Preface

Writing a popular account of the three-body problem is a special challenge. The problem is as old as science in general, and contributions towards its solution have been made by an untold number of scientists. Yet we are not yet at the stage where we could declare that the problem has been solved. Another challenge is to try to convey some of the excitement of this problem to the general reader without the use of mathematics. For a problem which is studied in mathematics departments of many universities, it means, by necessity, major simplifications and often appealing to applications in the field of astronomy. Astronomical systems may be easier to visualize than purely mathematical constructions.

We have taken the historical approach. The three-body problem, the description of the motion of three celestial bodies under the action of their mutual gravitational pull, was first studied by Isaac Newton. In Chap. 1, we give a brief history of the problem prior to Newton and only to the extent that is relevant to Newton's work. There is much astronomical and mathematical science before Newton that we are not able to describe here. Some additions to the historical background come in Chap. 7 after we have learnt concepts that are important to the problem, such as the idea of chaos.

Newton's law of gravity is accurate enough for most astronomical calculations. However, the more accurate Einstein's law of gravity is necessary in many modern applications. In fact, the need to improve Newton's law became apparent only in the late nineteenth century, when it was realized that planet Mercury did not behave as expected by the solution of the three-body problem in Newton's theory. The last chapter describes more drastic changes to Newton's law, such as the laws governing black holes. They cannot be understood without Einstein's General Relativity, as his law of gravity is called.

Chapter 3 follows some steps in the evolution of the three-body problem. It includes, among others, the famous pre-Nobel competition for finding the answer and describes Poincaré and Sundman as leaders of two schools of thought on the nature of the solution. For Poincaré it was statistical at best, while Sundman claimed a fully deterministic solution. Both lines of enquiry have correspondence in the current work. Poincaré's solution leads to chaos theory, and further to enquiries about the nature of time, the subject of the fourth chapter.

In the next two chapters, we meet astronomical applications on two different scales, the Solar System and galaxies. Both of these systems have more than three bodies, more than three by far. However, it is possible to get first-order estimates of many processes using just three bodies in the calculations, as numerous scientists have demonstrated over the years. For example, two galaxies may be understood as two rigid bodies which provide the variable gravitational field for the study of the motion of a third body, such as a star. Repeating the process for many stars gives us an overview of how galaxies made of billions of stars may change their shape and other properties.

After the first round review, we take a more detailed look at the steps involved in the history of the three-body problem. It includes new frontiers and some of the recent results. Among frontiers are the systems involving black holes which are found in the final chapter. It takes us straight to the current efforts to prove black hole theorems. That is, we try to verify the concept of black holes that is derived from General Relativity.

It is not clear where one should start the history of the three-body problem. Pythagoras probably understood that Earth, Moon, and the Sun are three spherical celestial bodies whose exact alignments produce eclipses, lunar and solar. But only after the introduction of the force law between them did the three-body problem in the modern sense emerge. Newton's attempts to solve the three-body problem filled a good part of his famous work *Principia*. The three-body problem of today, with Einstein's law of gravity, may be used to test the so-called no-hair theorem of black holes. The no-hair theorem was first formulated by Israel, Carter, and Hawking, and a distant quasar composed of two black holes and a cloud of gas is the system currently under study. This represents an enormous range of scale. At the lower end we have the mass of the Sun and at the upper end more than ten billion suns. The objects of study can be near to us, like the Sun about 8 light minutes away, or 3.5 billion light years away in the case of the binary black hole system OJ287.

Vladimir Titov from St. Petersburg State University (Russia) has prepared animations illustrating choreographies in the three-body systems. These animations and other add-on materials can be found on the book web page http://extras.springer.com.

We would like to thank Sverre Aarseth, Nick Ourusoff, and Renate and Henrik Appelqvist for reading the manuscript and for helpful comments. We thank also Laura Garbolino, Heike Hartmann, Göran Östlin, Mark Hurn, Kathryn Shaw, Alan Harris, Annika Augustsson, Jaana Tegelberg, Sylvio Ferraz Mello, Joerg Waldvogel, Arthur Chernin, Gene Byrd, Harry Lehto and Martin Gutknecht for help with illustrations.

We also acknowledge the Free Software Foundation for the permission to use the GNU Free Documentation License in reproducing several of our figures. They are indicated in the figure captions as originating from Wikimedia Commons. These figures are published under the terms of the GNU General Public License Version 3 (GNU GPLv.3) which permits use, duplication, adaptation, distribution, and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source and indicate if changes were made. If you remix,

transform, or build upon this figure or a part thereof, you must distribute your work under the same license as the original. Figure captions give bibliographic data of the source with name(s) of the originator(s); it may also be an http address.

We hope that this small review will stimulate interest in the reader, and for those with mathematical knowledge, further enquiries to the mysteries of the three-body problem.

Turku, Finland — Mauri Valtonen
Austin, TX — Joanna Anosova
St. Petersburg, Russia — Konstantin Kholshevnikov
Grenada, West Indies — Aleksandr Mylläri
St. Petersburg, Russia — Victor Orlov
Tokyo, Japan — Kiyotaka Tanikawa
September 2015

http://www.springer.com/978-3-319-22725-2

The Three-body Problem from Pythagoras to Hawking
Valtonen, M.; Anosova, J.; Kholshevnikov, K.; Mylläri, A.;
Orlov, V.; Tanikawa, K.
2016, XI, 173 p. 96 illus., 70 illus. in color., Softcover
ISBN: 978-3-319-22725-2