

Chapter 1

Introduction: Nonlocal or Unreal?

Abstract Via Bell's inequality, it is shown that a world described by quantum mechanics must be either nonlocal or unreal, and what that even means.

Quantum mechanics with its counterintuitive claims has created many controversies. In particular, many physicists could not accept the idea that a value measured in an experiment was not determined prior to the measurement but arises spontaneously at the moment of observation. This has led to speculations about “**hidden variables**” (also called “**hidden parameters**”) which render a measurement deterministic.

Only in 1964, 38 years after the formalism of quantum mechanics was formulated, John Bell could show that hidden variables, if they exist at all, must have a certain “ugly” property which makes them quite unattractive to a majority of physicists: they must be “nonlocal”, i.e. the change of a variable in one place has instantaneous effects on the rest of the world, without respecting the limit given by the speed of light for the transmission of signals.

In general we assume the **reality** and **locality** of physical phenomena. Here, **reality** means that the properties of an object are valid independent of whether or not we observe them in that moment. In particular, a property is not created by its measurement. The result that a measurement *would* give corresponds to a real property of the object, which exists independent of whether or not the measurement *actually* takes place. **Locality** means that the consequences of events can only propagate through space, and that maximally with the speed of light.

Bell has demonstrated that quantum mechanics cannot be simultaneously real and local, i.e. that at least one of the two assumptions has to be false. For the proof he made up an inequality, **Bell's inequality**, which has to be valid in a real and local world, and then showed that this inequality is violated for quantum phenomena. We want to present his argument for the example of photons passing through polarization filters.

An electromagnetic wave moves in z -direction and is polarized in x -direction. In the (xy) -plane a polarization filter is set up which lets only that part of the wave pass which is polarized in the direction of $\mathbf{r}$, where $\mathbf{r}$ encloses with the x -axis an angle ϕ . The transmitted amplitude $\mathbf{E}'$ is the projection of the original amplitude $\mathbf{E}$

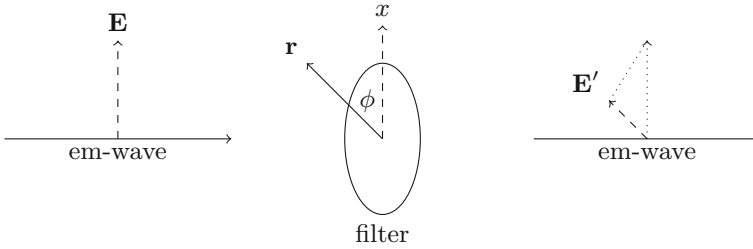


Fig. 1.1 Electromagnetic wave at a polarization filter in $\mathbf{r}$ -direction

in $\mathbf{r}$ -direction; its norm is therefore $E' = E \cos \phi$, and the intensity is $I' = I \cos^2 \phi$ (Fig. 1.1).

But taking a closer look, we find that the wave actually consists of photons, the quanta of the electromagnetic field. This circumstance alone has far-reaching consequences. For each single photon can pass the filter either completely or not at all (Fig. 1.2).

If N is the number of photons before and N' the number of photons after the filter, then $N' = N \cos^2 \phi$ is required for the equation regarding intensity to hold (the intensity is proportional to the number of photons). Therefore each photon has the probability $\cos^2 \phi$ to pass the filter. The question arises if this is decided in an “absolutely random” way for each photon, i.e. not just in a seemingly random way, due to our missing knowledge about the exact state of the photon. In other words: whether the photon’s destiny is only decided at the moment when it reaches the filter, or whether there are “hidden parameters” which determine the decision already prior to that, or which could explain it.

Since the arrival of the photon at the filter can be understood as a measurement with two possible outcomes, a proponent of the reality principle has to assume that such hidden parameters exist. For in his view, the ability of a photon to pass the filter is a property that is not just created at the moment of measurement, but must be given already before that.

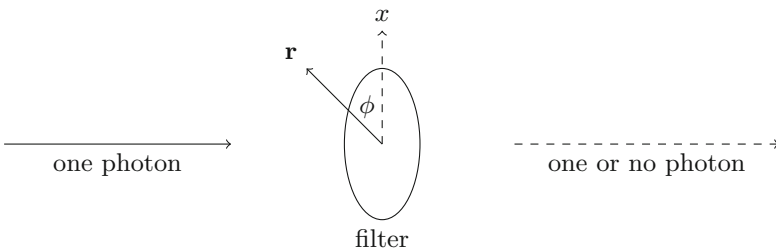


Fig. 1.2 Single photon at a polarization filter in $\mathbf{r}$ -direction

To decide the question, we have to investigate the combination of measurements of *several* properties, more precisely their **correlation**. Consider a set of objects all of which have three binary properties A , B and C , where binary means that for each object, each of A , B and C can either have a value of true or false. In the example of photons with hidden variables, A could be the ability of a photon to pass a polarization filter in the direction of $\mathbf{r}_A$, and B and C similarly for filters in the directions $\mathbf{r}_B$ and $\mathbf{r}_C$, respectively. It doesn't matter if there is an actual polarization filter filtering in any of these directions. A only means that *if* a filter is set up in direction $\mathbf{r}_A$, the photon will pass it. We denote the negation of A (photon will not pass the filter) by nA , and similarly for B and C .

In the set under consideration, some objects will have the property A , others will not. If an arbitrary object is picked up, there will be a certain probability $p(A)$ that A is true for it, and a certain probability $p(A, B)$ that both A and B are true for it etc. The variant of Bell's inequality we are going to use here reads:

Bell's Inequality

$$p(A, B) \leq p(A, C) + p(B, nC) \quad (1.1)$$

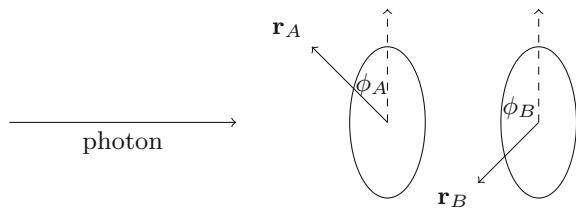
It follows from a simple set theoretical consideration: one has $p(A, B) = p(A, B, C) + p(A, B, nC)$, since for each object with the properties A and B , C is either true or not. Similar relations hold for the other two terms. In this way, the inequality can be rewritten to

$$\begin{aligned} & p(A, B, C) + p(A, B, nC) \\ & \leq p(A, B, C) + p(A, nB, C) + p(A, B, nC) + p(nA, B, nC). \end{aligned}$$

The two terms on the left hand side also appear on the right hand side. Since probabilities are always ≥ 0 , this inequality always holds, and (1.1) is thus proven.

The experimental check of Bell's inequalities is however nontrivial. The problem is that one cannot simultaneously measure the polarization of a photon in two different directions. If a filter in $\mathbf{r}_A$ -direction is set up, one cannot simultaneously filter in $\mathbf{r}_B$ - or $\mathbf{r}_C$ -direction. The subsequent application of two filters does not help either, since the measurement in the first filter influences the photon, so that the effect of the second filter is not the same as if the first filter would be missing (Fig. 1.3).

Fig. 1.3 A measurement of polarization in two directions $\mathbf{r}_A$ and $\mathbf{r}_B$ is only possible subsequently. But the first measurement influences the state of the photon



Assume a ray of photons is polarized in x -direction. If the first filter encloses an angle of 45° with the x -axis, exactly half of the photons pass. If behind that a second filter is set up in y -direction, again half the photons pass, altogether a quarter of the original ray. However, if already the first filter is set up in y -direction, no photons pass at all. This can be easily understood in the wave picture: at the first filter, the wave is projected into the direction of the filter. Before, it was polarized in x -direction, afterwards in the direction of the angle bisector between x - and y -axis. During the projection at the second filter, the amplitude has now a contribution in y -direction, and so a part of the wave passes the second filter. The intensity decreases at each filter by a factor $\cos^2 45^\circ = \frac{1}{2}$, so altogether it remains $\frac{1}{4}$ of the original intensity. If instead the wave meets the filter in y -direction already in the beginning, it is annihilated by the projection, since it has no contribution in y -direction.

Therefore the filter in y -direction, if placed behind the filter in x -direction, cannot be used to measure the *original* ability of a photon to pass a filter in y -Richtung.

However, Bell's inequalities can be checked using **entangled** photons. Entangled photons are created by certain atomic transitions, for example in calcium. Thereby two photons are emitted in opposite directions such that the following holds: If on opposite sides of the photon source polarization filters are set up, both of them filtering in the same direction $\mathbf{r}$, where each of the two photons runs into one of the two filters, then either both photons pass their respective filter, or none of them.

In our terminology for assumed hidden parameters this implies: the second photon has the property A if and only if the first one does. The measurement of such a property on one of the photons is thus equivalent to the same measurement on the other photon.

This observed behavior is remarkable and seems to be an argument *in favor* of hidden variables. This was first pointed out by Einstein in 1935 (together with his colleagues Podolsky and Rosen), by means of a variant of the phenomenon described here. Without hidden variables, photons would make a spontaneous decision at the moment of measurement whether or not to pass their filter. But how should one photon know what the other one has decided? It is as if the first photon shouted to the second one: "Hey, here was a filter in x -direction, and I passed it, so you have to do the same." Einstein spoke of a "**spooky action at a distance**", an impossibility (**Einstein-Podolsky-Rosen paradox**), and concluded there must be hidden parameters.

So, let's consider a source which emits entangled photons in $\pm z$ -direction. Assume for the moment that Einstein's conclusion is correct, i.e. that hidden parameters exist. As before, we denote by A , B and C the ability of a photon to pass a polarization filter in some chosen directions $\mathbf{r}_A$, $\mathbf{r}_B$ and $\mathbf{r}_C$, respectively. These properties are assumed to exist independently of whether there is *actually* a filter in any of these directions. Due to entanglement, we can consider A , B and C as properties not of an individual photon, but of a photon pair (since both photons always give the same result at the same kind of filter). Now one can measure two properties simultaneously and therefore determine correlations (Fig. 1.4).

For checking Bell's inequalities (1.1) it is not necessary to measure three properties simultaneously. One only has to measure the combinations (A, B) , (A, C) and (B, C) sufficiently many times to infer the probabilities $p(A, B)$, $p(A, C)$ and $p(B, C)$ from the relative frequencies.

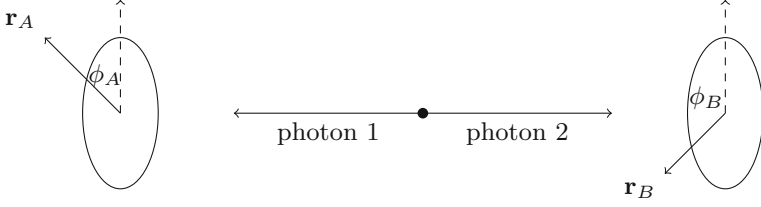


Fig. 1.4 Two entangled photons are emitted in opposite directions. On one photon, the polarization is measured in direction $\mathbf{r}_A$, on the other one in direction $\mathbf{r}_B$

Before that, one can also set up both filters in the same direction $\mathbf{r}_D$ and vary $\mathbf{r}_D$ to verify that the photons are really entangled: either both photons pass their respective filter or none of them. It turns out that independent of $\mathbf{r}_D$ one always has $p(D) = 1/2$, i.e. the ray of photons emitted by the source is completely unpolarized: for any filter direction, half of the pairs pass, the other half does not.

Now the probabilities occurring in Bell's inequalities can be determined. To measure $p(A, B)$, one photon is filtered in direction $\mathbf{r}_A$, the other one in direction $\mathbf{r}_B$. The other probabilities are determined similarly. The measurement yields

$$p(A, B) = \frac{1}{2} \cos^2 \phi_{AB}, \quad (1.2)$$

where ϕ_{AB} is the angle between $\mathbf{r}_A$ and $\mathbf{r}_B$. Equivalently one finds

$$p(A, C) = \frac{1}{2} \cos^2 \phi_{AC}, \quad (1.3)$$

$$p(B, nC) = p(B) - p(B, C) = \frac{1}{2}(1 - \cos^2 \phi_{BC}) = \frac{1}{2} \sin^2 \phi_{BC}. \quad (1.4)$$

Plugged into Bell's inequality (1.1) this gives

$$\cos^2 \phi_{AB} \leq \cos^2 \phi_{AC} + \sin^2 \phi_{BC}. \quad (1.5)$$

If $\mathbf{r}_B$ is chosen as the bisection between $\mathbf{r}_A$ and $\mathbf{r}_C$, e.g. with $\phi_{AB} = \phi_{BC} = 30^\circ$, $\phi_{AC} = 60^\circ$, one gets

$$\frac{3}{4} \leq \frac{1}{4} + \frac{1}{4}, \quad (1.6)$$

which is obviously not true. Bell's inequality is not fulfilled! Our assumption of hidden parameters has led us to a contradiction.

This contradiction can be avoided in two ways:

1. **Giving up reality:** Prior to the measurement, the photons have none of the properties of type A , B , C . Only at the moment of measurement (arrival at the filter) a photon decides spontaneously to pass the filter or not. In the case of an entangled photon pair, the pair decides as a whole, so that both individual measurements are consistent with each other.
2. **Giving up locality:** the measurement on one photon instantaneously influences the other photon, without respecting the limit given by the speed of light. The second photon is thereby “perturbed”, and the second filter no longer measures the original property (e.g. B) but a modified one, similar to the subsequent measurements on a single photon.

No matter which variant one chooses, quantum mechanics must be quite a “crazy” theory if it is able to describe such a reality (or a lack of it). Nowadays, a majority of physicists chooses the first variant (giving up reality), but there are also theories with nonlocal hidden variables. In addition, there is an interpretation of quantum mechanics which cannot be assigned to any of these two categories, because it sheds a new light on the entire matter: the Many Worlds Interpretation. Alone the fact that a scientific theory leaves so much room for interpretations is remarkable. We will come back to this issue in Chap. 4.

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