

Preface

Quantum mechanics (QM) is still quite a mysterious theory. In contrast to the theory of relativity, for example, it did not evolve out of some basic idea, some physical principle. Only reluctantly, physicists had to accept its claims which are completely counterintuitive. Many aspects regarding its interpretation are still a matter of debate nowadays.

I tried to write a textbook I would have liked to read as a student; a book that avoids many misunderstandings, unclear and vague ideas, and the questions that have settled in my mind for quite a while, at that time; or confirms that some of my questions belong to the issues of interpretation which are considered unclear by the entire physics community and are under heavy debate. The resulting book differs in some points from most of the other QM textbooks.

First, the mysterious properties of the theory are emphasized and discussed in detail. For that reason, it starts with Bell's inequalities as a foretaste of what we embark on. Other QM books often treat QM as something that became common-place and self-evident, just as a part of the physics canon. But this is not the case. The conceptual basis of QM has been under steady research and fiery debates throughout all the decades from 1900 until today (in contrast to special and general relativity whose foundations have been understood for 90 years), with new and surprising insights coming up again and again. By now, there is a whole bunch of interpretations of the theory which fundamentally differ in their world view. A separate chapter is dedicated to these interpretations.

Second, the book develops its matter from the general to the specific. At first, the general postulates of QM are presented and discussed in detail. The wave function as a special case of a quantum state follows later. This has the advantage that the main hurdle is taken right in the beginning, and a double run-up is avoided. For many books develop the matter twice: first by means of Schrödinger's wave mechanics and later again under the title of "abstract formulation." From my point of view, it is better to begin with the abstract part and thereby avoid the misunderstanding that QM is only a theory of wave functions.

Third, the general postulates and basic notions are all explained by means of the simplest nontrivial quantum system: the two-dimensional state space of the electron spin, also called qubit.

Also for wave functions we stick to our principle of the simplest example. Instead of following most books which directly jump from one to three dimensions, we take a stopover in two dimensions. For there it is easier to explain the separation of variables and in particular some features of a rotation symmetric potential (namely without spherical harmonics).

Fourth, there is a strong focus on the clarity of notions and understanding of the mathematical background. This clarity helps to avoid misunderstandings from the beginning.

For example, a large part of QM takes place in the infinite-dimensional space of wave functions. Many theorems known from linear algebra for finite-dimensional vector spaces are no longer valid there. These peculiarities are discussed in detail.

Tensor products also get enough space, since they play an enormously important role in QM, e.g., for the notion of an entangled state, and for the combination of wave function and spin state.

Fifth, the exercises are not gathered at the end of each chapter, but are embedded in the main text. This is supposed to encourage the reader to think about and to recalculate things during his read. The solutions to the exercises are gathered at the end of the book.

The focus of this book is on the general postulates of QM, its interpretation, its basic notions, and its mathematical formulation. The first and most extensive part of the book is dedicated to this topic.

In the second part, an important special case is treated: the QM of wave functions in one, two, and three spatial dimensions, under the assumption that the Hamiltonian operator consists only of a kinetic term and a time-independent potential. Here, the most important examples are the harmonic oscillator and the hydrogen atom. Scattering theory is also discussed within this context.

The third part encompasses further topics which belong to the canonical material of a lecture on QM: combination of spin and angular momentum, QM with electromagnetism, perturbation theory, and systems with many particles. Here we only develop the basic ideas and methods, as well as some simple examples. For applications like the fine and hyperfine structure of hydrogen, or the theory of atomic transitions, we refer the reader to the literature. Finally, we provide a short explanation of the notion of a path integral and discuss the relativistic theory of the electron (Dirac equation).

The target audience of this book is, of course, in the first place students of physics who study QM in the context of a course on theoretical physics. But due to the axiomatic, deductive approach and the detailed discussion of the mathematical background, it is also very well suited for mathematicians who want to understand QM. In some places we explicitly try to overcome the “cultural barrier” between mathematicians and physicists.

Some topics which are exciting but not mandatory for the further reading (and for exams) are placed in a dedicated text environment—the so-called “nerd’s corner”—for curious readers.

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