

Preface

During the past decade, reduced order modeling has attracted growing interest in computational science and engineering. It now plays an important role in delivering high-performance computing (and bridging applications) across industrial domains, from mechanical to electronic engineering, and in the basic and applied sciences, including neurosciences, medicine, biology, chemistry, etc. Such methods are also becoming increasingly important in emerging application domains dominated by multi-physics, multi-scale problems as well as uncertainty quantification.

This book seeks to introduce graduate students, professional scientists, and engineers to a particular branch in the development of reduced order modeling, characterized by the provision of reduced models of guaranteed fidelity. This is a fundamental development that enables the user to trust the output of the model and balance the needs for computational efficiency and model fidelity. The text develops these ideas by presenting the fundamentals with a gradually increasing complexity; comparisons are made with more traditional techniques and the performance illustrated by means of a few carefully chosen examples. The book does not seek to replace review articles on the topics (such as [1–5]) but aims to widen the perspectives on reduced basis methods and to provide an integrated presentation. The text begins with a basic setting to introduce the general elements of certified reduced basis methods for elliptic affine coercive problems with linear compliant outputs and then gradually widens the field, with extensions to non-affine, non-compliant, non-coercive operators, geometrical parametrization and time-dependent problems.

We would like to point out some original ingredients of the text. Chapter 3 guides the reader through different sampling strategies, providing a comparison between classic techniques based on singular value decomposition (SVD), proper orthogonal decomposition (POD), and greedy algorithms. In this context it also discusses recent results on a priori convergence in the context of the concept of the Kolmogorov N -width [6]. Chapter 4 contains a thorough discussion of the computation of lower bounds for stability factors and a comparative discussion of the various techniques. Chapter 5 focuses on the empirical interpolation method (EIM) [7], which is emerging as a standard element to address problems exhibiting non-affine

parametrizations and nonlinearities. It is our hope that these last two chapters will provide a useful overview of more recent material, allowing readers who wish to address more advanced problems to pursue the development of reduced basis methods for applications of interest to them. Chapter 6 offers an overview of a number of more advanced developments and is intended more as an appetizer than as a solution manual.

Throughout the text we provide some illustrative examples of applications in computational mechanics to guide readers through the various topics. All of the main algorithmic elements are outlined by graphical boxes to assist the reader in his or her efforts to implement the algorithms, emphasizing a matrix notation. An appendix with mathematical preliminaries is also included.

This book is loosely based on a Reduced Basis handbook available online [8], and we thank the co-author of this handbook, our colleague Anthony T. Patera (MIT), for his encouragement, support, and advice during the writing of the book. It benefits from our long-lasting collaboration with him and his many co-workers. We would like to acknowledge all those colleagues who contributed at various levels in the preparation of this manuscript and the related research. In particular, we would like to thank Francesco Ballarin and Alberto Sartori for preparing representative tutorials and the new open-source software library available as a companion to this book at <http://mathlab.sissa.it/rbnics>. An important role, including the provision of useful feedback, was also played by our very talented and motivated students attending regular doctoral and master classes at EPFL and SISSA (and ICTP), tutorials in Minneapolis and Savannah, and several summer/winter schools on the topic in Paris, Cortona, Hamburg, Udine (CISM), Munich, Sevilla, Pamplona, Barcelona, Torino, and Bilbao.

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