Preface

"Thus it follows that $d^{\frac{1}{2}}x$ will be equal to $x\sqrt{dx:x}$, an apparent paradox, from which one day useful consequences will be drawn." This first remark on the idea of a non-integer order derivative was found in a letter from Gottfried Wilhelm Leibnitz to the Guillaume de l'Hôpital dated 1695.[1] It has become a motivation for future generations of mathematicians to create basics of the non-integer order (fractional) calculus. And since the mid-twentieth century this mathematical apparatus has been used for creation of increasingly better models of simple and very complex physical phenomena systems and processes. As we know from numerous studies, fractional order models can depict the physical plant better than the classical integer order ones. This covers different research fields such as modeling of insulator properties, visco-elastic materials, electrodynamic, electrothermal, electrochemical, economic processes, etc.

Despite a huge increase of research activities and many remarkable theoretical achievements in the area of fractional calculus, we still face theoretical and practical challenges. The complicity of the non-integer order calculus causes most of the work to be theoretically oriented. This can also be seen in the contents of the previous[2,3] and this conference volume. All this shows that we are still in the early stages of development of non-integer order control systems. However, increasing potentialities offered by modern automation equipment gives hope of developing effective control techniques that could be applied and implemented also for non-integer order modeled processes.

[1]Oldham, K.B. and Spanier, J.: The Fractional Calculus. Academic Press, 1974.

[2]Mitkowski, W., Kasprzyk, J., Baranowski, J.: Advances in the Theory and Applications of Non-integer Order Systems, the 5th Conference on Non-integer Order Calculus and Its Applications, Cracow, Poland; Springer, Lecture Notes in Electrical Engineering, vol. 257.

[3]Latawiec, J.K., Łukaniszyn, M., Stanisławski, R.: Advances in Modelling and Control of Non-integer Order Systems, the 6th Conference on Non-integer Order Calculus and Its Applications, Opole, Poland; Springer, Lecture Notes in Electrical Engineering, vol. 320.

Some new ideas and examples of modeling, synthesis, and practical realizations of fractional order systems may be found in this study. This volume contains 24 papers divided into four parts covering: mathematical fundamentals, modeling and approximations, controllability, observability and stability problems, and practical applications of fractional control systems.

Part 1 expands the base of tools and methods of the mathematical basis for non-integer order calculus.

Malgorzata Klimek ("Fractional Sturm-Liouville Problem in Terms of Riesz Derivatives") formulates a regular fractional Sturm-Liouville problem on a bounded domain in terms of Riesz derivatives. The considered case includes vanishing Dirichlet boundary conditions. They prove that its eigenvalues are real, eigenfunctions are continuous, and form orthogonal sets of functions in the respective Hilbert spaces. In addition, boundedness results for eigenvalues are derived and a connection between the discussed fractional Sturm-Liouville equations and Euler-Lagrange equations for the corresponding action functionals is established.

Agnieszka B. Malinowska and Tatiana Odzijewicz ("Multidimensional Discrete-Time Fractional Calculus of Variations") introduce a discrete-time multidimensional fractional calculus of variations. They define fractional operators in the sense of Grünwald–Letnikov. Then they derive necessary optimality conditions and give examples illustrating the use of obtained results.

Dominik Sierociuk, Wiktor Malesza, and Michal Macias ("On a New Symmetric Fractional Variable Order Derivative") present particular definitions of symmetric fractional variable order derivatives. The AD and DA types of the fractional variable order derivatives and their properties are introduced. Additionally, they show the switching order schemes equivalent to these types of definitions. Finally, the theoretical considerations are validated on numerical examples.

Piotr Ostalczyk ("Linearization of the Non-linear Time-Variant Fractional-Order Difference Equation") discusses a linearization procedure of the fractional-order nonlinear time-variant discrete system. Starting from the nonlinear fractional-order difference equation he derives its equivalent state-space form and assuming the knowledge of the nominal trajectory evaluates the linear state-space model. The investigations are supported by numerical examples.

Ewa Girejko, Ewa Pawłuszewicz, and Małgorzata Wyrwas ("The Z-Transform Method for Sequential Fractional Difference Operators") discuss the linear Caputo–type sequential difference fractional-order systems. They use a classical $\mathcal{Z}$-transform method to show the general solutions of sequential systems in the form $\left(\Delta_*^\alpha(\Delta_*^\alpha x)\right)(n)+b\left(\Delta_*^\alpha x\right)(n)+cx(n)=0$, where $b,c\in\mathbb{R}$. In proofs they base on the formula for the image of the discrete Mittag-Leffler function in the $\mathcal{Z}$-transform.

Part 2 focuses on new methods and developments in process modeling and fractional derivative approximations.

Wojciech Mitkowski and Krzysztof Oprzędkiewicz ("An Estimation of Accuracy of Charef Approximation") present a new accuracy estimation method for Charef approximation. Charef approximation allows them to describe fractional-order systems with the use of an integer-order, proper transfer function.

They estimate the accuracy of approximation by comparing step responses of the plant and Charef approximation. The step response of the plant was calculated with the use of an accurate analytical formula and it can be interpreted as a standard. The presented approach can be applied for effective tuning of Charef approximant for a given plant. The use of the proposed method does not require knowing the step response of the modeled plant. The proposed methodology can be easily generalized to other known approximations.

Wieslaw Krajewski and Umberto Viaro ("A New Method for the Integer Order Approximation of Fractional Order Models") concern themself with the finite-dimensional approximation of a fractional-order system represented in state-space form. To this purpose, resort is made to the Oustaloup method for approximating a fractional-order integrator by a rational filter. They reduce the dimension of the resulting integer-order model using an efficient algorithm for minimization of the L_2 norm of a weighted equation error. Two numerical examples are worked out to show how the desired approximation accuracy can be ensured.

Jerzy Baranowski, Waldemar Bauer, and Marta Zagórowska ("Stability Properties of Discrete Time-Domain Oustaloup Approximation") present an analysis of discrete time domain realization of Oustaloup approximation. They present the scheme for realization along with a method of implementation of discretization formulas. The authors analyze also the stability considering influences of sampling frequency, order, and bandwidth. Analysis is illustrated with behavior of spectral radius of the discretized system.

Konrad Andrzej Markowski and Krzysztof Hryniów ("Digraphs Minimal Positive Stable Realisations for Fractional One-Dimensional Systems") present a method of the determination of positive stable realization of the fractional continuous-time positive system. The algorithm finds a complete set of all possible realizations instead of only a few realizations. They show that all realizations in the set are minimal and stable. The method proposed by them uses a parallel computing algorithm based on a digraphs theory, which is used to gain much needed speed and computational power for a numerical solution. The presented procedure is illustrated with a numerical example.

Marek Rydel, Rafał Stanisławski, Grzegorz Bialic, and Krzysztof J. Latawiec ("Modeling of Discrete-Time Fractional-Order State Space Systems Using the Balanced Truncation Method") present a new method of approximation of linear time-invariant discrete-time fractional-order state space systems by means of the Balanced Truncation Method. This reduction method is applied to the rational form of fractional-order system in terms of expanded state equation. As an approximation result the authors obtain rational and relatively low-order state space system. Simulation experiments show very high accuracy of the introduced methodology.

Dominik Sierociuk, Michal Macias, and Pawel Ziubinski ("Experimental Results of Modeling Variable Order System Based on Discrete Fractional Variable Order State-Space Model") present experimental results of modeling fractional variable order system using Discrete Fractional Variable Order State-Space Model. Experimental results given were obtained on the basis of modified multi-order switching analog realization of the constant parameter case introduced in this work. During the identification process two algorithms were used: direct and dual. Finally they present joint estimation results for parameter estimation, in order to verify constant parameters for the proposed analog model.

Part 3 provides a bunch of papers which raise problems of controllability, observability, and stability of non-integer order systems.

Tadeusz Kaczorek ("Positivity and Stability of a Class of Fractional Descriptor Discrete-Time Nonlinear Systems") proposes a method of analysis of the fractional descriptor nonlinear discrete-time systems with regular pencils of linear part. The method is based on the Weierstrass-Kronecker decomposition of the pencils. He establishes necessary and sufficient conditions for the positivity of the nonlinear systems. Then he proposes a procedure for computing the solution to the equations describing the nonlinear systems. Using an extension of the Lyapunov method to positive nonlinear systems, he derives sufficient conditions for the asymptotic stability.

Małgorzata Wyrwas and Dorota Mozyrska ("Stability of Linear Discrete–Time Systems with Fractional Positive Orders") study the problem of the stability of the Grünwald–Letnikov-type linear discrete-time systems with fractional positive orders. The method of reducing the considered systems by transforming them to the multi-order linear systems with the partial orders from the interval (0,1] is presented. For the reduced multi-order systems the authors formulate conditions for stability based on the $\mathcal{Z}$-transform as an effective method for stability analysis of linear systems.

Yassine Boukal, Michel Zasadzinski, Mohamed Darouach, and Nour-Eddine Radhy ("Robust H_∞ Observer-Based Stabilization of Disturbed Uncertain Fractional-Order Systems Using a Two-Step Procedure") consider the problem of robust H_∞ observer-based stabilization for a class of linear Disturbed Uncertain Fractional-Order Systems (DU-FOS) using H_∞-norm optimization. Based on the H_∞-norm analysis for FOS, they establish a new design methodology to stabilize a linear DU-FOS using robust H_∞ Observer-Based Control (OBC). The existence conditions are derived, and using the H_∞-optimization technique, the stability of the estimation error and stabilization of the original system are given in an inequality condition, where all the observer matrices gains and the control law can be computed by solving a single inequality condition in two steps. Finally, the authors give a simulation example to illustrate the validity of their results.

Zbigniew Zaczkiewicz ("Relative Observability for Fractional Differential-Algebraic Delay Systems within Riemann-Liouville Fractional Derivatives") presents the problems of relative R-observability for linear stationary fractional differential-algebraic delay system (FDAD). FDAD system consists of fractional differential equation in the Riemann-Liouville sense and difference equations. He introduces the determining equation systems and their properties. Applying the Laplace transformation he obtains solution representations into series of their determining equation solutions and presents effective parametric rank criteria for relative R-observability. He also formulates a dual controllability result.

Part 4 is devoted to the presentation of different fractional order control applications.

Jerzy Klamka ("Minimum Energy Control of Linear Fractional Systems") considers the minimum energy control problem of infinite-dimensional fractional-discrete time linear systems. He establishes necessary and sufficient conditions for the exact controllability of the system and gives sufficient conditions for the solvability of the minimum energy control of infinite-dimensional fractional discrete-time systems. Finally, he proposes a procedure for computation of the optimal sequence of inputs that minimizes the quadratic performance index.

Adam Makarewicz ("Use of Alpha-Beta Filter to Synchronization of the Chaotic Ikeda Systems of Fractional Order") considers a problem of signal filtering used in synchronization of two-fractional delay Ikeda systems, combined linearly by coupling. The synchronization uses Alpha-Beta filter, which operates on predicting the next value, based on the measured signal in a current point in time. He uses numerical simulations to investigate effects of fractional order and coupling rate on synchronization. Simulations are performed using Ninteger Fractional Control Toolbox for MatLab.

Paweł Dworak ("On Dynamic Decoupling of MIMO Fractional Order Systems") considers problems with a dynamic decoupling of multi-input multi-output fractional order systems. He shows their similarities and differences to integer order decoupling methods. Basing on a few examples he carries out simulations of decoupled fractional order systems to show the applicability of the considered methods. He ends his work with some final remarks on a practical implementation of decoupling methods for fractional order systems.

Łukasz Wach and Wojciech P. Hunek ("Perfect Control for Fractional-Order Multivariable Discrete-Time Systems") analyze the perfect control method for multi-input/multi-output MIMO fractional-order discrete-time systems in state space. The presented simulation example for nonsquare MIMO system carried out in a Matlab/Simulink environment confirms the correctness of the proposed algorithm.

Waldemar Bauer ("Implementation of Non-Integer Order Controller Using Oustaloup Parallel Approximation for Air Heating Process Trainer") presents a new implementation method of non-integer order controller. This controller is designed and analyzed for the model of air heating process trainer system. The author shows that the proposed controller is suitable for control of the inertial system with time-delay and time-varying gain. He also presents a new method of implementation of Oustaloup approximation and shows it usefulness, which allows operation of non-integer order controller in real-time environment.

Bogdan Broel-Plater, Krzysztof Jaroszewski, and Paweł Dworak ("Classical Versus Fractional Order PI Current Controller in Servo Drive") compare the fractional order PI current controller of the servo drive with its classical counterpart. They analyze structures of such a fractional order controller without as well as with antiwindup blocks. They present and discuss results of simulations carried out in Matlab/Simulink environment.

Marta Zagórowska ("Parametric Optimization of Non-Integer Order PD^{μ} Controller for Delayed System") analyzes a new tuning method for PD^{α} controller using approximation with Laguerre functions. She performs the optimization for various sets of parameters and also analyzes the convergence of chosen optimization parameters. The results are tested for a first-order system with delay.

Jerzy Baranowski, Waldemar Bauer, Marta Zagórowska, Aleksandra Kawala-Janik, Tomasz Dziwiński, and Paweł Piątek ("Adaptive Non-Integer Controller for Water Tank System") consider a new method of designing adaptive controller for non-integer order systems. The theoretical approach was verified with a computer simulation of three-tank system.

Stefan Domek ("Model-Plant Mismatch in Fractional Order Model Predictive Control") shows the effect of various plant-model mismatches on the performance of fractional order model predictive control (FOMPC) systems. He presents an algorithm of a FOMPC and describes different types of plant-model mismatches for fractional-order systems. His analysis is illustrated by results obtained from simulation tests.

This volume is a result of fruitful and stimulating discussions during the RRNR'2015, the 7th Conference on Non-integer Order Calculus and Its Applications organized by the Faculty of Electrical Engineering, West Pomeranian University of Technology, Szczecin, Poland. The conference gathered a number of researchers active in the fields of fractional calculus, here those interested in theoretical aspects of mathematical fundamentals, modeling, and approximations and those focused on the practical issues that have to be solved during control system implementation. Such a wide spectrum of interests displayed by the outstanding participants exploded in stimulation of lively discussions across the field and contributed to the success of the conference. We are grateful to the conference participants for sharing their research results and active and inspiring discussion. We would also like to acknowledge the contribution of the anonymous referees, whose comments allowed us to improve the final form of the papers. Finally, we wish to thank Dr. Thomas Ditzinger and Holger Schäpe from Applied Sciences and Engineering at Springer for their assistance and support in this editorial work.

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