

A Novel Reconfigurable 7R Linkage with Multifurcation

Ketao Zhang, Andreas Müller and Jian S. Dai

Abstract This paper investigates constraint singularity induced multifurcation of a novel 7R (R for revolute joint) linkage. Kinematic structure of the 7R linkage is described first for the purpose of geometric analysis. According to geometric properties of the kinematic structure, D-H parameters and kinematics equations in dual quaternion are derived subsequently. The study further explores analytical form of mechanism constraint-screw systems corresponding to distinct motion branches reconfigured from the 7R linkage based on reciprocity in screw theory. The constraint analysis reveals inherent properties of motion branch changes induced by constraints variation and geometric restriction of joints in these motion branches. This leads to identification of multifurcation of the reconfigurable 7R linkage, meaning motion-branch transitions between the non-overconstrained 7R linkage and overconstrained 6R and 4R mechanisms.

Keywords 7R linkage · Overconstrained 6R linkage · Reconfiguration · Multifurcation · Kinematics

1 Introduction

Reconfigurable mechanisms with peculiar capability to change their global mobility and motion characteristics can be developed into multifunctional robot devices to deal with complicated tasks. This kind of mechanism with reconfigurability has

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been broadly investigated in last two decades. Wohlhart [1] presented a type of linkage that is able to permanently change its global mobility by passing a singular position and termed the peculiar property kinematotropy. In the mean time, Dai [2] developed reconfigurable assemblies and packaging systems for folding decorative gifts and boxes, leading to exploration of mechanisms with metamorphosis [3, 4] and capability of changing their structure and subsequently motion characteristics. Following the study of constraint singularity, parallel mechanisms with dramatically different operation modes [5] were identified. This type of parallel mechanism can undergo a variety of transformations by passing constraint singularities. The study of this type of parallel mechanism mainly focuses on the variation of operation modes [6, 7] of the platform, a particular selected output link of the mechanism.

Within the reconfigurable mechanisms, constraint singularity induced configuration transition between distinct motion branches of typical spatial mechanisms has raised great interests. Wohlhart discussed changes of finite mobility of Wren's platform mechanism [1]. Zlatanov et al. [5] presented a typical reconfigurable parallel mechanism, the 3-URU DYMO which has five different types of platform motion corresponding to multiple 3-dimensional regions of the configuration space. Gogu [8] investigated the constraint singularity induced bifurcation in parallel mechanisms and mobility changes of the presented mechanisms. Zhang et al. [9] investigated constraint singularity induced bifurcation of a 3PUP parallel mechanism. Zeng et al. [10] developed a 4-DOF kinematotropic hybrid parallel mechanism which can reconfigure the platform mobility without changing its topological structure. Qin et al. [11] revealed the multifurcation of the Q-Square, a plane symmetric multi-loop spatial linkage, in terms of screw systems.

Comparing to the broad study of multi-loop spatial mechanisms with reconfigurability and single-loop linkages with invariant kinematic structure, mobile single-loop linkages that are capable of reconfiguring their kinematic structure had rarely been explored. Galletti and Fanghella [12] presented a systematic approach to form four basic kinematotropic single-loop chains. Kuo and Yan [13] investigated the configuration synthesis and analysis of mechanisms with variable topologies and presented typical single-loop linkages with different stationary configurations. Kong and Huang [14] synthesized and constructed single-loop mechanisms with two operation modes employing the basic single-DOF linkages such as planar 4R, spherical 4R, Bennett 4R linkage and paradoxical 5R and 6R linkages. This study further extended to single-loop 7R mechanisms with multi-operation mode [15]. In light of metamorphosis, Zhang et al. [16] presented a novel metamorphic 8R linkage inspired by artistic kirigami. Distinct motion branches were derived from the 8R linkage resorting to both physical and geometric constraints. This study further revealed two specific types of line-symmetric overconstrained 6R linkages. Chen and You [17] investigated bifurcations of two-fold symmetrical 6R deployable linkages using singular value decomposition of the Jacobian matrix of closure equations. Chen and Chai [18] put forward the study further on the bifurcated motion of a special line and plane symmetric Bricard linkage. Most recently, Zhang and Dai [19] explored trifurcation of the Bennett plano-spherical hybrid linkage,

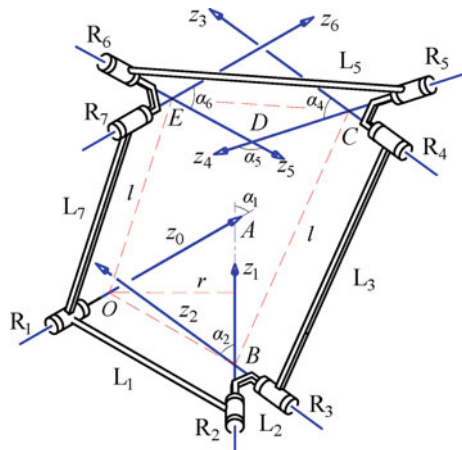
which is a plane-symmetric overconstrained 6R linkage, based on reciprocal screw systems. The 6R linkage allows distinct motion branches, in each of which the reconfigured linkage has only one-DOF, and transition between these motion branches by passing the singular position.

This paper presents a reconfigurable 7R linkage, which is able to change its structure and evolve to distinct overconstrained 6R linkages and planar 4R linkages. The geometry of the 7R linkage is first revealed. According to the geometric analysis, analytical expressions of the mechanism constraint-screw systems for distinct motion branches are derived. This leads to identification of constraint variation of the linkage and the induced motion branch change. An overconstrained 6R linkage that generates planar translational motion is explored in particular and the ruled surfaces of motion screws are illustrated.

2 Geometry and Kinematics of a Novel 7R Linkage

The focus of the present paper is a 7R linkage in Fig. 1, which only employs revolute joints. The links of the 7R linkage are labeled L_i ($i = 1, 2, \dots, 7$) and the commonly used revolute joints are labeled R_i . Every two adjacent joint axes at the two distal ends of a link are coplanar. The axes of adjacent joints R_3 and R_4 are parallel as well as that of joints R_1 and R_7 . The remaining axes of adjacent joints are intersecting. The common points of intersecting axes are denoted by A, B, C, D and E , respectively, and the angle between each pair of intersecting joint axes is denoted by α_i ($i = 1, 2, 4, 5, 6$) as illustrated in Fig. 1. In the particular design of this 7R linkage, $\alpha_1 = \alpha_2 = \alpha_4 = \alpha_6 = 45^\circ$ while $\alpha_5 = 90^\circ$. The projection of common point B on the axis of joint R_1 is denoted by O .

Fig. 1 Kinematic structure of the 7R linkage



2.1 Geometric Parameters

To facilitate the analysis, a Cartesian coordinate frame is attached to each link in terms of the Denavit and Hartenberg convention [20] with consideration of geometric properties of the 7R linkage in Fig. 1. The z_i -axis is aligned with the $(i + 1)$ th joint axis, and the x_i -axis is defined along the common normal between the i th and $(i + 1)$ th joint axes, pointing from the i th to the $(i + 1)$ th joint axis.

Parameters of the 7R linkage, including the offset distance between two adjacent joint axes a_i , the translational distance between two incident normals of a joint axis d_i and the twist angle between two adjacent joint axes α_i , are uniquely determined by geometry of the kinematic structure. Joint variables of revolute joints are denoted by θ_i . Accordingly, the invariant parameters a_i , α_i and d_i of the 7R linkage are given in Table 1. The design parameter l denotes length of identical links L_3 and L_4 and parameter r denotes the distance between point O and the axis of joint R_2 .

2.2 Kinematics Equations in Dual Quaternion

With the D-H parameters in Table 1, the transformation between two adjacent coordinate frames $i - 1$ and i is able to be described in dual quaternion [21–24] and expressed as

$$\hat{D}^{i-1} = \hat{S}(\hat{\theta}^i) \hat{S}(\hat{\alpha}^{i-1}) \quad (1)$$

where $\hat{S}(\hat{\theta}^i)$ and $\hat{S}(\hat{\alpha}^{i-1})$ are two dual quaternions, each of which represents a spatial displacement consisting of a rotation by θ_i (α_{i-1}) and a slide by d_i (a_{i-1}) around and along a screw axis $\mathbf{S}_z^{i-1}$ ($\mathbf{S}_x^i$), and are written as

$$\hat{S}(\hat{\theta}^i) = \cos\left(\frac{\hat{\theta}^i}{2}\right) + \sin\left(\frac{\hat{\theta}^i}{2}\right) \mathbf{S}_z^{i-1} \quad (2)$$

Table 1 D-H parameters of the 7R linkage

i	a_i	d_i	α_i (deg)
1	0	$\sqrt{2}r$	45
2	0	$2r$	45
3	l	0	0
4	0	0	45
5	0	r	90
6	0	r	45
7	l	0	0

in which

$$\hat{\theta}^i = \theta^i + \epsilon d^i \text{ and } \mathbf{S}_z^{i-1} = \mathbf{s}_z^{i-1} + \epsilon \mathbf{o}^{i-1} \times \mathbf{s}_z^{i-1} \quad (3)$$

and

$$\hat{\mathbf{S}}(\hat{\alpha}^{i-1}) = \cos\left(\frac{\hat{\alpha}^{i-1}}{2}\right) + \sin\left(\frac{\hat{\alpha}^{i-1}}{2}\right) \mathbf{S}_x^i \quad (4)$$

in which

$$\hat{\alpha}^{i-1} = \alpha^{i-1} + \epsilon a^{i-1} \text{ and } \mathbf{S}_x^i = \mathbf{s}_x^i + \epsilon \mathbf{o}^i \times \mathbf{s}_x^i \quad (5)$$

Thus, the dual quaternion kinematics equation for the 7R linkage in Fig. 1 can be derived as

$$\hat{\mathbf{D}}^1 \hat{\mathbf{D}}^2 \hat{\mathbf{D}}^3 \hat{\mathbf{D}}^4 \hat{\mathbf{D}}^5 \hat{\mathbf{D}}^6 \hat{\mathbf{D}}^7 = 1 \quad (6)$$

Given the D-H parameters in Table 1 and one of the joint variables θ_i , the remaining six joint variables can be derived based on the procedure addressed for a general case of 7R linkage in Ref. [25]. In general, for complex linkages, the explicit solution of (6) is unavailable. The mobility analysis can then resort to the higher-order kinematic analysis reported in [26, 27].

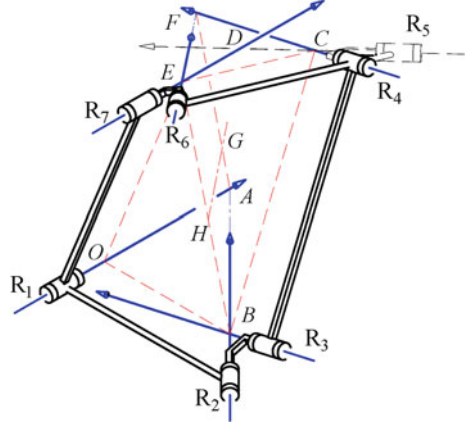
3 Constraint Variation Induced Multifurcation

For a general 7R linkage, the mechanism motion-screw system dimension [28, 29], d , equals to 6 and the general Kutzbach-Grübler mobility criterion stands. In other words, the mobility of the 7R linkage is calculated from the mobility criterion is $m = 6(7 - 7 - 1) + 7 = 1$. However, the mechanism motion-screw system dimension varies at typical configuration where singularity occurs, leading to distinct motion branches with different effective revolute joints and links. This section analyzes constraint variation induced changes of mechanism motion-screw system dimension and the structure reconfiguration.

3.1 Motion Branch of Line-Symmetric Overconstrained 6R Linkage

When the 7R linkage moves to a configuration in Fig. 2 where the axes of joints R_4 , R_5 and R_6 are coplanar, the disposition of joint axes except that of R_5 are with

Fig. 2 Kinematic structure of the evolved line-symmetric 6R linkage with R_5 geometrically restricted



rotational symmetry of order two [16] respect to line GH , where G and H are the mid-points of line BE and AF , respectively, and F is the common point of axes of joints R_4 and R_6 .

With consideration of rotational symmetry of the linkage with respect to line GH in this particular configuration, a global coordinate frame $O-XYZ$ can be set up by aligning Z - and X -axis to GH and BE , respectively. The mechanism constraint-screw system, in which the constraint screw is reciprocal to motion screws of the linkage in this configuration, expressed in the frame $O-XYZ$ is

$$\mathbb{S}^c = \mathbb{S}^r = \begin{bmatrix} \frac{d_0(d_0^2 + r^2 + l^2)}{2lr \sin \theta_1 \sqrt{d_0^2 + (l \sin \theta_1)^2}} & 1 & 0 & \frac{d_0 \sqrt{d_0^2 + (l \sin \theta_1)^2}}{2(l \cos \theta_1 - r)} & \frac{d_0^2}{2l \sin \theta_1} & 0 \end{bmatrix}^T \quad (7)$$

in which d_0 is the distance between points B and E and θ_1 is the angle measured between OB and OE .

The above constraint screw spans a screw system of order one. It implies that the mechanism motion-screw system of the 7R linkage in this line-symmetric configuration in Fig. 2 degenerates and $d = \dim(\mathbb{S}_m) = 6 - \dim(\mathbb{S}^c) = 6 - 1 = 5$. Under such a condition, the mobility of this evolved linkage can be calculated using the modified mobility criterion [29], that is $m = 5(7 - 7 - 1) + 7 = 2$. This means the infinitesimal mobility of the linkage in this configuration is two.

Since axes of revolute joints except that of joint R_5 are symmetric with respect to line GH , the linkage is able to work as a line-symmetric overconstrained 6R linkage with the joint R_5 geometrically restricted as the motion screw of joint R_5 is dependent on motion screws of the remaining six joints in Fig. 2. Hence, there are only six joints of the 7R linkage active and the finite mobility of the overconstrained 6R linkage is $m = 5(6 - 6 - 1) + 6 = 1$.

3.2 Motion Branch of an Overconstrained 6R Linkage with Planar Translational Motion

When the 7R linkage moves to a configuration in Fig. 3 where axes of joints R_5 , R_6 and R_7 are coplanar, lines OB and CE are perpendicular to axes of joints R_1 and R_7 , respectively. As a result, the four points B , O , E and C become coplanar in this configuration since line OE is the common normal of axes of joints R_1 and R_7 . According to the design of the 7R linkage, the length of links L_3 and L_7 are equal. It further derives that the four points B , O , E and C form a parallelogram.

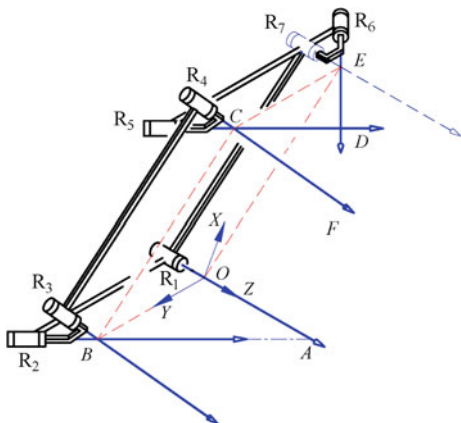
A coordinate frame O - XYZ for this motion branch is set up by aligning Z -axis to the axis of joint R_1 and Y -axis to the line OB as illustrated in Fig. 3. The X -axis is perpendicular to the triangle ΔAOB . In this configuration, the mechanism constraint-screw system expressed in the frame O - XYZ is

$$\mathbb{S}^c = \mathbb{S}^r = \begin{bmatrix} -am_3 & bm_3 & 0 & 0 & \sqrt{2}r(am_3 + b(l_3 - n_3)) & \sqrt{2}rbm_3 \end{bmatrix}^T \quad (8)$$

where $[l_3, m_3, n_3]^T$ is the unit vector pointing in the direction of joint axis R_3 and $[a, b, 0]^T$ is the position vector of point E .

The above constraint screw spans a screw system of order one. Under such a condition, the motion-screw system of the linkage in the configuration in Fig. 3 degenerates and $d = \dim(\mathbb{S}_m) = 6 - \dim(\mathbb{S}^c) = 6 - 1 = 5$. The linkage is able to work as another overconstrained 6R linkage with the joint R_6 geometrically restricted as the motion screw of joint R_6 is dependent on motion screws of the remaining six joints in Fig. 3. Hence, there are only six joints of the 7R linkage are active and the finite mobility is $m = 5(6 - 6 - 1) + 6 = 1$. The ruled surfaces of motion screws of all the revolute joints are illustrated in Fig. 4. The ruled surfaces further demonstrate that the three points B , C and E move in a unique plane perpendicular to the axis of joint R_1 . Further, the ruled surfaces of joints R_6 and R_7 in Fig. 4c prove that the surface by the axis of joint R_7 is a cylinder while the motion of joint R_6 is locally restricted with respect to joint R_7 .

Fig. 3 Kinematic structure of the evolved 6R linkage with planar translation



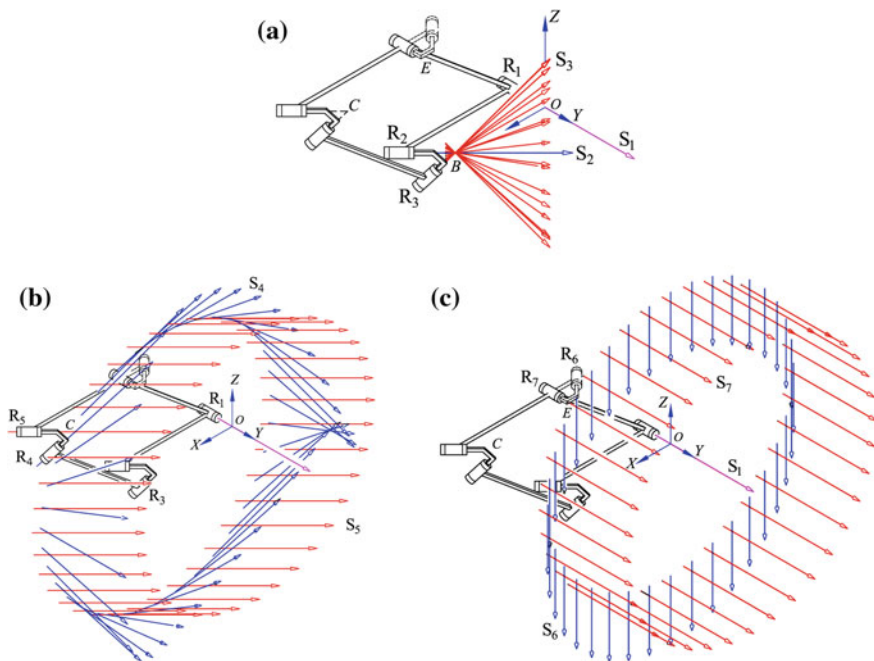


Fig. 4 Ruled surfaces of motion-screws of all revolute joints: **a** ruled surface of joint R_3 , **b** ruled surfaces of joints R_4 and R_5 **c** ruled surfaces of effective joint R_7 and restricted joint R_6

3.3 Motion Branch of Planar 4R Linkage

When the 7R linkage moves to a configuration in Fig. 5 where the axes of joints R_4 , R_5 , R_6 and R_7 are coplanar and the axes of joint R_1 , R_2 and R_3 are parallel to the plane formed by the former four coplanar joints. Particularly, axes of the joints R_1 , R_3 , R_4 and R_7 are parallel and points B , O , E and C become coplanar in this configuration. According to the design parameters of the 7R linkage, the four points B , O , E and C form a parallelogram.

The motion-screw system of the linkage in the configurations in Fig. 5 degenerates and $d = 3$. The linkage is able to work as planar four-bar linkage with joints R_2 , R_5 and R_6 geometrically restricted.

These two motion branches can be realized when the mobile loop passes a singular positions where axes of all seven joints are coplanar. One motion branch is a parallelogram 4R linkage in Fig. 5a and another is a anti-parallelogram 4R linkage in Fig. 5b.

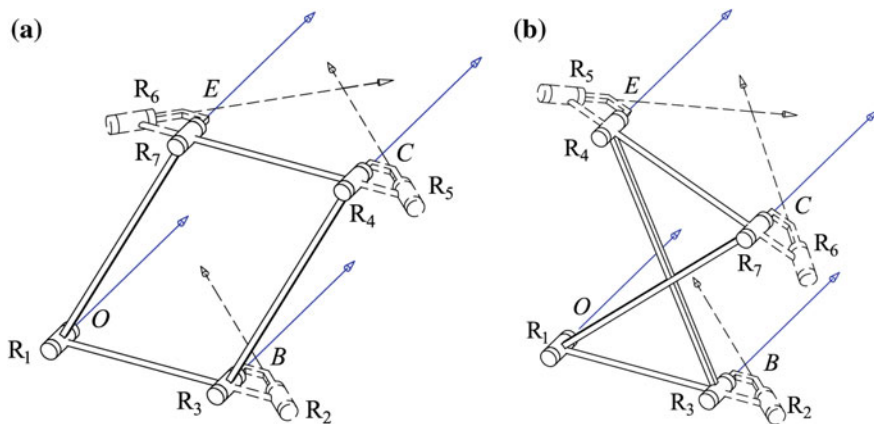


Fig. 5 The evolved planar 4R linkage with effective joints R_1 , R_3 , R_4 and R_7 : **a** parallelogram 4R linkage, **b** anti-parallelogram 4R linkage

4 Conclusions

This paper presented a 7R linkage with reconfigurability. The geometry of the seven-bar 7R kinematic structure was first described. Following this, D-H parameters and dual quaternion kinematics equations of the 7R linkage were subsequently derived. According to the kinematics analysis, analytical expressions of the constraint-screw system in typical configurations of the 7R linkage were investigated in accordance with geometric properties. The constraint analysis revealed that the mechanism motion-screw system degenerates in these typical configurations. The mechanism motion-screw system degeneration led to multifurcation of the non-overconstrained 7R linkage, which is able to reconfigure its kinematic structure to overconstrained 6R mobile loops and planar 4R linkages. An overconstrained 6R linkage that generates planar translational motion was explored and the ruled surfaces of motion-screws were illustrated.

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