

Maintenance Optimization Using Multi-attribute Utility Theory

A.H.S. Garmabaki, Alireza Ahmadi and Mahdieh Ahmadi

Abstract Several factors such as reliability, availability, and cost may consider in the maintenance modeling. In order to develop an optimal inspection program, it is necessary to consider the simultaneous effect of above factor in the model structure. In addition, for finding the optimal maintenance interval it is necessary to make trade-offs between several factors, which may conflicting each other as well. The study comprises of mathematical formulating an optimal interval problem based on Multi-Attribute Utility Theory (MAUT). The aim of the proposed research is to develop a methodology with supporting tools for determination of optimal inspection in a maintenance planning to assure and preserve a desired level of performance measure such as reliability, availability, risk, etc. For verification and validation purposes, the proposed methodology (analysis approach) and tools (models) will be applied in a real case which given by the literature.

Keywords Maintenance optimization • Multi-attribute utility theory • Reliability • Availability • Risk

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1 Introduction

Multi-criteria decision making (MCDM) is one of the most well-known branches of decision making. According to many authors (see, for instance, [1]) MCDM is divided into multi-objective decision making (MODM) and multi-attribute decision making (MADM). MCDM is concerned with the methods and procedure by which multiple criteria can be formally incorporated into the analytical process [2]. There are several methods proposed by literature. The weighted sum model (WSM) is the earliest and probably the most widely used methods. The weighted product model (WPM) can be considered as a modification of the WSM, and has been proposed in order to overcome some of its weaknesses. The analytic hierarchy process (AHP), as proposed by Saaty [3], is a later development and it has recently become increasingly popular in different area. Belton and Gear [4] modified AHP method and the new approach is more consistent than the original AHP. Some other popular methods proposed by literature are the VIKOR and the TOPSIS methods. These methods are based on an aggregating function representing “closeness to the ideal, which originated in the compromise programming method”. Both TOPSIS and VIKOR are based on the calculation of distances from the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS). Chu et al. [5] are in favors of using VIKOR when there are a larger number of decision makers (DM), and otherwise they recommend the use of TOPSIS. Recently, Ahmadi et al. [6] show that application of the combined AHP, TOPSIS, and VIKOR methodologies are applicable and verified the proposed methodology through a case study for an aircraft system.

Maintenance decision making is a complex task and may take place in several contexts with different types of systems in terms of technology, repairability, reliability and availability requirements, etc. For optimal time determination of the maintenance plan, maintenance management may present scenarios, including several objectives which often competing or conflicting with each other. The objectives can be represented by a set of appropriate measures or attributes, which are used to represent system characteristics. Here, the decision maker not only required to choose the best solution among alternatives, but also have to trade-off between the objectives.

Kralj and Petrovic [7] used multiple objective function to tackle costs and reliability in preventive maintenance. In another study, an optimal interval for preventive maintenance was obtained based on the PROMETHEE method [8]. Gopalaswamy et al. [9] argued for strict selection and lexicographical approaches applied to preventive maintenance, taking into account criteria such as costs, availability and reliability. Most research on preventive maintenance problems in the literature is based on a multi-criteria approach to analyze particular problems using multi-criteria approaches that do not incorporate the most useful advantage of multi attribute utility theory (MAUT). However, some decision models for maintenance are based in MAUT. See [10–12].

Here, we propose an optimal maintenance inspection model based on MAUT. In order to determine optimal time, different criteria such as cost, reliability, and

availability are considered in the model framework. In order to provide insight into the problem, a utility function is assessed for each of the relevant objectives. This allows for an appropriate multiple objective utility functions that are used to identify tradeoffs and compare the various objectives in a consistent manner. The basis of utility theory and its underlying quantitative axioms were initially established by Keeney and Raiffa [13]. The decision model has been applied on a real case in an electric power company. The decision level and weight parameter are selected, subjectively and sensitivity analysis is conducted to identify the most sensitive parameter.

The rest of the paper is organized as follows. The proposed model based on MAUT is discussed in Sect. 2. Section 3 shows the numerical example and verified the proposed methodology through a real case study. In addition, a sensitivity analysis is discussed in Sect. 4. Finally, conclusions are given in Sect. 5.

2 Multi Attribute Utility Theory

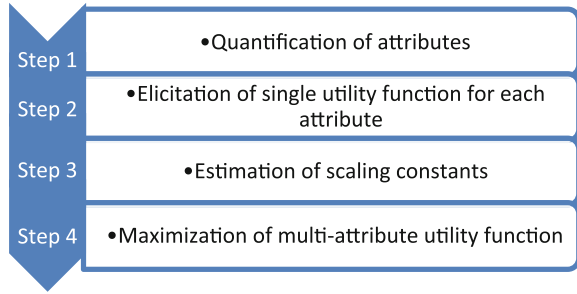
Multi-attribute utility theory (MAUT) [13] is concerned with expressing the utilities of multiple-attribute outcomes or consequences as a function of the utilities of each attribute taken singly. This approach has been used for choosing the most “desirable alternative” (or project) among many different alternatives. It has been used in a broad range of fields including energy, manufacturing and services, public policy, health care, etc. MAUT can help in these situations by creating a decision model through the elicitation process of expert practitioners.

The theory specifies several possible functions (additive, multiplicative and multi-linear) and the conditions (independence conditions to be met) under which each would be appropriate. As a practical matter, Keeney and Raiffa [13] suggest that for four or more attributes the reasonable models are the additive and the multiplicative. Since our problem contains less than four attributes, we restrict our attention to the additive form. The MAUT process provides a framework through which multiple objectives and uncertainty can be combined to aid managers in making decisions. In order to create a MAUF Problem, single utility functions must be assessed for every identified objective. In our case, we have identified three separate attribute. The objective list utilized for this preliminary analysis is minimization of cost and maximization of reliability and availability. Generally, a MAUF is defined as:

$$\begin{aligned} U(x_1, x_2, \dots, x_n) &= f[u_1(x_1), u_2(x_2), \dots, u_n(x_n)] \\ &= \sum_{i=1}^n w_i \cdot u_i(x_i) \end{aligned} \quad (1)$$

where, $\sum_{i=1}^n w_i = 1$

Fig. 1 The structure of MAUT for the determination of optimal inspection time



where, U is a multi-attribute utility function over all utility functions; $u_i(x_i)$ is a single utility function measuring the utility of attribute i ; x_i is level of i th attribute. w_i represent the relative importance weights for the utilities. By maximizing the multi-attribute utility function, the best alternative is obtained, under which the attractiveness of the conjoint outcome of attributes is optimized. The main reason for the selection of MAUT in our problem is that scenarios of management can be appropriately represented by the structure of this technique. Furthermore, MAUT has strong theoretical foundations based on the expected utility theory.

In order to obtain structure for utility functions, first we need to make assumptions regarding utility independence and the additive independence. The procedure of the use of it in our problem is discussed in detail by [13]. The utility functions are assessed in the following four steps [13, 14] (Fig. 1).

2.1 *Quantification of Attributes*

In our case study, cost, availability and reliability are selected as attribute to find out the optimal maintenance policy. The attributes and their mathematical structure are discussed in following subsection.

2.1.1 *Cost Modeling*

In the preventive replacement age policy subject to breakdown, instead of making a preventive replacement at fix time interval T , the preventive replacement depends on the age of the item. In addition, failure replacement is performed if the system fails before T and the time clock is reset to zero, see [15] for more details. The average cost per unit time based on optimal preventive replacement is given by:

$$\begin{aligned}
C(T) &= \frac{c_p R(T) + c_f (1 - R(T))}{T \cdot R(T) + M(T) \cdot (1 - R(T))} \\
&= \frac{c_p R(T) + c_f F(T)}{T \cdot R(T) + M(T) \cdot F(T)}
\end{aligned} \tag{2}$$

where $M(T) = \int_{-\infty}^T \frac{tf(t)}{(1-R(t))} dt$

and T is a replacement age at which a preventive replacement takes place, c_p and c_f ($c_f > c_p$) are the cost of a preventive and failure replacement. In both cases, replacement cost includes all costs resulting from the failure and its replacement.

In this model, the numerator equals to the total expected cost per cycle and the denominator equals to the expected cycle length; $F(t)$ and $R(t)$ are the cumulative distribution and reliability functions, respectively. The optimal value of T corresponds to the minimum cost, $C(T)$, can be derived by the first derivation of $C(T)$ with respect to T . This model is discussed in details by Jardine and Tsang [16].

Cost Attribute

The average cost per unit time given by Eq. (2) has a unique minimum C_{Min} which occurs at T_C . Since small value of $C(T)$ is preferred, we define the cost attribute function as:

$$U_{Cost} = \frac{C_{Min}}{C(T)} \tag{3}$$

2.1.2 Availability Modeling

Availability is defined as the long run probability of the system being available for use at any point in time [17]. This is expressed as a point estimate and calculated from the mean delay and reliability point estimates. There are several different forms of steady state availability depending on the definition of uptime and downtime. The Inherent availability is most common definition in the literature:

$$A_I = \frac{MTTF}{MTTF + MRT} \tag{4}$$

where MRT is the mean repair time and $MTTF$ is the mean time-to-failure.

In our decision problem, optimal preventive replacement age policy subject to breakdown are considered. For above standard definition, the following structure can be derived for single unit.

$$A(T) = \frac{\int_0^T R(t)dt}{\int_0^T R(t)dt + t_p R(T) + t_f(1 - R(T))} \quad (5)$$

where t_p and t_f are the require time of performing a preventive and a failure replacement, respectively. A large value of $A(T)$ is preferred.

Availability Attribute

The average availability per unit time given by Eq. (5) has a unique maximum A_{Max} which occurs at T_A . Since a large value of $A(T)$ is preferred, the availability attribute may be define as:

$$U_{Ava} = \frac{A(T)}{A_{Max}} \quad (6)$$

2.1.3 Reliability Modeling

Reliability is closely associated with the quality of the product. This criteria is one of the main concerns during different stage of product development such as design, testing and operation. Reliability is defined as probability that a system will function over the time period. Reliability can be expressed as

$$\begin{aligned} R(t) &= \Pr(T \geq t) \\ R(t) &= 1 - F(t) \end{aligned} \quad (7)$$

where $R(t) \geq 0, R(0) = 1$ and $\lim_{t \rightarrow \infty} R(t) = 0$.

Reliability Attribute

The reliability level of the product at time T , is depend to failure distribution and the interval which is our aim to study. Reliability per unit time given by Eq. (7), has a unique maximum R_{Max} which occurs at T_R . Since a large value of $R(T)$ is preferred, the reliability attribute is given by:

$$U_{Rel} = \frac{R(T)}{R_{Max}} \quad (8)$$

2.2 Elicitation of Single Utility Function for Each Attribute

The single utility function for each attribute represents management's satisfaction level towards the performance of each attribute. It is usually assessed by a few particular points on the utility curve [18, 19].

More specifically, suppose that the best and worst values of availability are selected first as A^B and A^W . At these boundary points, we have $U(A^W) = 0$ and $U(A^B) = 1$. For cost utility function, highest and lowest budget consumption requirement values are selected as C^W and C^B , respectively. Also, at these boundary points, we have $U(C^W) = 0$ and $U(C^B) = 1$.

To elicit the single utility function the exponential or linear function, may suggested for each attribute given by Eq (9).

$$\begin{cases} U(x) = k_1x + k_2 & \text{Linear function} \\ U(x) = k_3 \cdot \exp(-\frac{k_4}{x}) & \text{Exponential function} \end{cases} \quad (9)$$

where k_i are constants which secure $U(x_i) \in [0, 1]$. Unknown parameter for utility functions, $U(A)$, $U(R)$ and $U(C)$ can be obtained using linear (exponential) form of single utility function with the help of boundary conditions.

The linear utility function is applied for availability and cost attribute. The linear function is applicable when the DM is risk neutral [13]. That is, the DM is neither risk prone nor risk averse. For reliability, the logistic utility function is found to be suitable. This function presents a risk aversion for higher values of R and prone risk for lower values of R , which is the DM's risk behavior for increasing utility function.

2.3 Estimation of Scaling Constants

The following step is the estimation of the scaling constants w_A, w_R and w_C . They indicate the importance weights that management team allocates for each attribute [18, 20]. There are two common methods to assess the scaling constants:

1. Certainty scaling and
2. Probabilistic scaling

Given that the number of attributes considered in our problem is three and we will use probabilistic scaling technique.

Consider three attributes A , R and C as availability, reliability and cost. Let (A^B, R^B, C^B) and (A^W, R^W, C^W) denote the best and worst possible consequence, respectively (Fig. 2). There is a certain joint outcome (A^B, R^B, C^W) comprised three attribute A , R and C at the best and worst level with probability p and $(1-p)$, respectively. In these situations, the weight for attribute C equals p , where p is the indifference probability between them, see [18].

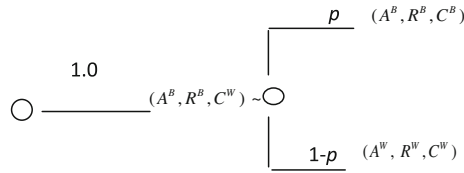


Fig. 2 Assessing scaling constants

2.4 Maximization of Multi-attribute Utility Function

Based on the previously estimated single utility functions and scaling constants, the additive form of the multi-attribute utility function in our problem can be obtained. That is

$$\begin{aligned} \text{Max : } U(A, R, C) &= w_A \times U(A) + w_R \times U(R) + w_C \times U(C) \\ w_A + w_R + w_C &= 1 \end{aligned} \quad (10)$$

where w_A , w_R and w_C are the weight parameters for attribute A, R and C, respectively. $U(A)$, $U(R)$ and $U(C)$ are the single utility function for availability, reliability and cost attribute. It may note that the $U(A, R, C)$ function is *Maximum* type and it has been written in terms of A, R and C. By maximizing this multi-attribute utility function, the optimal inspection, T^* will be obtained. It is worth noting here that the additive form of multi-attribute utility function is based on the utility independence and the additive independence assumptions.

3 Numerical Example

This numerical application is conducted to verify MAUT in maintenance application. Assume that 2-parameter Weibull model is selected as failure distribution which are given by Eq (11) and the parameter of the model and attributes are given in Table 1.

Table 1 Estimated parameter from real application [21]

β	3	Shape parameter
η	1200	Scale parameter
t_p	0.2	Time of performing preventive maintenance
t_f	0.4	Time of performing corrective maintenance
c_p	600	Cost of preventive maintenance
c_f	1200	Cost of corrective maintenance

$$F(T) = 1 - \exp(-(T/\eta)^\beta)$$
$$f(T) = \frac{\beta}{\eta} \cdot \left(\frac{T}{\eta}\right)^{\beta-1} \cdot \exp(-(T/\eta)^\beta)$$

(11)

In addition, the best and worse level for each attribute are given in Table 2. The linear utility function is applied for availability and cost attribute. In addition, the logistic utility function is considered for reliability attribute. For each attribute, the constant coefficients are calculated and given in Table 2. The availability, reliability and cost attribute are plotted in Figs. 3, 4 and 5.

Table 2 Attributes function and coefficients

Attributes	Best	Worse	Function	Coefficient value
Availability attribute	$A^B = 0.95$	$A^W = 0.25$	$U(x) = k_1A + k_2$	$k_1 = 1.428;$ $k_2 = -0.357$
Reliability attribute	$R^B = 0.9$	$R^W = 0.3$	$U(x) = k_3 \cdot \exp(-\frac{k_4}{R})$	$k_3 = 9.985;$ $k_4 = 2.0718$
Cost attribute	$C^B = 0.35$	$C^W = 1$	$U(x) = k_5A + k_6$	$k_5 = 1.5384;$ $k_6 = -0.538$

Fig. 3 The availability attribute

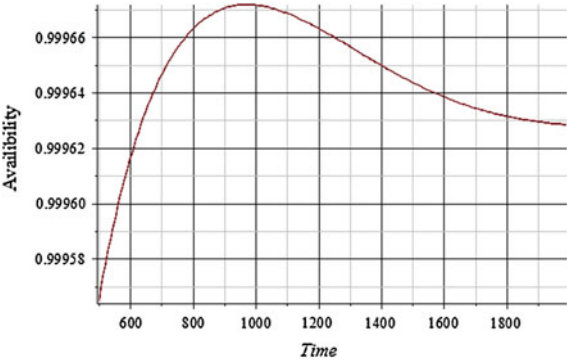


Fig. 4 The reliability attribute

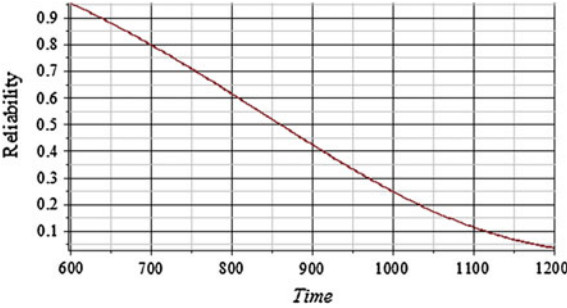
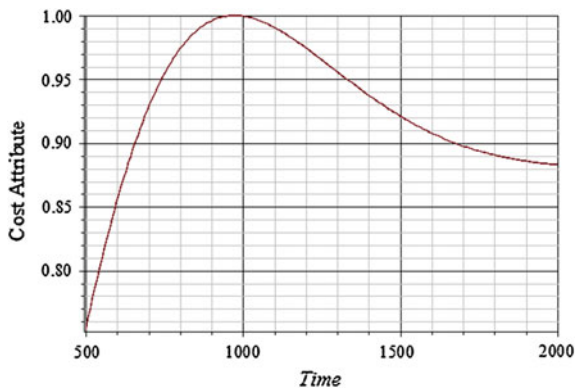
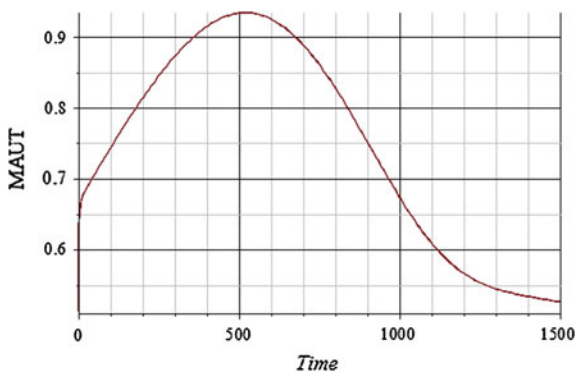


Fig. 5 The cost attribute**Fig. 6** Multi attribute utility function

The behavior of MAUF function is given in Fig. 6. The optimal inspection time by considering three attribute with above weight occur at $t \in [490, 550]$. More specifically, when we consider only cost for determination of optimal inspection time, we get $t = 950$ which seems is more delay for inspection time.

4 Sensitivity Analysis of the Model Parameters

From the discussion given in the preceding section, it is good to know that the optimal decision-making depends on various parameters that may not be precise.

The use of sensitivity analysis will help the analyst to understand how changing the parameters of the model will affect the decision outcome. The decision model is then rerun by holding all other parameters constant. We have conducted sensitivity analysis by calculating the relative change of optimal time based different parameters given in Table 3. The sensitivity of the optimal inspection time with respect to

Table 3 Sensitivity analysis results based on model parameter

	$p\%$					
$\Delta_{\rho,\theta}^{u^*}$	-30 %	-20 %	-10 %	10 %	20 %	30 %
$\Delta_{\rho,A^B}^{u^*}$	30 %	10 %	1 %	NA	NA	NA
$\Delta_{\rho,A^W}^{u^*}$	4 %	2 %	1 %	1 %	1 %	2 %
$\Delta_{\rho,C^B}^{u^*}$	6.5 %	6 %	5 %	NA	NA	NA
$\Delta_{\rho,C^W}^{u^*}$	8 %	7 %	4 %	2 %	2 %	2 %
$\Delta_{\rho,R^B}^{u^*}$	NF	NF	NF	10 %	NA	NA

Note: Na, impossible change; Nf, infeasible solution

a model parameter, can be quantified by $\Delta_{\rho,\theta}^{u^*}$, which are the relative changes of optimal utility level, $u^*(\theta)$ when θ is changed by 100p%, i.e.,

$$\Delta_{\rho,\theta}^{u^*} = \left| \frac{u^*(\theta + p\theta) - u^*(\theta)}{u^*(\theta)} \right| \quad (12)$$

In addition, different weight are assign to the attribute and the results are plotted in Fig. 7. The values of different weight are given in Table 4.

It can be seen that the sensitivity of optimal interval with respect to model parameters A^W and positive effect of C^W is at acceptably low levels, e.g., when $A^W(C^W)$ increases by 30 % (decreases by -30 %) the relative changes in Δ are 2 and 4 %, respectively. Results in Table 3 reveal that A^B and negative part of C^B and C^W are slightly more sensitive parameter than other parameters.

In addition, negative change of w_R did not reveal the high level of sensitivity and positive effect of w_R will reduce inspection time.

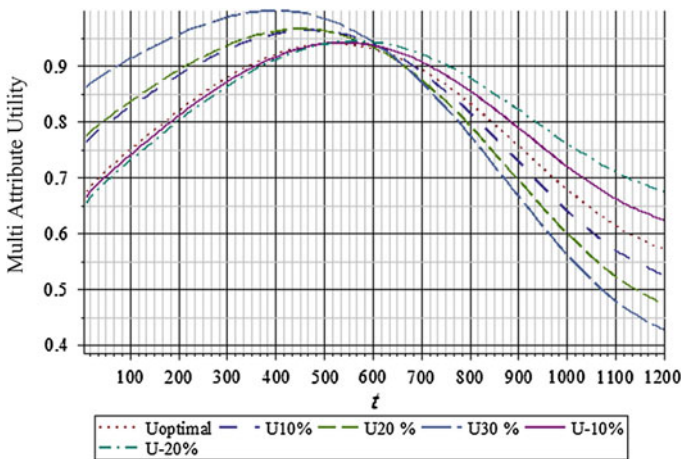
**Fig. 7** Sensitivity analysis on weight parameters

Table 4 Different weight of w_R in optimization problem

	$U_{-20} \%$	$U_{-10} \%$	U_{Optimal}	$U_{10} \%$	$U_{20} \%$	$U_{30} \%$
w_R	0.35	0.4	0.45	0.5	0.55	0.6
w_A	0.35	0.3	0.25	0.25	0.2	0.2
w_C	0.3	0.3	0.3	0.25	0.25	0.2

5 Conclusion

In this paper, we have developed a multi attribute utility model for the preventive replacement age policy subject to breakdown. Reliability, availability and cost are considered as three main attribute in our decision problem. By using MAUT, it is possible to make trade-offs between several factors, which may conflicting each other as well. In addition, the optimal solution depends not only on the failure distribution and the cost ratio, but also on the maintenance time ratio as well as the relative importance of the attributes. The MAUT is important for the maintenance and reliability community when a context of service production systems is to be taken into account due to disturbances caused by failures in the system. A numerical application has illustrated the use of the decision model and the procedure.

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