

Preface

Since their introduction in the pioneering work by Schoenberg [73], splines have become one of the powerful tools in mathematics [2, 44, 74, 75, 94] and, for example, in computer-aided geometric designs [20, 27, 43, 45, 56, 103]. In recent decades, splines have served as a source for the wavelet [1, 3, 4, 10, 12, 15, 29, 37, 38, 57, 68, 78, 87, 90, 91, 95, 100, 101, 102], multiwavelet [11, 41, 72, 80], and wavelet frame constructions [14, 17, 19, 35, 36, 42, 64, 69]. Splines and spline-based wavelets, wavelet packets, and frames have been extensively used in signal and image processing applications [5, 6, 9, 13, 16, 22, 24, 25, 31, 32, 46, 49, 51, 52, 63, 79, 84, 86, 88, 89], to name a few.

An excellent survey for the state-of-the-art (as of year 1999) on spline theory and applications is given in [85]. This survey motivated us in writing the present book. Another motivation was the emergence in recent years of new contributions of splines to wavelet analysis and its applications. In addition, we believe that the so-called discrete splines and their applications deserve a systematic exposure.

Discrete splines [30, 50, 58, 59, 60, 61, 65, 75, 92], whose properties mimic the properties of polynomial splines, are the discrete-time counterparts of polynomial splines. They provide natural tools for handling discrete-time signal processing problems and serve as a source for the design of wavelet transforms [7, 8, 54, 66] and frames transforms [14, 18, 105], whose properties perfectly fit signal/image processing applications.

The goal of this book is to provide a universal toolbox accompanied by a MATLAB software for manipulating polynomial and discrete splines, spline-based wavelets, wavelet packets, and wavelet frames for signal/image processing applications. The book is divided into two volumes. In Volume I ([26]), periodic splines and their diverse signal processing applications are discussed. The current Volume II deals with non-periodic splines. In this book, known and new contributions of splines to signal and image processing are described from a unified perspective, which is based on the Zak transform (ZT) [28, 93]. Being applied to B-splines, the ZT produces sets of so-called exponential splines (in Schoenberg [74]

sense), which are similar to Fourier exponentials. The ZT of discrete B-splines produces exponential discrete splines.

Periodic exponential splines form orthogonal bases of periodic spline spaces which are very similar to periodic Fourier exponentials. Representation of periodic splines via orthonormal bases produces the so-called Spline Harmonic Analysis (SHA) [99, 101], which combines the approximation abilities of splines with the computational strength of the Fast Fourier transform (FFT). It introduces the harmonic analysis methodology into periodic spline spaces. SHA is a basic working tool in Volume I of the book. Non-periodic exponential splines generate integral representations of polynomial and discrete splines, which have much in common with the Fourier integral.

The ZT approach provides explicit constructions of different types of splines such as interpolating, quasi-interpolating, and smoothing splines, best-approximation splines, and orthonormal bases for spline spaces. Constructions and utilization of various spline-wavelets and spline-wavelet packets have become natural.

Coupled with the Lifting scheme [82] of a wavelet transform, the ZT methodology utilizes polynomial and discrete splines for the design of a versatile library of biorthogonal wavelets, multiwavelets, and wavelet and multiwavelet frames (framelets) in signal space [7, 8, 10, 11, 12, 14, 15, 17, 18, 19, 105]. Properties of the designed wavelets and framelets, such as symmetry, flat spectra, vanishing moments, and good localization in either time or frequency domains, are valuable for signal/image processing applications. For example, the so-called Butterworth biorthogonal wavelets and wavelet frames, which originate from discrete splines, have proved to be especially efficient in signal/image processing applications. Digital filters, which have been produced during wavelets design process, give birth to subdivision schemes for the fast explicit computation of splines values at dyadic and triadic rational points [20, 103, 104], which is needed for interpolation, resampling, and geometric transformations of images.

The following topics are covered in Volume II of the book:

Introduction: Two introductory chapters briefly outline necessary facts about digital signals and images and their Fourier and z -transforms, which are needed for further constructions. In addition, digital filters and filter banks are outlined. Computation of many splines and spline-wavelets to be presented use filters whose impulse responses are infinite. Recursive implementation of such filtering is described in Chap. 2.

Zak transform (ZT): The ZTs of functions which belong to the space L_1 are introduced and their properties are outlined. In particular, they include the Poisson summation formulas (for example, [67, 77]). Realizations of the ZT in spline spaces produce integral representations of polynomial and discrete splines, which are extensively used in the spline and spline-wavelet design.

Elements of spline theory and design: Different types of polynomial and discrete splines with equidistant nodes are presented and their properties are outlined.

The design of interpolating, smoothing, shift-orthogonal splines becomes straightforward due to the ZT methodology. Constructions of these spline types utilize filters with infinite impulse response (IIR). Due to their recursive implementation, the computational complexity of filtering with IIR is competitive to the complexity of filtering with finite impulse response (FIR) filters. However, an advantage of FIR filtering is that it can be utilized for real-time processing. For this purpose, local quasi-interpolating and smoothing splines, which are constructed by filtering data samples with FIR filters, can be used, [15, 96, 98]. Their properties are close to the properties of global interpolating and smoothing splines. These splines are presented in detail in Chap. 5 of this volume.

Cubic local splines on non-uniform grid: In the whole book, except for Chap. 6, we deal with splines constructed on uniform grids. Chapter 6, which is based on [81, 97], describes two types of local cubic splines on non-uniform grids: (1) Variation-diminishing splines and (2) Quasi-interpolating splines. These splines are computed by simple fast computational algorithms, which utilize the relation between cubic splines and cubic interpolation polynomials. These splines can serve as an efficient tool for real-time processing of arbitrarily sampled signals. The capability to adapt the grid to the structure of an object and consuming low operating memory are great advantages for offline processing of signals and multidimensional data arrays.

Spline subdivision and signals (images) upsampling: If a spline's samples at grid points are given, then the computation of its values between the grid points is called a spline subdivision. Fast subdivision algorithms, which explicitly derive spline values at dyadic and triadic rational points from the samples taken at integer grid points in one and two dimensions, are described. The computer-aided geometric design is a main field of application for subdivision schemes (for example, see [27, 70]). However, these techniques suit well signals and images upsampling to restore sparsely sampled signals and images at intermediate points. These upsampling procedures increase the objects resolution. On the other hand, when data are corrupted by noise, upsampling from a sparse grid can significantly reduce the noise level. Appears in Chap. 7 of this volume.

Design of polynomial spline-wavelets: Constructions of different types of spline-wavelets in an explicit form and fast implementation of the corresponding transforms using recursive filtering are described. Generators of spline-wavelet spaces are presented such as B-wavelets and their duals and the Battle-Lemarié wavelets whose shifts form orthonormal bases of the spline-wavelet spaces. The spline-wavelets construction utilizes the integral representation of splines and simple two-scale relations between exponential splines from different resolution scales. The Fourier spectra of spline-wavelets from different resolution scales partition the frequency domain in a logarithmic way. The shapes of the magnitude spectra of the Battle-Lemarié wavelets tend to be rectangular as the spline's order increases. Appears in Chap. 8 of this volume.

Discrete splines: The space of discrete splines, which is a subspace of the signal space, is described. Properties of the discrete splines mirror the properties of polynomial splines. The Zak transform applied to discrete B-splines produces

exponential discrete splines and an integral representation of discrete splines. The integral representation simplifies manipulations with the discrete splines. In particular, it provides explicit expressions for interpolating and smoothing discrete splines in one and two dimensions and fast algorithms for calculation of discrete splines values from grid samples. This is a useful tool for upsampling signals and images. Appears in Chap. 9 of this volume.

Discrete splines wavelets: Similarly to the polynomial splines case, the wavelet transforms are introduced to the discrete spline space. These transforms are based on the relations between the exponential discrete splines from different resolution scales. Practically, the wavelet transforms of signals are implemented by multirate filtering of signals by two-channel filter banks with the downsampling factor 2 (critically sampled filter banks). The filtering implementation is accelerated by switching to the polyphase representation of signals and filters. Appears in Chap. 10 of this volume.

Design of biorthogonal wavelets: The polynomial and discrete splines may contribute to wavelet analysis in another way. They are a source for a family of filters, which generate biorthogonal wavelets, whose properties are valuable for signal processing. Although these wavelets originate from splines, they, unlike the spline-wavelets, do not belong to spline spaces. Design of biorthogonal wavelets and efficient implementation of the signals' transforms is carried out through the Lifting scheme [82]. The idea is to split the signal into even and odd subarrays. Then, the even subarray is filtered using some *prediction* filter in order to predict the odd subarray. The predicted subarray is extracted from the original odd subarray. The difference array is filtered by an *update* filter and it is used to update the even subarray in order to eliminate aliasing. These operations are then applied to the updated even subarray and so on. Thus, multiscale wavelet decomposition is achieved. Reconstruction is implemented in the reverse order. The key point is a proper choice of the prediction and update filters. Naturally, odd samples can be predicted from midpoint values of either polynomial or discrete splines, which interpolate or quasi-interpolate the even samples of the signal. In this way, a number of linear phase IIR and FIR prediction filters are designed. Being properly modified, they are used for the update step as well. By using these filters, a diverse library of biorthogonal wavelets is constructed [7, 8, 10]. Exclusive properties are demonstrated by the so-called Butterworth wavelets, which originate from discrete interpolatory splines. They are related to the Butterworth filters [62] that are widely used in signal processing. Appears in Chaps. 11 and 12 of this volume.

Data compression: This is a critical area in signal processing. Data compression is needed for efficient transmission and storage of huge data including rich multimedia context, seismic, and hyperspectral data. The compression ratio should be as high as possible without damaging the decoding capabilities and without degrading the quality of the source data after decompression. Generally, a lossy compression consists of three procedures: appropriate transform, which corresponds with the structure of the compressed object (lossless), quantization (lossy), and entropy coding (lossless).

- Wavelet transforms have successful history in achieving high compression ratios for still images and some types of 3D objects. Coding schemes, which are associated with wavelet transforms that rely on the space-frequency localization of the wavelets and on the tree structure of the coefficients arrays in its multi-scale representation, were developed. Examples are EZW [76], SPIHT [71], and JPEG-2000 standard [83] that are based on wavelet transforms. A proper wavelet transform, which retains the essential contents of the object in a small number of coefficients, is crucial for the compression success. The Butterworth biorthogonal wavelets described in Chap. 12 ([9, 13]) demonstrate excellent performance in comparison with other known wavelets such as, for example, the most popular 9/7 biorthogonal wavelets [6].
- Some types of data, such as seismic, hyperspectral data and many multimedia images have a mixed structure. They are piecewise smooth in one direction(s) and have oscillating events in the other direction. A hybrid algorithm which combines wavelet and local cosine transform (LCT) [39], proved to be highly efficient to compress such data. If, for example, the data array is piece-wise smooth in the horizontal direction and has oscillations in the vertical direction (that is typical for seismic data), then wavelet transform is applied to the horizontal direction while LCT is applied to the vertical direction. In this way, near optimal sparsity of the data representation is achieved. To apply the coding schemes to a mixed coefficients array, reordering of the LCT coefficients takes place. This algorithm outperforms other algorithms that are based only on 2(3)D “pure” wavelet transforms. Its compression capabilities are also demonstrated on multimedia images that have a fine texture. The wavelet part in the mixed transform of the hybrid algorithm utilizes the library of Butterworth wavelet transforms ([21, 23]). Appears in Chap. 13 of this volume.

Wavelet frames (framelets). Design and implementation: Recently frames, which provide redundant expansions of signals, have attracted considerable interest from researchers working in signal processing. When the requirement of one-to-one correspondence between the signal and its transform coefficients is dropped, there is more freedom to design and implement frame transforms. In addition, frame transforms demonstrate resilience to the data corruption and loss.

Wavelet transforms use critically sampled filter banks. On the other hand, wavelet frame transforms are implemented by the application of oversampled perfect reconstruction (PR) pairs of filter banks. It means that the number of channels in the filter banks exceeds the downsampling factor. Moreover, translations of the impulse responses of filters, which constitute such filter banks, form wavelet frames in the signal space [33, 40, 55]. Generally, the synthesis filter bank in the PR pair differs from the analysis filter bank. In the case when both filter banks are the same, the corresponding frame is tight. Tight frames can be regarded as redundant counterparts of orthogonal bases.

The design of a variety of three- and four-channel PR filter banks, which generate tight frames in the space of periodic signals, is described in Chap. 14. The filter banks comprise one low-pass, one high-pass, and either one or two band-pass

filters. All these filters are derived from spline-based prediction filters which were used for the design of biorthogonal wavelet transforms. The so-called semi-tight frames are introduced, where the low- and high-pass filters in the synthesis filter bank are the same as in the analysis filter bank, while the band-pass filters are different. The utilization of a wide range of IIR and FIR filter banks with a relaxation of the tightness requirement provides additional design opportunities. Properties such as symmetry, interpolation, and flat spectra combined with fine time-domain localization of framelets as well as a high number of vanishing moments can be easily achieved. The transforms are implemented using recursive filtering.

Multiwavelets originated from Hermite splines: The wavelet frame transforms are generalizations of wavelet transforms. Another generalization of wavelet transforms, which enables us to achieve a high approximation accuracy by using filters with very short supports, is the multiwavelet transform. Like wavelet transforms (and unlike frame transforms), the multiwavelet transforms retain one-to-one correspondence between signals and sets of their transform coefficients, although they use more filters than wavelet transforms. Chapter 15 presents multiwavelet transforms for the manipulation of discrete-time signals. The transforms are implemented in two phases: (1) Pre (post)-processing, which transforms a scalar signal into a vector signal (and back). (2) Wavelet transforms of the vector signal using multifilter banks. Both phases are performed in a lifting manner. The cubic interpolating Hermite splines ([34], for example) are used as a predicting aggregate in the vector wavelet transform. The presented pre(post)-processing algorithms do not degrade the approximation accuracy of the vector wavelet transforms. The transform results in signals expansion over biorthogonal bases that consist of translations of a few discrete-time wavelets which are symmetric and have short supports.

Multiwavelet frames: The Hermite spline-based multiwavelets design is extended to the construction of multiwavelet frames in the signal space. The frames are generated by three-channel perfect reconstruction oversampled multifilter banks. The design of the multifilter bank starts from a pair of interpolating multifilters, which originate from the cubic Hermite splines. The remaining multifilters are designed by factorizing of polyphase matrices. The input to the oversampled analysis multifilter bank is a vector signal, which is produced from an initial scalar signal by the same preprocessing algorithms as in the multiwavelets processing. The postprocessing algorithms convert the vector output from the synthesis multifilter banks into the scalar signal. The discrete-time framelets, generated by the designed filter banks, are (anti)symmetric and have short support. Note that most multiframe constructions reported in the literature are based on the multiresolution analysis in the space L_2 that provide continuous-time frames in L_2 ([47, 48, 53], for example). On the contrary, the design presented in Chap. 16, which is based on oversampled multifilter banks, provides discrete-time frames in the signals' space l_1 .

All the presented methods are accompanied by MATLAB codes. A software guide is given in Appendix.

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Tel Aviv, Israel
Jyväskylä, Finland
Tel Aviv, Israel
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Amir Z. Averbuch
Pekka Neittaanmäki
Valery A. Zheludev

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