d}_L	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
Sleptons, leptons	L	$\tilde{L} = (\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	\bar{e}	$\tilde{\bar{e}} = \tilde{e}_L^\dagger$	\bar{e}_L	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, Higgsinos	H_1	$H_1 = (H_1^0, H_1^-)$	$(\tilde{H}_1^0, \tilde{H}_1^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	H_2	$H_2 = (H_2^+, H_2^0)$	$(\tilde{H}_2^+, \tilde{H}_2^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

⁹In literature the two Higgs doublets are often called $H_u \equiv H_2$ and $H_d \equiv H_1$. For the Higgs doublets we use the same notation for the chiral supermultiplets and for its scalar entry.

Table 2.2 Vector supermultiplets in the MSSM

	Spin 1/2	Spin 1	$(SU(3)_C, SU(2)_L, U(1)_Y)$
Gluino, gluon	\tilde{g}	g	$(\mathbf{8}, \mathbf{1}, 0)$
Wino, W -boson	$\tilde{W}^\pm, \tilde{W}^0$	W^\pm, W^0	$(\mathbf{1}, \mathbf{3}, 0)$
Bino, B -boson	\tilde{B}	B	$(\mathbf{1}, \mathbf{1}, 0)$

where $Q, \bar{u}, \bar{d}, L, \bar{e}, H_1, H_2$ are the chiral supermultiplets from Table 2.1. The gauge indices and generation indices are suppressed. $\mathbf{y}_u, \mathbf{y}_d$ and \mathbf{y}_l are the dimensionless Yukawa coupling parameters, which are 3×3 matrices in family space. Here one can see that in the MSSM (contrary to the SM) indeed two Higgs doublets are needed to give mass to up- and down-type fermions: In the term $\bar{u}\mathbf{y}_u Q H_2$ one cannot replace H_2 by $H_1^C \propto H_1^*$, since W is a complex analytic (or holomorphic) function and therefore no complex conjugates may appear. Two Higgsinos (and therewith Higgs doublets with opposite hypercharge) are also needed for a successful cancellation of the anomaly that would result from only one Higgsino fermion. To get the supersymmetric Lagrangian of the MSSM, the chiral and vector supermultiplets and the MSSM superpotential must be inserted in Eq. (2.30).

2.2.3 *R-Parity*

Lepton and baryon number conservation have experimentally been probed precisely and searches (e.g. proton decay) have not shown deviations at the present level of sensitivity. Whereas in the SM these symmetries are an accidental consequence of the field content and the gauge symmetry, in supersymmetric models lepton and baryon number can be violated, which would lead to an unstable proton. One way to prevent a too rapid proton decay is to require that every coupling in the MSSM preserves R parity

$$R = (-1)^{3B+L+2S} = \begin{cases} +1 & \text{for SM particles} \\ -1 & \text{for SUSY particles} \end{cases} \quad (2.33)$$

where B is the Baryon number (quarks have baryon number $+\frac{1}{3}$, the antiquarks have baryon number $-\frac{1}{3}$), L the Lepton number (leptons have lepton number $+1$, the antileptons have lepton number -1) and S is the spin. R parity conservation can also theoretically be motivated, it can e.g. be a remnant of a $U(1)$ gauge symmetry. The conservation of R parity implies that supersymmetric particles can only be produced in pairs and that the lightest supersymmetric particle (LSP) is stable. This has important phenomenological consequences, since the LSP can be a suitable dark matter candidate.

2.2.4 SUSY Breaking

SUSY particles have yet to be observed experimentally. Given that SUSY particles would have the same mass as their SM partners in an unbroken supersymmetric theory and that no experimental signal has been seen yet, supersymmetry, if existing, cannot be an exact symmetry but must be broken. Spontaneous breaking of supersymmetry in a hidden sector can be mediated to the visible sector by different mechanisms. The SUSY breaking can generally be parameterised at low scale without being restricted to a particular SUSY breaking mechanism. The breaking is described phenomenologically by explicitly adding terms, called soft breaking terms, to the Lagrangian density. The term ‘soft’ means that the relations between the dimensionless couplings are not modified and thus no quadratic divergencies are reintroduced. The soft breaking terms in the MSSM are [16]

$$\begin{aligned}
 -\mathcal{L}_{\text{soft}}^{\text{MSSM}} = & \frac{1}{2} \left(M_1 \tilde{B} \tilde{B} + M_2 \tilde{W}^a \tilde{W}^a + M_3 \tilde{g}^a \tilde{g}^a + c.c \right) \\
 & + \left(\tilde{u} \mathbf{a}_u \tilde{Q} H_2 - \tilde{d} \mathbf{a}_d \tilde{Q} H_1 - \tilde{e} \mathbf{a}_e \tilde{L} H_1 + c.c \right) \\
 & + \mathbf{m}_{\tilde{Q}}^2 \tilde{Q}^\dagger \tilde{Q} + \mathbf{m}_{\tilde{L}}^2 \tilde{L}^\dagger \tilde{L} + \mathbf{m}_{\tilde{U}}^2 \tilde{u}^\dagger \tilde{u} + \mathbf{m}_{\tilde{D}}^2 \tilde{d}^\dagger \tilde{d} + \mathbf{m}_{\tilde{E}}^2 \tilde{e}^\dagger \tilde{e} \\
 & + \tilde{m}_2^2 H_2^\dagger H_2 + \tilde{m}_1^2 H_1^\dagger H_1 - \left(m_{12}^2 H_2 H_1 + c.c \right)
 \end{aligned} \tag{2.34}$$

where M_1 , M_2 and M_3 are the bino, wino and gluino mass terms; in the term $M_3 \tilde{g}^a \tilde{g}^a$ the gauge index a runs from 1 to 8 and in the term $M_2 \tilde{W}^a \tilde{W}^a$ from 1 to 3. \mathbf{a}_u , \mathbf{a}_d and \mathbf{a}_e (3×3 matrices in family space) are the trilinear sfermion couplings and $\mathbf{m}_{\tilde{Q}}^2$, $\mathbf{m}_{\tilde{U}}^2$, $\mathbf{m}_{\tilde{D}}^2$, $\mathbf{m}_{\tilde{L}}^2$, $\mathbf{m}_{\tilde{E}}^2$ (3×3 matrices in family space) are the sfermion squared mass matrices. The parameters in the last line are the Higgs soft SUSY breaking parameters \tilde{m}_2^2 , \tilde{m}_1^2 , and m_{12}^2 .

2.2.5 Constrained Models: CMSSM and pMSSM

A remarkable feature of the MSSM is that it allows for gauge coupling unification (provided that the SUSY particles are at the TeV scale) at a high scale $M_{\text{GUT}} \sim 10^{16}$ GeV, which is called Grand Unification or GUT scale. The running of gauge couplings is discussed in Sect. 3.2.3. Therefore it is a popular assumption that also the gaugino masses unify at that scale, which leads to the relation

$$M_1 = \frac{3}{5} \frac{s_W^2}{c_W^2} M_2 \approx \frac{1}{2} M_2. \tag{2.35}$$

This assumption is used throughout this work. No relation is assumed for M_3 unless stated otherwise.

Going one step further one can assume that at the GUT scale the theory is described by only a few parameters. The constrained MSSM (CMSSM) is a SUSY model, which contains only five parameters: the universal scalar (soft) mass m_0 , the universal gaugino (soft) mass $m_{1/2}$, the universal trilinear coupling A_0 (all at GUT scale), the ratio of the vacuum expectation values of the two Higgs doublets $\tan \beta$ and the sign of the Higgsino mass parameter. The weak scale parameters are then obtained by renormalization group running (see Sect. 3.2.3).

On the other hand low-scale models, which are not directly derived from some high-scale (GUT) theory, are termed pMSSM models. In this bottom-up approach no assumptions about the mechanism of SUSY breaking are made.

The soft breaking terms introduce plenty of new parameters: in total the MSSM involves 105 new parameters (masses, mixing angles and phases). However many of these new parameters lead to new sources of flavour mixing and \mathcal{CP} -violation, both strongly constrained by experiments (see Ref. [17] and references therein). In phenomenological studies of the MSSM one often makes experimentally motivated and simplifying assumptions, reducing the number of MSSM parameters significantly. In the following we always assume

$$\mathbf{m}_{\tilde{Q}, \tilde{L}}^2 = \begin{pmatrix} M_{\tilde{Q}_1, \tilde{L}_1}^2 & 0 & 0 \\ 0 & M_{\tilde{Q}_2, \tilde{L}_2}^2 & 0 \\ 0 & 0 & M_{\tilde{Q}_3, \tilde{L}_3}^2 \end{pmatrix}, \quad \mathbf{m}_{\tilde{U}, \tilde{D}, \tilde{E}}^2 = \begin{pmatrix} M_{\tilde{U}_1, \tilde{D}_1, \tilde{E}_1}^2 & 0 & 0 \\ 0 & M_{\tilde{U}_2, \tilde{D}_2, \tilde{E}_2}^2 & 0 \\ 0 & 0 & M_{\tilde{U}_3, \tilde{D}_3, \tilde{E}_3}^2 \end{pmatrix} \quad (2.36)$$

and

$$\mathbf{a}_u = \begin{pmatrix} A_u y_u & 0 & 0 \\ 0 & A_c y_c & 0 \\ 0 & 0 & A_t y_t \end{pmatrix}, \quad \mathbf{a}_d = \begin{pmatrix} A_d y_d & 0 & 0 \\ 0 & A_s y_s & 0 \\ 0 & 0 & A_b y_b \end{pmatrix}, \quad \mathbf{a}_\ell = \begin{pmatrix} A_e y_e & 0 & 0 \\ 0 & A_\mu y_\mu & 0 \\ 0 & 0 & A_\tau y_\tau \end{pmatrix} \quad (2.37)$$

where the y_f are the Yukawa couplings. The Yukawa couplings of the first two generations are small and often neglected. These assumptions already significantly reduce the number of free parameters. In this work (if not stated otherwise) we allow the parameters M_1, M_2, M_3, A_f ($f = u, d, c, s, t, b, e, \mu, \tau$) and μ to be complex. The phase of either M_1, M_2 or μ (we usually choose M_2) can be rotated away.

2.2.6 Sfermion Sector

Putting together the terms of the form $\tilde{f}_L^\dagger \tilde{f}_L$, $\tilde{f}_R^\dagger \tilde{f}_R$ and $\tilde{f}_L^\dagger \tilde{f}_R$, $\tilde{f}_R^\dagger \tilde{f}_L$ ($\tilde{f}_{L/R}$ denoting the superpartner of a left/right-handed fermion f) appearing in the F-term, the D-term and the soft SUSY breaking terms, one can write the sfermion mass part of the MSSM Lagrangian as

$$- \begin{pmatrix} \tilde{f}_L^\dagger & \tilde{f}_R^\dagger \end{pmatrix} \mathbf{M}_{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} \quad (2.38)$$

Neglecting flavour violation in the sfermion sector (assuming Eq. (2.36)), the 2×2 sfermion mass matrix for each flavour can be written as

$$\mathbf{M}_{\tilde{f}} = \begin{pmatrix} M_{\tilde{f}_L}^2 + m_f^2 + M_Z^2 \cos 2\beta (I_f^3 - Q_f s_w^2) & m_f X_f^* \\ m_f X_f & M_{\tilde{f}_R}^2 + m_f^2 + M_Z^2 \cos 2\beta Q_f s_w^2 \end{pmatrix}, \quad (2.39)$$

where I_f^3 is the third component of the weak isospin, Q_f the electric charge ($Q = I^3 + Y/2$ where Y is the hypercharges), m_f is the corresponding fermion mass and $M_{\tilde{f}_L}^2$ and $M_{\tilde{f}_R}^2$ are defined by

$$\begin{aligned} M_{\tilde{f}_L}^2 &= \begin{cases} M_{\tilde{Q}_i}^2 & \text{for left-handed squarks} \\ M_{\tilde{L}_i}^2 & \text{for left-handed sleptons} \end{cases} \\ M_{\tilde{f}_R}^2 &= \begin{cases} M_{\tilde{U}_i}^2 & \text{for right-handed up-type squarks} \\ M_{\tilde{D}_i}^2 & \text{for right-handed down-type squarks} \\ M_{\tilde{E}_i}^2 & \text{for right-handed sleptons} \end{cases} \end{aligned} \quad (2.40)$$

where i indicates the generation. The mixing parameter X_f is defined by

$$X_f = A_f - \mu^* \{\cot \beta, \tan \beta\}, \quad (2.41)$$

where $\cot \beta$ applies to up-type squarks and $\tan \beta$ for down-type squarks and charged sleptons. In the MSSM with complex parameters, the trilinear couplings $A_f = |A_f| \exp(i\phi_{A_f})$ and the μ parameter $\mu = |\mu| \exp(i\phi_\mu)$ can have non-zero complex phases. Diagonalizing the mass matrix by a complex 2×2 unitary matrix $U_{\tilde{f}}$ (which can be parameterised by an angle $\theta_{\tilde{f}}$ plus a complex phase) gives the sfermion mass eigenstates

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = U_{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}, \quad (2.42)$$

In the following we will use the convention $m_{\tilde{f}_1} \leq m_{\tilde{f}_2}$. The explicit mass eigenvalues are then given by

$$m_{\tilde{f}_{1,2}}^2 = m_f^2 + \frac{1}{2} \left[M_{\tilde{f}_L}^2 + M_{\tilde{f}_R}^2 + I_3^f M_Z^2 \cos 2\beta \right. \\ \left. \mp \sqrt{[M_{\tilde{f}_L}^2 - M_{\tilde{f}_R}^2 + M_Z^2 \cos 2\beta (I_3^f - 2Q_f s_w^2)]^2 + 4m_f^2 |X_f|^2} \right]. \quad (2.43)$$

2.2.7 Chargino Sector

The electrically charged Higgsinos and gauginos mix into charginos $\tilde{\chi}_{1,2}^\pm$. Defining the gauge-eigenstates as

$$\tilde{g}^+ = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \end{pmatrix}, \quad \tilde{g}^- = \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_1^- \end{pmatrix} \quad (2.44)$$

the chargino mass terms in the MSSM Lagrangian can be written as

$$\frac{1}{2} [\tilde{g}^{+T} \mathbf{M}_{\tilde{\chi}^\pm}^T \tilde{g}^- + \tilde{g}^{-T} \mathbf{M}_{\tilde{\chi}^\pm} \tilde{g}^+] + h.c. \quad (2.45)$$

with

$$\mathbf{M}_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}, \quad (2.46)$$

with the soft breaking parameter M_2 . The mass eigenvalues are obtained by diagonalizing the mass matrix using two unitary matrices U and V

$$U^* \mathbf{M}_{\tilde{\chi}^\pm} V^{-1} = \text{diag}(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}) \quad (2.47)$$

with the chargino masses $m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^\pm}$. The eigenvalues are

$$m_{\tilde{\chi}_{1,2}}^2 = \frac{M_2^2 + |\mu|^2 + 2M_W^2}{2} \mp \sqrt{\left(\frac{M_2^2 + |\mu|^2 + 2M_W^2}{2} \right)^2 - |\mu M_2 - M_W^2 \sin 2\beta|^2}. \quad (2.48)$$

2.2.8 Neutralino Sector

The neutral Higgsinos and gauginos in the MSSM mix (as a result of electroweak symmetry breaking), the resulting mass eigenstates are called neutralinos. Defining the gauge-eigenstate base as

$$\tilde{G}^0 = \begin{pmatrix} \tilde{B} \\ \tilde{W}^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix} \quad (2.49)$$

one can rewrite the mass terms of the MSSM Lagrangian containing \tilde{G}^0 as

$$\frac{1}{2} \tilde{G}^{0T} \mathbf{M}_{\tilde{\chi}^0} \tilde{G}^0 + h.c. \quad (2.50)$$

The neutralino mass matrix is given by

$$\mathbf{M}_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z s_W \cos \beta & M_Z s_W \sin \beta \\ 0 & M_2 & M_Z c_W \cos \beta & -M_Z c_W \sin \beta \\ -M_Z s_W \cos \beta & M_Z c_W \cos \beta & 0 & -\mu \\ M_Z s_W \sin \beta & -M_Z c_W \sin \beta & -\mu & 0 \end{pmatrix}. \quad (2.51)$$

The neutralino masses are obtained by a diagonalization of the mass matrix using a single, complex, unitary matrix N

$$\text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}) = N^* \mathbf{M}_{\tilde{\chi}^0} N^{-1} \quad (2.52)$$

The neutralinos are ordered in mass such that $m_{\tilde{\chi}_i^0} \leq m_{\tilde{\chi}_j^0}$ for $i < j$. The gaugino masses M_1 and M_2 may (in addition to μ) be complex. However, there are in total only two physically relevant phases. One phase, usually taken to be that for M_2 , can therefore be rotated away (as we already discussed in Sect. 2.2.5).

2.2.9 Gluino Sector

The gluino is a colour octet fermion and cannot mix with any other MSSM particle. Its mass

$$m_{\tilde{g}} = |M_3| \quad (2.53)$$

is directly given by the mass term in the soft breaking Lagrangian.

2.2.10 Electroweak Symmetry Breaking and the MSSM Higgs Sector

Writing the components of the two Higgs doublets (with opposite hypercharge $-Y_{H_1} = Y_{H_2} = 1$) as $H_1 = (H_{11}, H_{12}) = (H_1^0, H_1^-)$ and $H_2 = (H_{21}, H_{22}) = (H_2^+, H_2^0)$, the scalar potential of the MSSM can be written as

$$V_H^{\text{MSSM}} = m_1^2 H_{1i}^* H_{1i} + m_2^2 H_{2i}^* H_{2i} - \epsilon^{ij} \left(m_{12}^2 H_{1i} H_{2j} + m_{12}^{2*} H_{1i}^* H_{2j}^* \right) + \frac{1}{8} (g_1^2 + g_2^2) (H_{1i}^* H_{1i} - H_{2i}^* H_{2i})^2 + \frac{1}{2} g_2^2 |H_{1i}^* H_{2i}|^2.$$

where the indices $\{i, j\} = \{1, 2\}$ refer to the respective Higgs doublet component and $\epsilon^{12} = 1$. Here $m_1^2 \equiv \tilde{m}_1^2 + |\mu|^2$ and $m_2^2 \equiv \tilde{m}_2^2 + |\mu|^2$, where \tilde{m}_1^2 and \tilde{m}_2^2 are the real soft breaking terms. The soft breaking parameter m_{12}^2 can a priori be complex, however its complex phase can be rotated away (see Refs. [18, 19]) and from here on we will treat m_{12}^2 as a real parameter. The terms proportional to $|\mu|^2$ are F-term contributions while the terms proportional to g_1 and g_2 arise from the D-terms. The terms proportional to \tilde{m}_1^2 , \tilde{m}_2^2 and m_{12}^2 are the last three terms of the soft breaking Lagrangian Eq. (2.34).

To get massive gauge bosons, V_H^{MSSM} must have a minimum which breaks the electroweak symmetry. Interestingly, the conditions to find such a minimum cannot be fulfilled for $\tilde{m}_1^2 = \tilde{m}_2^2$. This also means that $\tilde{m}_1^2 = \tilde{m}_2^2 = 0$ is not possible and therefore in the MSSM SUSY breaking is necessary for electroweak symmetry breaking.

GUT models often predict $\tilde{m}_1^2 = \tilde{m}_2^2$ at a high scale. In the evolution of the \tilde{m}_2^2 parameter down to the electroweak scale (RGE running is discussed in Eq. (3.2.3)), radiative corrections occur involving terms proportional to the squared masses of the SUSY particles. In order to fulfill the minimization conditions of the Higgs potential at the electroweak scale, the SUSY particle masses should be at the TeV scale. Otherwise unnaturally large cancellations (large ‘fine tuning’) would be necessary to trigger electroweak symmetry breaking. This mechanism to activate electroweak symmetry breaking via quantum corrections is termed ‘radiative electroweak symmetry breaking’.

When the electroweak symmetry is broken, the neutral components of the Higgs doublets get vevs

$$\langle H_1^0 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle H_2^0 \rangle = \frac{v_2}{\sqrt{2}} \quad (2.54)$$

while the charged components can (as in the SM) be chosen zero at the minimum of V_H^{MSSM} . The ratio between the two vevs defines the parameter

$$\tan \beta = \frac{v_2}{v_1}. \quad (2.55)$$

The two complex doublets can be expanded around the minimum as

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}} (\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad H_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}} (\phi_2 + i\chi_2) \end{pmatrix}. \quad (2.56)$$

ξ is a possible phase between the two Higgs doublets (we will see below that this phase must be zero and that the MSSM Higgs sector is \mathcal{CP} -concerning at tree-level).

The two MSSM Higgs doublets contain eight degrees of freedom. As in the SM, three degrees of freedom give the unphysical Goldstone bosons, which become the longitudinal polarization modes of the Z and W^\pm bosons. The remaining degrees of freedom give the five physical MSSM Higgs bosons. The generated gauge boson masses are given by

$$M_W^2 = c_W^2 M_Z^2 = \frac{1}{2} g_2^2 (v_1^2 + v_2^2). \quad (2.57)$$

We define

$$v \equiv \sqrt{v_1^2 + v_2^2} \sim 174 \text{ GeV}, \quad (2.58)$$

where the value is given by the measured gauge boson masses.

By rearranging the bilinear terms, the Higgs potential can be written as

$$V_H^{\text{MSSM}} = \frac{1}{2} (\phi_1, \phi_2, \chi_1, \chi_2) \mathbf{M}_{\phi\phi\chi\chi} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \chi_1 \\ \chi_2 \end{pmatrix} + (\phi_1^-, \phi_2^-) \mathbf{M}_{\phi^\pm\phi^\pm} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} + \dots, \quad (2.59)$$

with the mass matrices

$$\mathbf{M}_{\phi\phi\chi\chi} = \begin{pmatrix} M_{\phi\phi} & M_{\phi\chi} \\ M_{\phi\chi}^\dagger & M_{\chi\chi} \end{pmatrix} \quad (2.60)$$

and $\mathbf{M}_{\phi^\pm\phi^\pm}$. Non-zero off diagonal elements $M_{\phi\chi}$ of the mass matrix $\mathbf{M}_{\phi\phi\chi\chi}$, lead to a \mathcal{CP} -violating mixing between the \mathcal{CP} -even and \mathcal{CP} -odd states. The non-vanishing entries of $M_{\phi\chi}$ are proportional to $\sin \xi$.

The (tree-level) mixing of the gauge eigenstates into Higgs mass eigenstates is described by

$$\begin{pmatrix} h \\ H \\ A \\ G \end{pmatrix} = U_{\text{MSSM}}^N \begin{pmatrix} \phi_1 \\ \phi_2 \\ \chi_1 \\ \chi_2 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = U_{\text{MSSM}}^C \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}. \quad (2.61)$$

The condition that v_1 and v_2 are stationary points of the Higgs potential leads to the requirement that the phase ξ between the two Higgs doublets has to be zero. Therefore also $M_{\phi\chi}$ vanishes and there is no \mathcal{CP} violation in the MSSM Higgs sector at tree-level. The mixing matrices can then be parametrised by

$$U_{\text{MSSM}}^N = \begin{pmatrix} -\sin \alpha \cos \alpha & 0 & 0 \\ \cos \alpha \sin \alpha & 0 & 0 \\ 0 & 0 & -\sin \beta \cos \beta \\ 0 & 0 & \cos \beta \sin \beta \end{pmatrix}, \quad U_{\text{MSSM}}^C = \begin{pmatrix} \cos \beta \sin \beta \\ -\sin \beta \cos \beta \end{pmatrix}, \quad (2.62)$$

with

$$\tan 2\alpha = \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \tan 2\beta, \quad (2.63)$$

and one finds the tree-level mass relations

$$\begin{aligned} M_{H,h}^2 &= \frac{1}{2} \left(M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta} \right) \\ M_{H^\pm}^2 &= M_A^2 + M_{W^\pm}^2. \end{aligned} \quad (2.64)$$

The mass eigenstates correspond to the neutral Higgs bosons h, H (with $M_h < M_H$) and A , and the charged Higgs pair H^\pm . At tree level, where possible \mathcal{CP} -violating contributions of the soft supersymmetry-breaking terms do not enter, h and H are the light and heavy \mathcal{CP} -even Higgs bosons, and A is \mathcal{CP} -odd. At lowest order the MSSM Higgs sector is fully described by M_Z and two MSSM parameters, often chosen as the \mathcal{CP} -odd Higgs boson mass, M_A , and $\tan \beta$, the ratio of the two vacuum expectation values.

From the above expressions it follows that $M_h < M_Z$ (at tree level). Such a light Higgs boson would be excluded by LEP searches (unless it has strongly suppressed couplings to gauge bosons). But higher order corrections to the Higgs masses are known to be sizeable and must be included; particularly important are the one- and two-loop contributions from top quarks and their scalar top partners.

ATLAS and CMS discovered a Higgs boson at ~ 126 GeV. Within the framework of the MSSM the lighter \mathcal{CP} -even Higgs boson, h , can have a mass of about 126 GeV for sufficiently large M_A and sufficiently large higher-order corrections from the scalar top sector. However, also the interpretation of the discovered particle as the heavy \mathcal{CP} -even Higgs state, H , is, at least in principle, a viable possibility, see Refs. [20–26].¹⁰ The interpretation of the discovered Higgs in the MSSM will be discussed in detail in Chaps. 6 and 7.

2.3 The Next to Minimal Supersymmetric Standard Model

So far there exists no direct evidence for any particular theory beyond the Standard Model. Therefore it is important to examine also other supersymmetric extensions of the SM besides the MSSM. In the Next-to-Minimal Supersymmetric Standard Model (NMSSM) the Higgs sector of the MSSM is enlarged by an additional Higgs singlet, which entails 7 physical Higgs bosons and leads to a rich phenomenology which can differ significantly from the MSSM. In this section we motivate the NMSSM and discuss the relevant particle sectors. Since the matter content remains the same, the fermion sector of the NMSSM is unchanged with respect to the MSSM. Also the

¹⁰This scenario is challenged by the recent ATLAS bound on light charged Higgs bosons [27].

chargino sector of the NMSSM is identical to that in the MSSM (since no new charged degrees of freedom are introduced), whereas the neutralino sector is extended.

2.3.1 Motivation

The MSSM superpotential Eq. (2.65) contains the bilinear term $\mu H_2 H_1$. Since the dimensionful μ parameter is introduced in the supersymmetric theory it is not connected to the SUSY breaking scale. However a value for μ of the order of that scale is necessary to find an acceptable phenomenology. This issue is called the μ -problem of the MSSM [28]. In the NMSSM the corresponding term in the superpotential is replaced by a coupling of the two Higgs doublets to a new singlet field, and the μ parameter arises dynamically from the vev of the singlet and may therefore be close to the SUSY breaking scale.

The solution of the μ -problem is probably the most compelling motivation to study the NMSSM, however there are further phenomenological motivations. The singlet field modifies the Higgs mass relations compared to the MSSM, such that the tree-level mass of the lightest neutral Higgs boson gets an additional NMSSM contribution, which can increase its value. Consequently the radiative corrections needed to shift the mass of the lightest Higgs mass up to 126 GeV can be smaller. In the MSSM a large splitting in the stop sector is necessary to explain the LHC signal in terms of the lightest Higgs. This requirement is relaxed in NMSSM parameter regions, in which the tree-level Higgs mass is larger than the maximal MSSM value [29]. Another reason to study the NMSSM is the enriched dark matter phenomenology due to a fifth neutralino.

2.3.2 The NMSSM Superpotential and Soft Breaking Terms

The NMSSM involves the same supermultiplets as the MSSM listed in Tables 2.1 and 2.2, but in addition to the two Higgs doublets of the MSSM it also contains a scalar singlet S which couples only to the Higgs sector. The NMSSM¹¹ superpotential has the form

$$W^{\text{NMSSM}} = \bar{u} \mathbf{y}_u Q H_2 - \bar{d} \mathbf{y}_d Q H_1 - \bar{e} \mathbf{y}_l L H_1 + \lambda S H_2 H_1 + \frac{1}{3} \kappa S^3. \quad (2.65)$$

Similar to the Higgs doublets, our notation for S is the same for the supermultiplet as for its scalar component. Obviously the superpotential contains the supermultiplet S , while the scalar component occurs in the soft breaking terms. The new contributions of the Higgs singlet to the soft breaking terms are

¹¹We consider the Z_3 -symmetric version of the NMSSM, in which no linear or quartic terms in S appear.

$$\mathcal{L}_{\text{soft}}^{\text{NMSSM}} = \mathcal{L}_{\text{soft}}^{\text{MSSM,mod}} - m_S^2 |S|^2 - (\lambda A_\lambda S H_2 H_1 + \frac{1}{3} \kappa A_\kappa S^3 + h.c.), \quad (2.66)$$

where $\mathcal{L}_{\text{soft}}^{\text{MSSM,mod}}$ is the soft-breaking Lagrangian $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$ of the MSSM, given in Eq. (2.34), but without the term $m_{12}^2 H_2 H_1$.

2.3.3 Higgs Sector of the NMSSM

The additional contributions (and the modified μ term) in the superpotential and in the soft breaking terms lead to a modified Higgs potential in the NMSSM, which contains the additional soft breaking parameters m_S^2 , A_λ , A_κ , as well as the superpotential trilinear couplings λ and κ .

At tree level no \mathcal{CP} violation occurs exclusively within the Higgs doublet sector (as we saw in the discussion of the MSSM Higgs sector). The NMSSM doublet-singlet couplings can violate \mathcal{CP} already at tree-level. However we do not consider this possibility.

The minimum of the NMSSM Higgs potential triggers electroweak symmetry breaking. After electroweak symmetry breaking the Higgs doublets can be expanded around their minima in the same way as in the MSSM (see Eq. (2.56), with $\xi = 0$). The singlet scalar component can be expanded as

$$S = v_s + \frac{1}{\sqrt{2}} (\phi_s + i\chi_s), \quad (2.67)$$

where v_s is the (non-zero) vacuum expectation value of the singlet. The effective μ parameter is dynamically generated by

$$\mu_{\text{eff}} = \lambda v_s, \quad (2.68)$$

and is therefore of the order of the SUSY breaking scale, which solves the μ problem of the MSSM.

The bilinear part of the Higgs potential can be written as

$$\begin{aligned} V_H = & \frac{1}{2} (\phi_1, \phi_2, \phi_s) \mathbf{M}_{\phi\phi\phi} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_s \end{pmatrix} + \frac{1}{2} (\chi_1, \chi_2, \chi_s) \mathbf{M}_{\chi\chi\chi} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_s \end{pmatrix} \\ & + (\phi_1^-, \phi_2^-) \mathbf{M}_{\phi^\pm\phi^\pm} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} + \dots, \end{aligned} \quad (2.69)$$

with the mass matrices $\mathbf{M}_{\phi\phi\phi}$, $\mathbf{M}_{\chi\chi\chi}$ and $\mathbf{M}_{\phi^\pm\phi^\pm}$.

The mixing of the \mathcal{CP} -even, \mathcal{CP} -odd and charged Higgs fields into mass eigenstates is described by unitary matrices U^H , U^A and U^C , where

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = U^H \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_S \end{pmatrix}, \quad \begin{pmatrix} a_1 \\ a_2 \\ G \end{pmatrix} = U^A \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_S \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = U^C \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}. \quad (2.70)$$

The matrices U^H , U^A and U^C transform the neutral \mathcal{CP} -even, \mathcal{CP} -odd and charged Higgs fields, respectively, such that the resulting mass matrices are

$$\mathbf{M}_{hhh}^{\text{diag}} = U^H \mathbf{M}_{\phi\phi\phi} U^{H\dagger}, \quad \mathbf{M}_{aaG}^{\text{diag}} = U^A \mathbf{M}_{\chi\chi\chi} U^{A\dagger} \text{ and } \mathbf{M}_{H^\pm G^\pm}^{\text{diag}} = U^C \mathbf{M}_{\phi^\pm\phi^\pm} U^{C\dagger}. \quad (2.71)$$

The mass eigenstates h_1 , h_2 and h_3 (with $m_{h_1} \leq m_{h_2} \leq m_{h_3}$) are the three \mathcal{CP} -even Higgs bosons, a_1 and a_2 (with $m_{a_1} \leq m_{a_2}$) the two \mathcal{CP} -odd Higgs bosons, and H^\pm is (unchanged) the charged Higgs pair. Also unchanged from the MSSM is the presence of the unphysical Goldstone bosons, G and G^\pm . The charged Higgs mass is given by

$$M_{H^\pm}^2 = \hat{m}_A^2 + M_W^2 - \lambda^2 v^2 \quad (2.72)$$

where \hat{m}_A is the effective \mathcal{CP} -odd doublet mass

$$\hat{m}_A^2 = \frac{\lambda v_s}{\sin \beta \cos \beta} (A_\lambda + \kappa v_s). \quad (2.73)$$

It can be seen from Eq. (2.70) that the singlet component of h_i is given by entry U_{i3}^H (accordingly the singlet component of a_i by U_{i3}^A). As singlets do not couple to gauge bosons, a larger singlet component of h_i means a reduced coupling of h_i to gauge bosons (and SM fermions). This will be important later.

2.3.4 Neutralino Sector of the NMSSM

In the NMSSM, the singlino \tilde{S} , the superpartner of the additional singlet scalar enlarging the Higgs sector, extends the neutralino sector compared to the MSSM by a fifth mass eigenstate. In the basis $(\tilde{B}, \tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S})$ the neutralino mass matrix at tree-level is now given by

$$\mathbf{M}_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_{ZSW} \cos \beta & M_{ZSW} \sin \beta & 0 \\ 0 & M_2 & M_{ZCW} \cos \beta & -M_{ZCW} \sin \beta & 0 \\ -M_{ZSW} \cos \beta & M_{ZCW} \cos \beta & 0 & -\mu_{\text{eff}} & -\lambda v_2 \\ M_{ZSW} \sin \beta & -M_{ZCW} \sin \beta & -\mu_{\text{eff}} & 0 & -\lambda v_1 \\ 0 & 0 & -\lambda v_2 & -\lambda v_1 & 2K\mu_{\text{eff}} \end{pmatrix}. \quad (2.74)$$

where $K \equiv \kappa/\lambda$. As in the MSSM, the mass matrix can be diagonalised by a single unitary matrix N

$$\text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0}) = N^* \mathbf{M}_{\tilde{\chi}^0} N^\dagger, \quad (2.75)$$

which gives the mass eigenvalues ordered as $m_{\tilde{\chi}_i^0} \leq m_{\tilde{\chi}_j^0}$ for $i < j$.

2.3.5 MSSM and Decoupling Limit

Since the NMSSM is an extension of the MSSM, the MSSM is recovered for

$$\lambda \rightarrow 0, \quad \kappa \rightarrow 0, \quad K \equiv \kappa/\lambda = \text{constant}, \quad (2.76)$$

with all other parameters (including μ_{eff}) held fixed. This limit is referred to as MSSM limit. In this limit one \mathcal{CP} -even, one \mathcal{CP} -odd Higgs boson (not necessarily the heaviest ones) and one neutralino become completely singlet and decouple from the MSSM sector.

If additionally $M_{H^\pm} \gg M_Z$ is fulfilled and all superpartners are heavy, we are in the decoupling limit, in which the Higgs sector becomes SM-like (decoupled heavy doublet Higgs states, one light Higgs with SM-like couplings).

When the doublet decoupling condition $M_{H^\pm} \gg M_Z$ is fulfilled, but λ and κ have finite non-zero values (i.e. values that differ from the MSSM limit) we call it the SM+singlet limit.

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