

Chapter 2

Single Electron Transistor

The first single electron transistor (SET), made of small tunnel junctions, was realized in the Bell Laboratories in 1987 by Fulton and Dolan [1]. Since then, the fabrication of SETs became more and more sophisticated and allowed for operation at room temperature [2] or as sensors for electron spin detection [3]. In this thesis, a single electron transistor will be used to read-out the state of an isolated nuclear spin and, therefore, a basic knowledge of the transport properties in SETs and its associated effects such as Coulomb blockade, elastic and inelastic cotunneling, and the Kondo effect are necessary.

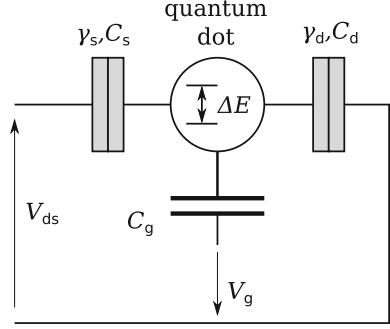
2.1 Equivalent Circuit

A single electron transistor consists of a conducting island or quantum dot, which is tunnel-coupled to the source and drain leads. Due to the small size of the dot the electronic energy levels E_n are discretized. In order to observe the characteristic single electron tunneling through the device, the resistance R_t of the tunnel barriers should be much higher than the quantum of resistance:

$$R_t \gg \frac{h}{e^2} \quad (2.1.1)$$

where h is the Planck constant and e the elementary charge. This condition ensures that only one electron at the time is tunneling in or out of the quantum dot. A simple model to describe the electron transport through the dot was developed by Korotkov et al. [4], and reviewed by Kouwenhoven [5], and Hanson [6]. Therein, the quantum dot is coupled via constant source, drain, and gate capacitors (C_s , C_d , C_g) to the three

Fig. 2.1 Equivalent circuit of an SET. The electrostatic behavior of the dot is modeled by capacitors to the source, drain, and gate terminals



terminals as shown in Fig. 2.1. By applying a voltage to the three different terminals, the electrostatic potential U_{es} of the quantum dot is modified as:

$$U_{es} = \frac{(C_s V_s + C_d V_d + C_g V_g)^2}{2C_\Sigma} \quad (2.1.2)$$

with $C_\Sigma = C_s + C_d + C_g$ and V_s , V_d , and V_g being the source, drain, and gate voltages, respectively. Furthermore, due to the Coulomb repulsion, adding an electron to the quantum dot with N electrons ($N > 0$) will cost an additional energy:

$$U_c = \frac{E_c}{2} = \frac{e^2}{2C_\Sigma} \quad (2.1.3)$$

with E_c being the charging energy. Accordingly, to observe single electron tunneling, temperatures smaller than E_c are required since, otherwise, the tunnel process can be activate thermally.

$$E_c \gg k_B T \quad (2.1.4)$$

Putting all contributions together results in the total energy U of the quantum dot with N electrons:

$$U(N) = \frac{(-e(N - N_0) + C_s V_s + C_d V_d + C_g V_g)^2}{2C_\Sigma} + \sum_1^N E_n(B) \quad (2.1.5)$$

where N_0 is the offset charge and $E_n(B)$ the magnetic field dependent single electron energies. Experimentally, it is more convenient to work with the chemical potential, defined as the energy difference between two subsequent charge states $\mu_{dot}(N) = U(N) - U(N - 1)$. Inserting Eq. 2.1.5 into this expression gives:

$$\mu_{dot}(N) = \left(N - \frac{1}{2}\right) E_c - \frac{E_c}{|e|} (C_s V_s + C_d V_d + C_g V_g) + E_N(B) \quad (2.1.6)$$

with E_N being the energy of the N th electron in the quantum dot. Notice that the chemical potential depends linearly on the gate voltage, whereas the total energy shows a quadratic dependence. Therefore, the energy difference between the chemical potentials of different charge states remains constant for any applied voltages. The energy to add an electron to the quantum dot is called addition energy E_{add} and is defined as the difference between two subsequent chemical potentials.

$$E_{\text{add}}(N) = \mu(N+1) - \mu(N) = E_c + \Delta E \quad (2.1.7)$$

with ΔE being the energy spacing between two discrete energy levels.

2.2 Coulomb Blockade

The transport through the quantum dot is very sensitive to the alignment of the chemical potential μ inside the dot with respect to those of the source μ_s and drain μ_d . If we neglect the level broadening and any excited states of the quantum dot for a moment, then the transport through the SET can be explained with Fig. 2.2. Notice that V_{ds} and V_g are in arbitrary units and $V_g = 0$ when $\mu_{\text{dot}} = \mu_s = \mu_d$.

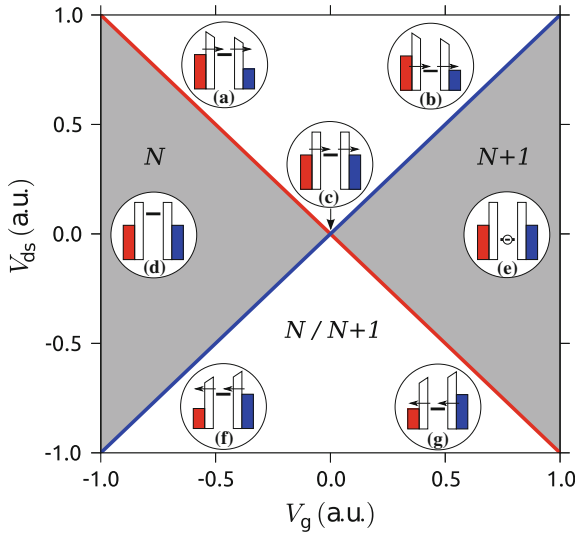


Fig. 2.2 Schematic of a stability diagram. Inside the *grey* regions the charge of the quantum dot is fixed to N (d) or $N+1$ (e), leading to the Coulomb blockade. Likewise, inside the *white* area electrons can tunnel in and out of the quantum dot. If the conductance dI/dV is measured instead of the current I , only the *red* and the *blue* line are visible, corresponding to a change in I . Along the *red* line the chemical potential of the quantum dot is aligned with the source chemical potential, whereas along the *blue* line it is aligned with the drain chemical potential

First we want to discuss what happens for zero bias $V_{ds} = V_d = V_s = 0$. If $V_g < 0$, the chemical potential of the dot is larger than the chemical potential of the leads, and the SET is in its off state (Fig. 2.2d). Increasing V_g to zero will align the three chemical potentials. Electrons can tunnel in and out of the dot from both sides leading to a finite conductance and a charge fluctuation between N and $N + 1$. This particular working regime is called the charge degeneracy point (Fig. 2.2d). A further increase of V_g will push the chemical potential of the dot below the ones of source and drain, and the SET is again in its off state, but having $N + 1$ electrons on the dot. Whenever the charge of the dot is fixed, the SET is in the Coulomb blockade regime since adding another electron would cost energy to overcome the electron-electron repulsion.

If we now increase the bias voltage to $V_{ds} \neq 0$, we shift the chemical potential between source and drain and open an energy or bias window of $\mu_s - \mu_d = eV_{ds}$, and a current is observed even for $V_g \neq 0$.

The red line in Fig. 2.2 corresponds to the situation where the chemical potential of the dot is aligned with μ_s (Fig. 2.2a, g). Crossing this line will turn the SET on or off, resulting in a conductance ridge along the line. The slope can be calculated from the equivalent circuit by setting the potential difference between dot and source to zero and is given by $-C_g/(C_g + C_s)$.

On the other hand, if μ_{dot} is aligned with the drain chemical potential, the SET turns also on or off, resulting in another conductance ridge (blue line in Fig. 2.2). Its slope is of opposite sign and calculated by setting the potential difference between drain and dot to zero, resulting in C_g/C_d . Therefore, inside the white region the transistor is turned on, whereas inside the grey region the SET is Coulomb blocked.

2.3 Cotunneling Effect

Up to now only transport through energetically allowed states was considered. This is usually sufficient if the tunnel barrier resistances are larger than $1 \text{ M}\Omega$. However, for smaller tunnel barrier resistances, the time to exchange an electron between the dot and the leads becomes fast enough to allow for transport through energetically forbidden states. This is possible due to the Heisenberg uncertainty relation, which states that a system can violate energy conservation within a very short time $\tau = \hbar/E$, where $E \approx E_c$ for quantum dots. Therefore within the time τ an electron can enter the quantum dot whereas another is tunneling into the leads. Since this process involves two electrons it is called cotunneling. Note that the entire tunnel process is considered to be a single quantum event. We distinguish in the following two different cases of cotunneling events, namely, elastic and inelastic cotunneling [7]. If the electron entering the quantum dot occupies the same energy level as the outgoing one, the cotunneling is elastic and requires no additional energy (Fig. 2.3a). Experimentally, it can be observed as a conductance background inside the Coulomb blocked region. If, however, the electron entering the dot occupies an excited state, separated by ΔE from the chemical potential of the electron leaving the dot, the transport is inelastic (Fig. 2.3b). This process requires energy and happens only at finite bias voltages with

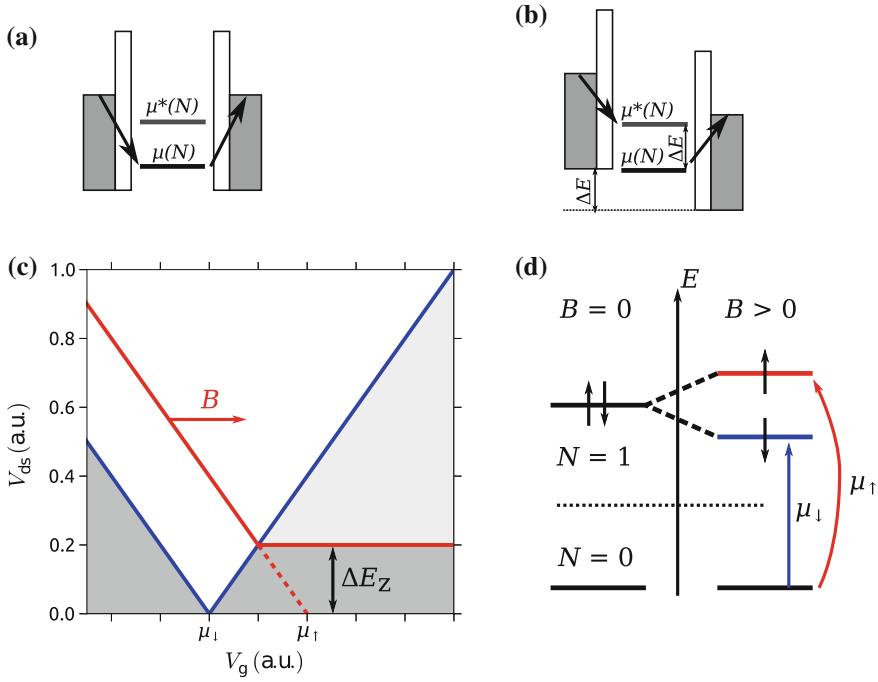


Fig. 2.3 Schematic showing the elastic **a** and inelastic **b** cotunneling process. **c** Stability diagram for a quantum dot whose ground and excited state are split by the Zeeman energy. The conductance step inside the Coulomb blocked region (*dark grey*) occurs at $e|V_{ds}| = \Delta E_Z$ and can be used to determine the Zeeman splitting. **d** Energy level diagram of a quantum dot with zero or one electron. Due to an external magnetic field, the degeneracy between *spin up* and *spin down* is lifted. The two lowest lying chemical potentials of the quantum dot correspond to the energy difference of the Zeeman split $N = 1$ doublet and the $N = 0$ singlet

$e|V_{ds}| > \Delta E$. The result is a conductance step inside the Coulomb blocked region. In the case of a very simple quantum dot as shown in Fig. 2.3c, d the conductance step can be used to determine the Zeeman splitting due to a magnetic field.

2.4 Kondo Effect

In the 1930s, de Haas et al. found out that while cooling down a long wire of gold, the resistance reaches a minimum at around 10 K and increases for further cooling [8]. Later, it was discovered that this effect was correlated to the presence of magnetic impurities, but a theoretical explanation of this phenomenon was only presented in the 1960s, by Jun Kondo [9]. In his model, an antiferromagnetic coupling between the conduction electrons and the residual magnetic impurities leads to the formation of a singlet state below a certain temperature T_K (Kondo temperature). This can be thought

of a cloud of conduction electrons, screening the magnetic impurity and therefore augmenting its effective cross section, which causes an increase in resistance.

The same effect can be found in quantum dots. If they are filled with an odd number of electrons, its total spin $S = 1/2$, which makes it an artificial magnetic impurity. If, furthermore, the coupling between the dot and the leads is large enough (tunnel resistances below $1 \text{ M}\Omega$), electrons from the leads try to screen the artificial impurity by continuously flipping its spin via a tunnel process (Fig. 2.4a, b). This allows for a hybridization between the leads and the quantum dot, resulting in the appearance of two peaks in the quantum dot's DOS: one at Fermi level of the source and one at the Fermi level of the drain (Fig. 2.4c). The conductance through the quantum dot can be explained by the convolution of the two peaks. Since at zero V_{ds} the source and drain Fermi level coincide, the conductance will have a maximum and drops to zero for higher bias voltages, resulting in a peak, or Kondo ridge. If the temperature becomes comparable to T_K , the antiferromagnetic coupling between the magnetic

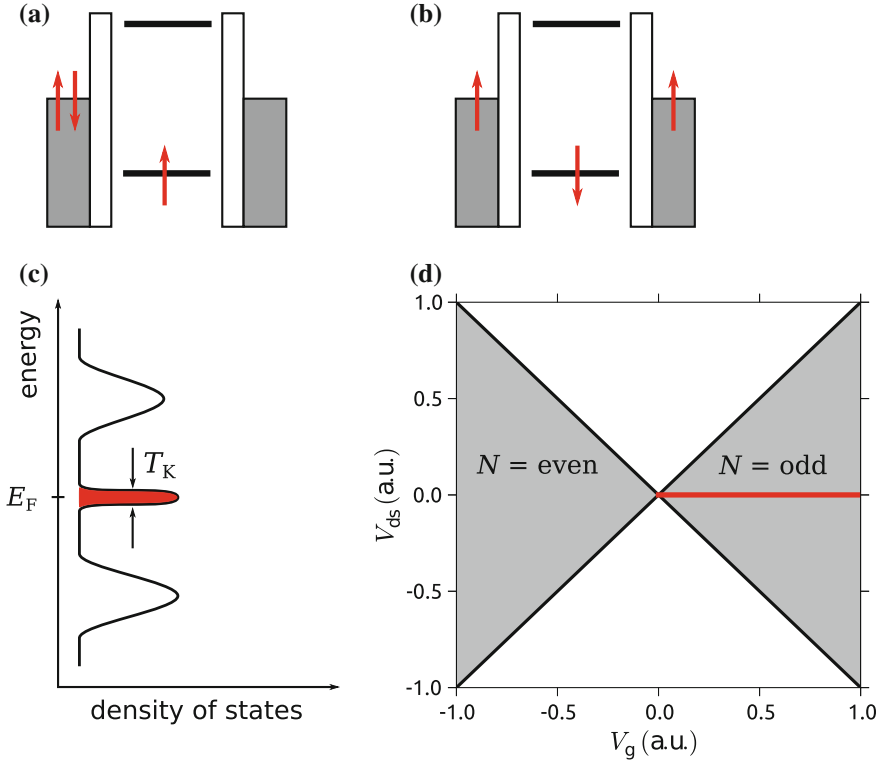


Fig. 2.4 Kondo transport mechanism with the initial state (a) and the final state (b). Note that the spin of the quantum dot flipped during the process. **c** The Kondo effect creates a peak in the density of states, whose width is given by the Kondo temperature. **d** Experimentally, the Kondo effect is observed as a conductance ridge or Kondo ridge (red line) inside the Coulomb blockade region of the stability diagram

impurity and the electrons in the leads is destroyed, resulting in the suppression of the conductance peak. The temperature dependence can be fitted by the empirical Goldhaber-Gordon equation [10]

$$G(T) = G_0 \left(\frac{T^2}{T_K^2} (2^{1/s} + 1) \right)^{-s} + G_c \quad (2.4.1)$$

and results in $G(T_K) = G_0/2$. The variable G_c accounts for a conductance offset caused by elastic cotunneling and $s = 0.22$.

Another possibility to study the Kondo effect is by applying a magnetic field. For a classical spin 1/2 in a magnetic field, the Zeeman effect will split the spin up and down levels by $g\mu_B B_c$. If this splitting becomes larger than the antiferromagnetic coupling given by $0.5k_B T_K$ [11], the Kondo ridge at zero bias is destroyed. However, applying a positive or negative bias voltage V_{ds} can compensate for the energy gap when $e|V_{ds}| = g\mu_B|B|$. This leads to the revival of the Kondo effect and is observed as two peaks, one at negative and one at positive bias. The separation of the Kondo peak as a function of the applied magnetic field is schematically displayed in Fig. 2.5a.

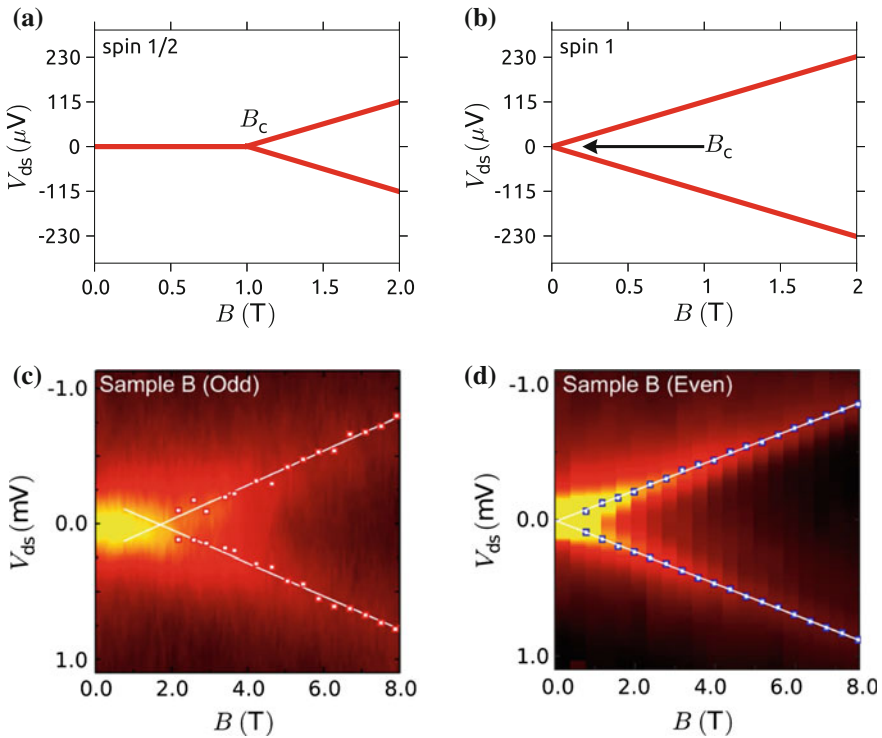


Fig. 2.5 Separation of the spin 1/2 (a) and spin 1 (b) Kondo peak as a function of the applied magnetic field. The critical field of 0.5 T was chosen arbitrarily. The slope corresponds to a g factor of 2. (c, d) Experimental data adapted from [12]

Using this model, the critical field can be used to estimate the Kondo temperature and the strength of the coupling.

If we add one electron to the quantum dot, the spin is either zero (singlet state) or one (triplet state). In case of zero spin, no Kondo effect will be observed. If, however, the triplet state becomes the ground state of the system, the situation changes. Similar to the spin 1/2 Kondo effect, electrons from the leads try to screen the artificial magnetic impurity, which now has a spin of 1. Therefore, the screening requires two conduction channels, one for each electron of the triplet.

In quantum dots, like the ones we used in our experiments, the coupling of different energy levels to the source and drain terminals is not symmetric in energy, resulting in two individual Kondo temperatures T_{K_1} and T_{K_2} . Hence, in the temperature window $T_{K_1} < T < T_{K_2}$ the screening of channel 1 is suppressed, whereas the screening of channel 2 is still working. This scenario is referred to as the underscreened Kondo effect. Its signature is such that the critical field needed to quench the conductance ridge is much smaller than $0.5k_B T_K$ [12]. This can be understood by a semi-classical consideration of the residual spin which was left unscreened. The ferromagnetic coupling between the two spins, which led to the formation of the triplet ground state, results in an effective magnetic field created by the unscreened spin at the site of the screen spin. This field weakens the antiferromagnetic coupling of the second spin to the electrons in the leads. Hence, already at very small external magnetic fields, the critical field is reached, leading to a shift of B_c towards zero (Fig. 2.5b). The magnitude of the shift is proportional to the ferromagnetic exchange coupling between the two spins but cannot be determined precisely due to the lack of knowledge of the fully screened B_c . However, it gives an estimate of its order of magnitude.

References

1. T. Fulton, G. Dolan, Observation of single-electron charging effects in small tunnel junctions. *Phys. Rev. Lett.* **59**, 109–112 (1987)
2. K. Matsumoto, M. Ishii, K. Segawa, Y. Oka, B.J. Vartanian, J.S. Harris, Room temperature operation of a single electron transistor made by the scanning tunneling microscope nanooxidation process for the TiOx/Ti system. *Appl. Phys. Lett.* **68**, 34 (1996)
3. J.M. Elzerman, R. Hanson, L.H. Willems Van Beveren, B. Witkamp, L.M.K. Vandersypen, L.P. Kouwenhoven, Single-shot read-out of an individual electron spin in a quantum dot. *Nature* **430**, 431–435 (2004)
4. A. Korotkov, D. Averin, K. Likharev, Single-electron charging of the quantum wells and dots. *Phys. B Condens. Matter* **165–166**, 927–928 (1990)
5. L.P. Kouwenhoven, C.M. Marcus, P.L. Mceuen, S. Tarucha, M. Robert, *Electron Transport in Quantum Dots* (Proceedings of the NATO Advanced Study Institute, Kluwer, 1997). ISBN 140207459X
6. R. Hanson, J.R. Petta, S. Tarucha, L.M.K. Vandersypen, Spins in few-electron quantum dots. *Rev. Mod. Phys.* **79**, 1217–1265 (2007)
7. M. Pustilnik, L. Glazman, Kondo effect in quantum dots. *J. Phys. Condens. Matter* **16**, R513–R537 (2004)

8. W. de Haas, J. de Boer, G. van den Berg, The electrical resistance of gold, copper and lead at low temperatures. *Physica* **1**, 1115–1124 (1934)
9. J. Kondo, Resistance minimum in dilute magnetic alloys. *Prog. Theor. Phys.* **32**, 37–49 (1964)
10. D. Goldhaber-Gordon, J. Göres, M. Kastner, H. Shtrikman, D. Mahalu, U. Meirav, From the Kondo regime to the mixed-valence regime in a single-electron transistor. *Phys. Rev. Lett.* **81**, 5225–5228 (1998)
11. T.A. Costi, Kondo effect in a magnetic field and the magnetoresistivity of Kondo alloys. *Phys. Rev. Lett.* **85**, 1504–1507 (2000)
12. N. Roch, S. Florens, T.A. Costi, W. Wernsdorfer, F. Balestro, Observation of the underscreened Kondo effect in a molecular transistor. *Phys. Rev. Lett.* **103**, 197202 (2009)

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