

Preface

Combinatorics is a research field driven by collaboration, with a large number of applications to different areas of pure and applied mathematics. The Institute for Mathematics and Its Applications (IMA) is an ideal setting for such collaborations and applications to develop.

The 2014–2015 Annual Thematic Program at the IMA was *Discrete Structures: Analysis and Applications*. The program was organized by Sergey Bobkov (University of Minnesota), Jerrold Griggs (University of South Carolina), Penny Haxell (University of Waterloo), Michel Ledoux (Paul Sabatier University of Toulouse), Benny Sudakov (University of California, Los Angeles), and Prasad Tetali (Georgia Institute of Technology).

Combinatorics was the focus during Fall 2014, and this volume presents some of the research topics discussed during this intense semester. We have particularly encouraged authors to write surveys of research problems, thus making state-of-the-art results more conveniently and widely available.

This volume is organized into parts, following the themes of the three workshops held during Fall 2014:

- *Probabilistic and Extremal Combinatorics*, held September 8–12, 2014, at IMA and organized by Penny Haxell (University of Waterloo), Eyal Lubetzky (Microsoft Research), Dhruv Mubayi (University of Illinois, Chicago), and Benny Sudakov (Eidgenössische TH Zürich-Zentrum).
- *Additive and Analytic Combinatorics*, held September 29–October 3, 2014, at IMA and organized by David Conlon (University of Oxford), Ernie Croot (Georgia Institute of Technology), Van Vu (Yale University), and Tamar Ziegler (Hebrew University).
- *Geometric and Enumerative Combinatorics*, held November 10–14, 2014, at IMA and organized by Zoltan Füredi (Hungarian Academy of Sciences MTA), Jerrold Griggs (University of South Carolina), Victor Reiner (University of Minnesota, Twin Cities), and Carla Savage (North Carolina State University).

Part 1: Extremal and probabilistic combinatorics. Extremal and probabilistic combinatorics are central to modern combinatorial theory, and both have developed

dramatically over the last few decades. Extremal combinatorics studies problems of finding the maximum or minimum possible cardinality of a set of finite objects satisfying certain requirements. Frequently such problems originate in other areas, such as computer science, information theory, analysis, number theory, and geometry. Probabilistic combinatorics, as the name suggests, blends combinatorics and probability. It is the foundation of the study of random graphs and other random discrete structures, and probabilistic arguments have been very powerfully applied to problems in other areas of combinatorics and in theoretical computer science. Major research topics in extremal and probabilistic combinatorics include extremal problems for graphs and set systems, Ramsey theory, random graphs, and application of probabilistic methods.

Part 2: Additive and analytic combinatorics. Additive combinatorics counts additive structures in sets; there have been exciting developments in recent years. Tools from Fourier and harmonic analysis have expanded the realm of additive combinatorics into the analytic while contributing to more effective applications. Many combinatorial ideas known to the combinatorics community can be used effectively to attack difficult problems in other areas of mathematics. For example, a famous theorem of Szemerédi on arithmetic progressions in dense sets is a key tool for the proof of the Green-Tao theorem on the existence of long arithmetic progressions in primes. The work of Breuillard, Green, and Tao, which established an analogue of the Freiman inverse theorem for noncommutative groups, is another example. This theorem was first stated and proved for integers by Freiman. Ruzsa's subsequent different proof was extended to abelian groups by Green and Ruzsa a few years ago. The extension to noncommutative groups is much more difficult. Research in additive and analytic combinatorics is also of interest to computer scientists; techniques and results have been applied to communication complexity, property testing, and the design of randomness extractors.

Part 3: Enumerative and geometric combinatorics. Geometric combinatorics studies discrete objects with geometric or topological structure, such as convex polytopes, arrangements of vectors, points, subspaces, triangulations, tilings, and partially ordered sets. Enumerative combinatorics, often called the mathematics of counting, has broad applications to probability, statistical physics, optimization, and computer science. Problems in geometric combinatorics give rise to counting problems that are sometimes difficult even to estimate and sometimes involve objects with interesting symmetry groups. Such problems often dovetail nicely with topics from enumerative combinatorics via calculations of partition functions, f -vectors, Ehrhart polynomials, and other quantities. Enumerative combinatorics also includes the study of permutation patterns, the complexity of tilings, and bijections between families of objects counted by the same numerical sequences or with related generating functions. In recent years, problems in both of these areas have stimulated the development of many new results and tools and enhanced connections with other areas of mathematics.

Discrete Structures: Analysis and Applications attracted intense interest from the mathematical sciences community, with each of the three workshops drawing more than 100 visitors and often filling Keller 3-180 to capacity. There are many

other aspects to an annual thematic year at the IMA besides workshops, with the relaxed but stimulating environment of the IMA fostering new collaborations and approaches to solving problems old and new. This program drew an eclectic mix of experts and junior researchers in various aspects of combinatorics together with numerous people who apply combinatorics to other fields. This volume reflects many of the aspects of the semester, with chapters drawn from workshop talks, annual program seminars, and research interests of the many visitors.

No single volume could possibly cover all the active and important areas of combinatorics research that were presented at the IMA, and we make no claim of comprehensiveness. But we think this volume presents a reasonable selection of interesting areas, written by leading experts who have surveyed the current state of knowledge and posed conjectures and open questions to stimulate further research. We thank the authors for their generous donations of time and expertise; needless to say, without them this volume would not have been possible.

We thank the IMA for wonderfully stimulating and productive long-term visits. We believe that the IMA is a critical national resource for mathematics. The *Discrete Structures: Analysis and Applications* program will have a lasting impact on research in combinatorics and related fields, and we hope this volume will enhance that impact. We are grateful for the opportunity to be part of it.

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Recent Trends in Combinatorics

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2016, XIII, 778 p. 77 illus., 14 illus. in color., Hardcover

ISBN: 978-3-319-24296-5