Chapter 2
The Curious History of Quantum Mechanics

We start with an account of the history of physics in the early 20th century as it applies to the development of quantum mechanics and its interpretation. This account is intended to show how the theory and interpretation were developed and came to be in the state in which we find them today.

Writing about the development of science can be frustrating to the writer, because there are so many people involved, so many false starts, so many mistaken ideas, so much bad data, and a tangled story-line of rejected ideas and falsified theories left behind. I will attempt to streamline the lines of development by focusing on the ideas that turned out to be better and more important. I will, for the most part, ignore the bad ideas, the misconceptions, and the minor contributions. This perhaps distorts history, but I am not a historian, and I feel no obligation to achieve historical precision. Rather, I want to communicate the feel of the intellectually turbulent times when quantum mechanics was emerging. Many individual contributions will be ignored or neglected, but a picture of the development of quantum mechanics, with all its triumphs, paradoxes, and problems, should emerge.

2.1 Atomic Theory in the Early 20th Century (1900–1924)

William Thompson, Lord Kelvin (Fig. 2.1), was one of the founders of thermodynamics and perhaps the most prominent British scientist of his day. In 1900, he is reputed to have told the British Association for the Advancement of Science that there was nothing new to be discovered in physics and that all that remained to do was to make more and more precise measurements. One of his contemporaries, confident of the universality of Newtonian mechanics, similarly asserted that if one knew the positions and velocities of all the particles in the universe at one particular moment, the entire past and future of the universe could be calculated. This confidence in the finality of the physics of the time prevailed, despite the presence of certain unexplained loose ends. For example, in 1900 there were no good explanations of the line structure of atomic spectra, the energy source that powered stars, or

© Springer International Publishing Switzerland 2016

J.G. Cramer, *The Quantum Handshake*, DOI 10.1007/978-3-319-24642-0_2

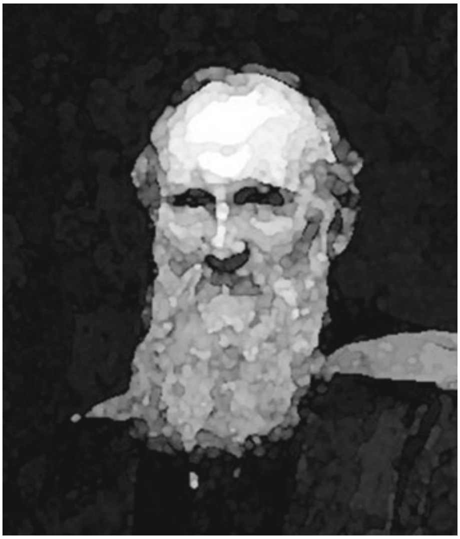

Fig. 2.1 William Thompson, Lord Kelvin (1824–1907), was one of the founders of thermodynamics

the nature of radioactivity. These unexplained "details" were clouds on the horizon that foreshadowed a great intellectual storm, a scientific revolution in the making.

The physics of the 19th century had firmly established that light was an electromagnetic wave. Thomas Young's two-slit experiment (see Sect. 6.1) had demonstrated in the early 1800s that light waves taking two paths to reach a screen could be made to interfere, canceling at some locations on a screen and reinforcing in others. In the 1860s James Clerk Maxwell, starting with a set of equations governing the behavior of stationary or slowly changing electric and magnetic fields, had derived a wave equation that described self-sustaining coupled electric and magnetic waves moving through space at the speed of light.

At the beginning of the new 20th century, this widely accepted wave picture of light was jarred by the work of Max Planck (Fig. 2.2) in 1901. Planck showed that heated objects could only emit light in "energy chunks" of an energy size given by the frequency f of the light multiplied by a new physical constant, which we now call Planck's constant and denote by the symbol h (see Appendix A.1).

Planck interpreted his results as demonstrating the peculiarities of the emitting system and insisted that they were *not* describing an intrinsic property of light itself. In 1905, however, this view was challenged by Albert Einstein (Fig. 2.3), who showed that the photoelectric effect, the emission of electrons from metals illuminated by light, can be consistently explained by assuming that light itself has particle-like properties, with each particle (or photon) of light carrying an energy E_γ equal to its frequency f multiplied by Planck's constant h (i.e., $E_\gamma = hf = \hbar\omega$, where $\hbar = h/2\pi$ and angular frequency $\omega = 2\pi f$).

Fig. 2.2 In 1918, Max Planck (1858–1947) received the Nobel Prize in Physics for his work on black-body radiation

Fig. 2.3 In 1921, Albert Einstein (1879–1955) received the Nobel Prize in Physics for his work on the photoelectric effect

The structure and behavior of the atom proved to be particularly vexing problems for the physicists of the early 20th century. J. J. Thompson (Fig. 2.4) discovered the negatively charged electron, a fundamental particle that somehow was a part of atoms. Ernest Rutherford (Fig. 2.5) demonstrated that the mass and the positive electric charge of atoms were both concentrated in a small central region (the atomic nucleus) much smaller than the size of the atom. These discoveries led to new insights.

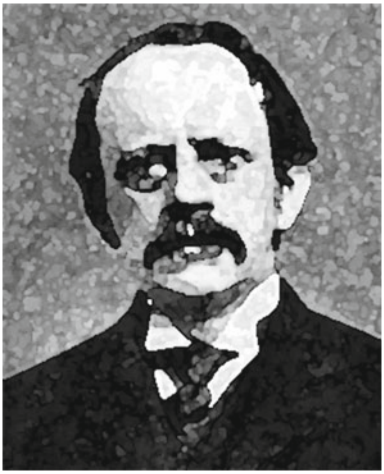

Fig. 2.4 In 1906, J. J. Thompson (1856–1940) received the Nobel Prize in Physics for the discovery of the electron

Fig. 2.5 In 1908, Ernest Rutherford (1871–1937) received the Nobel Prize in Chemistry for the discovery of the atomic nucleus

Rutherford had suggested that each atom might be a tiny solar system, with the negatively-charged electron "planets" orbiting a central positive nuclear "sun". However, electrons in such paths would be continually changing direction with large accelerations, and the accepted electromagnetic theory of Maxwell required that such accelerated charges must produce light waves and must radiate away their energy and angular momentum in microseconds. But instead of continuous light radiation from unstable atoms, experimentalists observed that atoms in electrical discharges produced light only at specific narrow frequencies or "spectral lines" and that atoms were otherwise stable.

In 1913 Niels Bohr (Fig. 2.6) solved a part of the problem by placing constraints on Rutherford's solar-system model of the atom. Bohr's model allowed electrons to

Fig. 2.6 In 1922, Niels Bohr (1885–1962) received the Nobel Prize in Physics for his atom model

orbit only in paths that had integer multiples of an angular momentum "quantum" given by Planck's constant h divided by 2π (which physicists now denote by the symbol h-bar or $\hbar$). To change from one such atomic orbit to another, an electron had to make a "quantum jump", disappearing from one orbit and appearing in the other while changing energy and angular momentum and emitting a light photon that made up the energy difference. Bohr's model accounted for the stability of atoms (the electron orbits were stable states that did not radiate) and for spectral lines (as the photons produced in the well-defined quantum jumps), and it worked very well in explaining the structure and light radiated from the hydrogen atom, which consisted of a single electron orbiting a proton nucleus. However, when an atom had two or more electrons present, Bohr's model failed miserably. Bohr's model worked only for hydrogen and for hydrogen-like ionized atoms with only one orbiting electron. It was telling a part of the story, but important pieces of the puzzle were still missing.

In his 1924 PhD thesis, the French nobleman Prince Louis de Broglie (Fig. 2.7) supplied another missing piece of the puzzle. He reasoned that since light had particle-like behavior, as shown by Einstein's analysis of the photoelectric effect, it was plausible that matter particles like electrons might show an analogous wave-like behavior. The wavelength λ of a photon can be calculated by dividing Planck's constant h by its momentum p ($\lambda = h/p$). If electrons showed wave-like behavior, de Broglie reasoned, they might have wavelengths given by the same relation. He applied his wavelength relation to Bohr's model of the hydrogen atom, and he found that a precisely integer number of electron wavelengths, as calculated from his formula, fitted into the circumference of each stable orbit of Bohr's model as "standing waves". In other words, Bohr's assumption that each orbit had an integer number of $\hbar$ units of angular momentum was completely equivalent to assuming that

Fig. 2.7 In 1929, Louis de Broglie (1892–1987) received the Nobel Prize in Physics for his work on particle wavelength

an integer number of de Broglie wavelengths of each electron fitted into its orbit. Each atomic electron is a stable "particle in a box" standing wave, with the box consisting of the electron's path bent into a closed circle or ellipse by the electric field of the nucleus. Later experimental work by Davisson and Germer in 1927 verified the concept by demonstrating that electrons could be made to show wave interference effects characteristic of their de Broglie wavelength when scattered from a crystal of nickel.

In the early 1920s, the stage was thus set for the development of a comprehensive theory of atomic structure and behavior, i.e., quantum mechanics. Some of the pieces of the puzzle had been provided by Einstein, Bohr, and de Broglie, while many others remained hidden. It would require at least two more major breakthroughs before the full theory of quantum mechanics, with all its power and peculiarities, could be realized.

2.2 Heisenberg and Matrix Mechanics (1925)

In late 1924, young Werner Heisenberg (Fig. 2.8) found that nothing seemed to make sense at Niels Bohr's Institute. The grateful Danish government, with the financial support of the Carlsberg Brewery, had provided their new Nobel Laureate with an endowed Institute for Theoretical Physics housed in a three-story building just outside Copenhagen. Here Bohr had gathered some of the world's brightest young theoretical physicists, including Hans Kramers, Wolfgang Pauli, and Werner Heisenberg, in an attempt to make sense of the rich data that were coming from the spectral lines of light emitted by excited atoms. They sought a way of generalizing the Bohr model so that it worked for all atoms instead of just for hydrogen.

Fig. 2.8 In 1932, Werner Heisenberg (1901–1976) received the Nobel Prize in Physics for his work on matrix mechanics

The data from studies of atoms in electrical discharges showed a bewildering array of spectral lines that changed dramatically from one element to the next. There were mysterious double values or "doublets" in some lines, and there were strange shifts and splittings that depended on electric and magnetic fields. Clearly, Nature was trying to send an important message about the way the universe worked, but Bohr and his best and brightest had so far been unable to decode it. Starting from Bohr's application of angular momentum quantization to the hydrogen atom and his more recent work with John C. Slater and Hans Kramers attempting to ignore photons and use "virtual oscillators", they had tried model after model that would have built on Bohr's initial success. However, all attempts to picture what might be going on inside the atom, and then to develop mathematics appropriate to that picture, had utterly failed.

By May of 1925, Werner Heisenberg had moved from Copenhagen back to Max Born's Institute in Göttingen and was feeling burned out. He had been modeling an atom as a not-quite-perfect mass-and-charge system, a little anharmonic oscillator that had exotic Bessel functions instead of sine waves as its vibration modes. He had generated reams and reams of math, to no avail. The resulting atom orbits made frequencies that looked nothing like the light from real atoms. Ugly experimental reality had killed another lovely theoretical idea. The pictures at the heart of the calculations were somehow wrong.

In desperation, Heisenberg tried another approach, working directly with "laundry lists" of values describing the frequencies and strengths of atomic transitions and focusing on hydrogen, the simplest of the atoms. Somehow, these concrete variables, based on direct measurements, seemed to have more meaning as objects of theoretical significance than did the more ephemeral "unseen" variables that were implicit in the pictures and models behind the calculations. Heisenberg was reaching the conclusion that one should perhaps dismiss models altogether and focus exclusively on relationships between observable quantities.

But just when this new approach seemed to be on the verge of making progress, disaster struck. Heisenberg had always had problems with allergies, and every year the spring hay fever season had been a time of particular difficulty. Further, this year was much worse than usual. There had been a warm winter, and the pollen-producing vegetation of Northern Europe had outdone itself in the late spring. Heisenberg, attempting to work with Max Born in Göttingen, had been laid low with a bout of hay fever that was worse than anything in his previous 23 years. He could not sleep, and he walked around with swollen half-closed eyes, feeling as if his head was trapped inside a specimen bottle. He was unable to concentrate on anything.

A friend recommended Helgoland, a barren grassless island off the northern coast of Germany. The air there was the purest in Europe, there were few pollen-producing plants, and the growing season occurred several months later due to the colder climate of the North Sea. Heisenberg was now ensconced for ten days on the 2nd floor of a cozy guest cottage, with a nice view of the southern coast of the island. He was alone with the barren boulders, the pure air, newly cleared sinuses, a book by Goethe, and his lists and tables of observables.

And then a miracle happened. In later life, Werner Heisenberg was never able to adequately describe the mental processes that led to his breakthrough. Somehow, he had discovered a "new math", a systematic set of procedures that allowed him to manipulate the lists of numbers that were the focus of his inquiry into new lists that predicted other experimental observations. The new lists agreed with known data from the atomic spectroscopy of hydrogen. But Heisenberg's procedures were peculiar. Among other oddities, they violated the usual commutation rule of mathematics. Multiplying P by Q gave a different result from multiplying Q by P.

Heisenberg wrote what he considered to be a "crazy" paper describing his new arcane procedures for producing new experimental results from other experimental results, and he pondered what to do with it. On his return to Göttingen, he gave a copy of the paper to his older friend and sometime employer, Max Born. Born and his mathematically skilled assistant, Pascal Jordan, immediately recognized the procedures Heisenberg had described as the manipulation of matrices. Heisenberg was completely unfamiliar with the mathematics of matrices, but nevertheless he had somehow invented it to fill the needs of his calculations. By November 1925, Heisenberg, Born, and Jordan had produced the matrix formulation of quantum mechanics, a powerful formal approach to making quantum mechanical predictions [1–3] that is still widely used in atomic and nuclear physics, particularly in shell-model calculations.

This new matrix approach was abstract (and nearly incomprehensible to the technically uninitiated). There was no underlying model to illustrate or justify the procedures used. Complex algebra was invoked, so that elements of matrices had both real and imaginary parts. Every variable and function of conventional atomic physics had to be reinterpreted as a matrix, a one- or two-dimensional list of values that was treated as a single object of manipulation. Continuous variables became infinite matrices. These elements were combined using matrix operations (addition, multiplication, inversion, diagonalization, extraction of eigenvalues, etc.) to yield "matrix elements" that were squared to make them real instead of complex and to obtain predictions for experimental observations.

The "why" of this matrix formalism was not apparent. The recommended approach, grounded in the logical positivism that was philosophically fashionable in the 1920s, was to "shut up and calculate". Heisenberg had much accumulated frustration from the focus on pictures and models at Bohr's Institute. He now found it rather liberating to reject such ephemeral pictures and models and to focus exclusively on experimental observations. And *matrix mechanics*, one flavor of the new standard theory of quantum mechanics, had emerged.

2.3 Schrödinger and Wave Mechanics (1926)

Erwin Schrödinger (Fig. 2.9) was a physicist on the move in post-WWI Europe. In early 1920, in rapid succession, he married Annemarie (Anny) Bertel and became an Assistant to Max Wien in Jena. Then in September, 1920, he became an "AO" (or associate) Professor in Stuttgart. Shortly afterwards, in early 1921, he became an "O" (or full) Professor in Breslau. Later in 1921 he moved to the University of Zurich and became an "O" Professor there. He stayed in Zurich for the next six years, and it was there that he made his breakthrough discovery.

In September, 1925 Schrödinger had obtained a copy of the 1924 French PhD thesis of Louis de Broglie, which outlined de Broglie's matter-wave hypothesis and showed that treating electrons as orbital standing waves led to the angular momentum quantization constraint that was at the heart of Bohr's model. Peter Debye, a Professor at Zurich's ETH, persuaded Schrödinger to give a joint colloquium for their two institutions describing de Broglie's work. At the end of this colloquium, which reportedly was a very clear and thoughtful presentation, Debye casually remarked that he considered de Broglie's way of discussing waves to be rather naïve. He had learned as a student of Arnold Sommerfeld in Munich, he said, that to properly deal with waves, one must have a wave equation. But no wave equation was apparent in de Broglie's work (Fig. 2.7).

Schrödinger puzzled over Debye's remark while on a ski vacation in Arosa, a village in the Swiss Alps, where he was on holidays with a young lady of his acquaintance (while his wife Anny stayed in Zurich). He started calculations at their hotel. On about the third day of the trip, Schrödinger derived the time-independent wave equation for matter waves, subsequently called the Schrödinger Equation, which became the foundation for his quantum wave mechanics. Returning to Zurich, he gave another colloquium in which he presented his new formalism of quantum mechanics to his colleagues.

He published a series of four papers laying out the new quantum formalism in detail in 1926. In the first of these [4], he presented a derivation of the time-independent Schrödinger equation, which is applicable to stationary states in which there is no change in energy. In the second paper [5], he re-derived the

Fig. 2.9 In 1933, Erwin Schrödinger (1887–1961) received the Nobel Prize in Physics for his work on wave mechanics

time-independent Schrödinger equation and solved the problems of the rigid rotor (spinning top) and the harmonic oscillator (mass + spring vibrations). The third paper [6] addressed the equivalence of his wave mechanics and Heisenberg's matrix mechanics, and he solved the problem of an atom in an external electric field (the Stark effect). The fourth paper [7] derived the time-dependent Schrödinger equation (permitting the investigation of systems with changing energy) and addressed the problems of atomic transitions and particle scattering.

What is it that Schrödinger did to obtain his equation? A wave equation is a differential equation (an equation stating a relation between some wave function and its space and time derivatives) that has as its solutions waves that travel through space (see Appendix B.4). For light waves, the wave equation of Maxwell required that the second time-derivative of the wave function was equal to the speed of light squared times the second space-derivative of the wave function. The solutions of Maxwell's wave equation are sinusoidal electromagnetic waves that travel through space at the speed of light (c). Schrödinger's problem was to produce a similar wave equation, but one that would have as its solutions matter waves that had a characteristic mass m and traveled through space at a slower speed v appropriate to the momentum $p = mv$ and kinetic energy $E = \frac{1}{2}mv^2$ of massive particles.

He accomplished this by comparing the relationships between kinetic energy and momentum that were appropriate for the particle-waves of light and of matter. For particle-waves of light, the photon's energy equals the speed of light times the photon's momentum ($E = cp$, or more relevant to Maxwell's wave equation, $E^2 = c^2p^2$). For particle-waves of matter, kinetic energy equals the momentum squared divided by twice the mass ($E = p^2/2m$). Starting with the space and time derivatives that would extract the energy and momentum from the wave function, Schrödinger constructed a differential equation that was the equivalent of the latter relation, and this became the Schrödinger equation (see Appendix B for further details).

We can see from this account that, unlike Heisenberg, Schrödinger started from a definite picture of the underlying physics when he formulated quantum wave mechanics. His picture was that matter waves were like Maxwell's electromagnetic waves. He rejected Bohr's idea of instantaneous quantum jumps and pictured matter particles as waves moving through space from one place to another, carrying energy and momentum with them, just as do photons of light. One could visualize such matter waves moving on little trajectories through space-time, connecting one interaction with the next. Schrödinger attempted to form his waves into "wave packets" that represented particles, but he found that his packets tended to come apart as they moved.

The problem with this naïve picture is that, when carefully compared with what is known about quantum behavior, it is not consistent. In September, 1926, Bohr arranged for Schrödinger to visit Copenhagen. There Bohr, aided by Heisenberg who came to Copenhagen for the occasion, held lengthy discussions with Schrödinger, attempting to convince him that while his wave-mechanics formalism was valid, the naïve picture that he saw behind it was not [8]. Bohr argued that many aspects of quantum wave behavior, particularly the phenomena of wave function reduction, probability, and uncertainty, were not consistent with Schrödinger's simple ideas of matter waves-in-space as matter analogs of classical electromagnetic waves.

Bohr persisted in these arguments so vigorously that Schrödinger actually became physically ill, and Bohr's wife Margrethe had to nurse him back to health. However, Schrödinger was not convinced that there was any interpretational problems in his views until after a lengthy correspondence with Bohr and Heisenberg that followed the visit. He finally acquiesced, but he retained a dislike for the Copenhagen view of quantum mechanics for the rest of his life. And his new wave mechanics had to go forward without any underlying picture of what lay behind it.

The upshot of these events was that quantum mechanics, while possessing two alternative formalisms, was left without any picture of what lay behind the mathematics or of the inner mechanisms that produced quantum behavior. A pictorial interpretation of the formalism was missing, and many believed that none was possible.

The British electrical engineer and mathematical physicist Paul Dirac, on reading the papers of Schrödinger and Heisenberg describing the two new quantum mechanics formalisms, quickly recognized the importance of the rival theories and investigated their relationship. By 1927, he was able to derive a more general quantum formalism and to show that both wave mechanics and matrix mechanics could be derived from it. The problem of the two competing formalisms had been resolved, but the question of their interpretation remained.

2.4 Heisenberg and Uncertainty (1927)

While Heisenberg was the originator of the matrix mechanics formalism of quantum mechanics, he also carefully studied the rival wave mechanics formalism originated by Schrödinger and generalized by Dirac and Jordan. In it, he found an interesting connection that its originators had not appreciated. The wave functions in the formalism typically depend on pairs of independent variables, position and momentum or energy and time, that are multiplied together as they appear in the formalism. Niels Bohr called these "complementary variables". This pairing is a precondition for Fourier analysis, in which a function of one of these variables can be converted into a function of the other variable by summation or integration.

As a modern example of the Fourier relation between complementary variables, consider an electronics laboratory experiment in which we use a pulse generator to produce a Gaussian-shaped electrical pulse of a certain time-width Δt. We view this pulse on an oscilloscope that has fast Fourier-transform (FFT) capabilities and can display both the time and the frequency distributions of a given signal. In the time domain, (see upper Fig. 2.10) the pulse rises smoothly to its peak and then drops to near zero, showing a definite peak amplitude and time width Δt. If we use the FFT capabilities of the oscilloscope to view the distribution of frequency components of the pulse, we find that in the frequency domain (see lower Fig. 2.10) the pulse also has a Gaussian shape, rising to a peak angular frequency ω_0 and then dropping to near zero, showing a maximum amplitude at ω_0 and showing an angular frequency width $\Delta\omega$ around this center. Now if we change the pulse generator to increase the time width of the pulse, we will find that the frequency width decreases. Conversely, if we decrease the time width, the frequency width increases.

Because the complementary time and frequency variables are connected through a Fourier transform, the time and frequency widths of the pulse have a "see-saw" relation, with one increasing as the other decreases and *vice versa*. Further, because of the Fourier algebra, the product $\Delta t \Delta\omega$ of the time and frequency widths of the pulse is an invariant constant, representing a sort of "uncertainty principle" for pulses. We can have a pulse that is as sharply defined in time as we wish, but only at the expense of having a very broad distribution of frequency components. Conversely, we can have a pulse that has a very narrow band of frequency components, but only at the expense of having a very broad time width. We emphasize that a pulse with a very narrow time width is composed of a broad spectrum of frequency components and *does not have a precise frequency.*

Heisenberg discovered that when quantum waves are "localized" (see Appendix B.2) just this relationship exists between the position and momentum and between the energy and time for a particle described by the formalism of wave mechanics. He used this relationship to derive the uncertainty principle, which he published in a 1927 paper [9]. In particular, the position width (or uncertainty) Δq of a localized particle described by wave mechanics is related to its momentum width (or uncertainty) Δp by the uncertainty relation $\Delta p \cdot \Delta q \geq \hbar$. Similarly, its time width (or uncertainty)

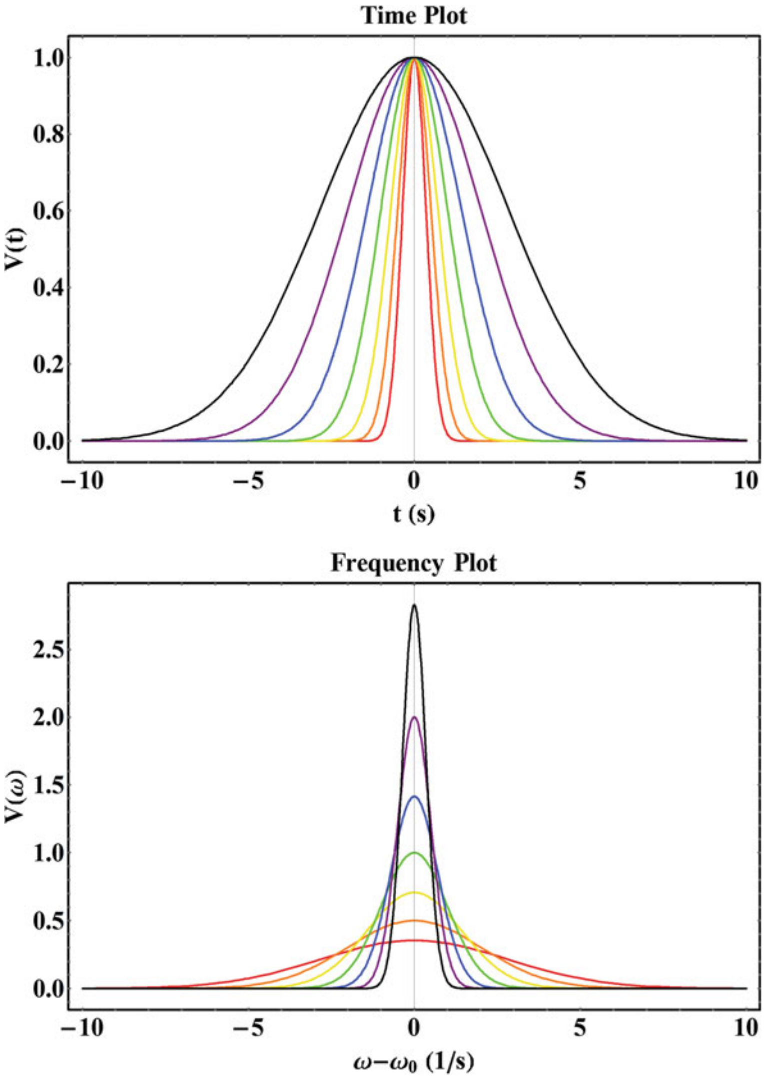

Fig. 2.10 Gaussian voltage pulses $V(t)$ produced periodically at angular frequency ω_0 with varying time widths (*narrow to broad*) and their Fourier frequency transforms $V(\omega)$ *(broad to narrow)*

Δt is related to its energy width (or uncertainty) ΔE by the uncertainty relation $\Delta E \cdot \Delta t \geq \hbar$. This is Heisenberg's uncertainty principle.[1]

[1]More precisely, if σ_x is the standard deviation of the position x probability distribution function (PDF) and σ_p is the standard deviation of the momentum p_x PDF, then $\sigma_x \sigma_p \geq \hbar/2$. Similarly, if σ_E is the standard deviation of the energy PDF and σ_t is the standard deviation of the time PDF, then $\sigma_E \sigma_t \geq \hbar/2$.

Heisenberg asserted that these uncertainty relations, derived for relatively simple wave function examples, were fundamental properties of all physical systems and that they represented fundamental limits on our possible knowledge of physical quantities. We cannot simultaneously know precisely the position of a particle and its momentum (or speed or wavelength). The more precisely we can determine the value of one variable, the less precisely we can know the value of its complement. A particle with a precise position *does not have* a precise momentum value, and *vice versa*. This situation is radically different from that of classical physics, where the position, momentum, time, and energy of objects are independent variables that can be separately determined to arbitrary precision.

2.5 Heisenberg's Microscope (1927)

If Heisenberg had stopped there, it would have been much better for future generations of physicists. However, in his paper on the uncertainty principle [9], he chose to illustrate its operation with a *gedankenexperiment* (i.e., thought experiment), often called "Heisenberg's microscope". He envisioned that there was a particle located somewhere in empty space, and we wished to measure its position. We could do this by shooting at it a set of photons that passed through various position possibilities. When we observed that one of the photons had been Compton-scattered by the particle of interest, we would know that the particle was at the location that corresponded to that photon. The position precision of such a measurement would be restricted to wavelength of the photon. If we used high frequency radio waves, we could locate the particle with a precision of a few centimeters. If we used light, we could locate the particle with a precision of a few hundred nanometers. If we used gamma rays, we could locate the particle with a precision of a few femtometers. But each of these scattering measurements has another effect on the particle. It changes the particle's momentum by recoil, giving it a new momentum that is uncertain because we do not know all the details of its initial momentum or of the Compton scattering collision. And the shorter the wavelength of the probing photon, that larger the momentum disturbance of the particle that is struck, so the more imprecisely we can know its momentum. Position precision in the experiment is achieved only at the expense of momentum imprecision (Fig. 2.11).

Heisenberg used the increasing precision in the determination of the particle's position and the decreasing precision in our knowledge of its momentum as an illustration of the operation of the uncertainty principle. This is often called the "disturbance model", and, as first pointed out by Bohr, it is *wrong*. Nevertheless, the disturbance model is still widely used in physics textbooks by unsuspecting authors who wish to illustrate the operation of the uncertainty principle, and many generations of physics students have been led to false conclusions through its use.

The problem with the disturbance model is that it has its head wedged in classical physics. It assumes that the particle of interest simultaneously possesses a well-defined position and a well-defined momentum, and that, due to the limitations of

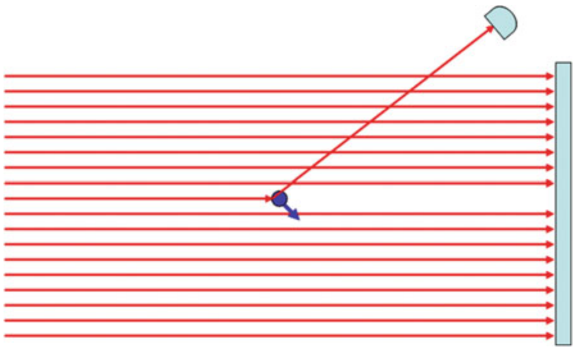

Fig. 2.11 Heisenberg's Microscope *gedankenexperiment*: a scattered photon provides the location of a particle while disturbing potential knowledge of its momentum. Photon trajectories are one wavelength λ apart; particle recoil is proportional to $1/\lambda$

our crude measurement capabilities, we are too clumsy to measure both of them properly at the same time.

That is not the message from the formalism of wave mechanics that Heisenberg had discovered. The complementary Fourier relations of wave mechanics tell us that a particle described by that formalism cannot possess a well-defined position and a well-defined momentum at the same time, just as our Gaussian pulse in the electronics lab illustration cannot simultaneously have a narrow time width and a narrow frequency width. They are complementary quantities, and precision in one domain makes precision in the other domain impossible. This is a fundamental property of the particle and has nothing to do with the choice or quality of measurements that we perform on it. Through measurement, we may choose to restrict the range of values that a variable may have to some arbitrary precision. The post-measurement particle, as described by the mathematical formalism, must have the precision of the complementary variable automatically expanded by the formalism to compensate, as described by Fourier algebra and the uncertainty principle. Mathematics, not measurement difficulties, is behind the uncertainty principle.

In Copenhagen, Heisenberg's paper, which had already been submitted for publication by the time Bohr read it, triggered strong and ongoing arguments at the Institute between the two theorists. Bohr argued that the aperture restricting the lateral range of the photon was of key importance, led in a simple way to the uncertainty relation, and had been ignored in favor of questionable scattering arguments. Reportedly at one point Heisenberg was reduced to tears by the strength of Bohr's arguments. As a result of the lengthy discussions at the Institute, when the page proofs for the uncertainty principle paper were received from the journal publisher, Heisenberg appended a "note added in proof", a long paragraph that rather vaguely outlined Bohr's objections to the disturbance model and admitted its inadequacies. That note, unfortunately, did not discourage later authors from lifting the disturbance model and Heisenberg's microscope *gedankenexperiment* from the publication and using them widely in the physics literature, particularly in textbooks.

2.6 The Copenhagen Interpretation (1927)

The quantum theoretical work of the 1924–27 period described in the previous sections had delivered a new theory of quantum mechanics that was unlike any previous physical theory. It had a well developed formalism (two, in fact), but there seemed to be no picture behind it that allowed practitioners to visualize the operation of the system they were describing with mathematics. More disturbing, the new quantum formalism had brought with it a number of unanswered questions and problems of interpretation that are still troubling physicists and philosophers, some eight and a half decades later.

Here we want to introduce these interpretational problems, not all of which were fully appreciated when Bohr, Heisenberg, and Born first developed and promoted their interpretation in late 1927. In no particular order, here is at least a partial list of such interpretational problems.

- **The problem of identity** What is the meaning of the wave function (or state vector) of wave mechanics and the matrix elements of matrix mechanics and where does it exist?
- **The problem of complexity** Why, unlike any other physical theory, are the wave function and matrix elements of quantum mechanics allowed to be complex, with both real and imaginary parts?
- **The problem of wave-particle duality** How can the mutually exclusive particle-like and wave-like behaviors described by quantum mechanical systems and observed in experiments be reconciled?
- **The problem of indeterminism** Why is the quantum formalism able to make only probabilistic, but not definite and deterministic, predictions of the outcome of well specified physical situations? How can identical conditions produce varying results? What is the source of the intrinsic randomness?
- **The problem of measurement and collapse** How and why does the wave function (or state vector) of wave mechanics change abruptly and discontinuously when a measurement is made? What is the mechanism behind state reduction?
- **The problem of nonlocality** How and why are separated but entangled parts of a quantum mechanical system nonlocally connected, so that measurements on one subsystem somehow influence the outcomes of measurements on the other subsystem, even when they are out of speed-of-light contact. What is the mechanism behind nonlocality?

As we observed in Sect. 2.2, the experience of Heisenberg with the intellectual traps implicit in pictorial models of physical systems had led him to distrust them, and to focus on the "real" variables of physical systems that could be measured in the laboratory. He carried over this approach to his interpretation of quantum mechanics. Logical positivism was fashionable in the philosophical circles of Berlin and Vienna in the late 1920s, and, perhaps influenced by his philosopher colleagues, Heisenberg adopted positivist thinking to his work [10]. He decided that one should not attempt to "look behind the scenes" at the inner workings of quantum mechanics that were inaccessible to physical measurement. These were “off limits”. One should focus on observables and on the outcome of real or, if necessary, *gedanken* measurements. His uncertainty principle reinforced this view by demonstrating that many of the “virtual variables” suggested by pictorial models were not only elusive, but were completely impossible to measure, even in principle. The emerging Copenhagen Interpretation became essentially a “don’t ask; don’t tell” approach to the quantum formalism that fulfilled the needs of those who wanted to calculate and make predictions, but frustrated those who wanted to understand what went on behind the scenes. In later years, this emphasis on positivism was somewhat attenuated in Heisenberg’s writings, but it never completely disappeared. These interpretational ideas were not explicitly called “The Copenhagen Interpretation” until 1955, when Heisenberg gave them that name [11], after which the term was widely adopted.

Niels Bohr focused on the wave-particle duality problem, emphasizing the relation between the complementarity of these aspects and the complementarity of the conjugate variables in the uncertainty principle [12]. He also emphasized the “oneness” of the system and the measurements performed on it, and insisted that these could not be separated and analyzed separately. His philosophy of complementarity, the idea that two seemingly contradictory descriptions together characterized the same phenomenon, was widely promoted and became an important part of the emerging Copenhagen Interpretation.

Max Born (Fig. 2.12), in working on the development of matrix mechanics and investigating its connection to wave mechanics, originated what became known as the “Born probability rule”, the assumption that the complex values of wave functions and matrix elements could be related to physical observables by multiplying the complex quantity by its complex conjugate [13]. In other words, one added the square of the real part of the variable to the square of its imaginary part, producing a real positive number that was interpreted as the probability of making the particular observation

Fig. 2.12 In 1954, Max Born (1882–1970) received the Nobel Prize in Physics in 1954 for his contributions to quantum mechanics

that had specified what went into the calculation. ($P = \psi\psi^*$) This became a central assumption of the Copenhagen Interpretation and an important guide for quantum mechanics practitioners who wished to relate calculations to observations. However, it led to the problem of indeterminism listed above, because, as a probability, it meant that precisely the same physical situation might have many different outcomes. The crisp deterministic character of Newtonian dynamics had been replaced by the fuzzy calculation of probabilities and by varying outcomes from identical initial conditions.

Werner Heisenberg addressed the problem of identity by asserting that the wave function was not a real wave moving through space-time, but rather was an evolving mathematical representation of *the knowledge of an observer*, real or potential, who was observing a quantum mechanical system and performing measurements on it [14, 15]. This was Heisenberg's "knowledge interpretation", which became a central element of the Copenhagen Interpretation.

At a stroke, the knowledge interpretation dealt with several of the other interpretational problems listed above: The wave function was allowed to be complex because it was an encoding of knowledge and only its absolute square, a real variable, could be directly observed. Wave-particle duality was allowed because the uncertainty principle prevented measurements that revealed particle-like and wave-like behaviors at the same time (see, however, the Afshar experiment [16] described in Sect. 6.15). The outcomes of measurements was indeterminate because of the mathematics of the formalism permitted a measurement to select a particular value from the distribution of possible values present in the wave function, that distribution representing the lack of knowledge of the observer as to the value of the measured quantity until a measurement was made. The wave function "collapsed" when the measured value became known and the knowledge of the observer changed.

The Freedman–Clauser experimental results [17], an experimental demonstration on quantum nonlocality, were published in 1972, four years before Heisenberg's death on February 1, 1976. To my knowledge, he never attempted to apply his knowledge interpretation to the emerging experimental multi-measurement demonstrations of EPR-type nonlocality, i.e. the enforced correlation of separate measurements on two separated but entangled subsystems (see Chap. 3), that implicitly would involve the knowledge of two separated observers. However, in 1984 I pressed the issue with the late Sir Rudolf Peierls (1907–1995), a Copenhagenist and skilled practitioner of Heisenberg's knowledge interpretation [18], who was visiting the University of Washington Physics Department at the time. Peierls addressed the problem of EPR nonlocality in the Freedman–Clauser experiment by observing that there was no real observer-to-observer communication involved in EPR nonlocality, only correlations. He went on to point out that the state vector *should* be allowed to be nonlocal when describing the observer's knowledge of the separated parts of the system, because the observer's knowledge must span all of the subsystems. I was looking for mechanisms, and I found this to be an interesting but unsatisfying answer.

In any case, the knowledge interpretation is a self-consistent viewpoint, even if it leaves many tantalizing quantum questions unanswered and raises many other issues (e.g., observer-created reality). The central tenets of the Copenhagen Interpretation can be summarized as follows:

- A system is completely described by a wave function ψ, which is a solution of a wave equation characteristic of the system; the wave function is a mathematical representation of an observer's knowledge of the system and changes when knowledge changes.
- One should focus on the observable quantities of a system and avoid asking questions about aspects that are not subject to measurement.
- The quantum mechanical description of nature is probabilistic and random. The probability of an event is the absolute square of the amplitude of the wave function related to it. Identical conditions can produce varying outcomes.
- It is not possible to know the precise values of all of the properties of a system at the same time; complementary variables are subject to the uncertainty principle; properties that are not known are described by probabilities.
- Matter and light exhibit wave-particle duality; an experiment can show the particle-like properties or wave-like properties, but *not* both at the same time. The uncertainty principle prevents wave versus particle conflicts.
- Measuring devices are essentially classical devices and measure classical properties such as position and momentum; the quantum system and the apparatus that makes measurements on it are parts of a unified whole and cannot be separated and analyzed separately.
- The quantum mechanical description of a system, in the limit of large quantum numbers, should closely correspond to its classical description.

The Fifth International Solvay Conference on Electrons and Photons, held in Brussels in October 1927, served as the coming-out party for this new way of interpreting quantum mechanics (later given the name Copenhagen Interpretation by Heisenberg [11]). Bohr, Heisenberg, and Born presented their new interpretation and defended it against all assaults. Einstein, Schrödinger, and others raised objections (see Sect. 6.2), but in the end, it was generally acknowledged that the Copenhagen Interpretation had become the standard way of approaching the quantum formalism, and that would-be practitioners of the formalism would be best guided by following its tenets.

At the Sixth International Solvay Conference on Magnetism held in Brussels in 1930, Einstein confronted the Copenhagen view by presenting his clock paradox, an ingenious *gedankenexperiment* involving a photon in a box that seemed to violate the uncertainty principle between time and energy. Leon Rosenfeld, a scientist who had participated in the Congress, described the event several years later: [19]

> It was a real shock for Bohr…who, at first, could not think of a solution. For the entire evening, he was extremely agitated, and he continued passing from one scientist to another, seeking to persuade them that it could not be the case, that it would have been the end of physics if Einstein were right; but he could not come up with any way to resolve the paradox. I will never forget the image of the two antagonists as they left the club: Einstein, with his tall and commanding figure, who walked tranquilly, with a mildly ironic smile, and Bohr who trotted along beside him, full of excitement.

The next morning brought Bohr's triumph. He presented his solution to the puzzle, an intricate account of the necessary measurements and their uncertainties that invoked Einstein's own equivalence principle to establish that the time-energy uncertainty principle was indeed preserved. Einstein conceded defeat on that occasion, but five years later in 1935 he made another assault on quantum mechanics and the Copenhagen Interpretation with the famous Einstein, Podolsky, and Rosen paper.

2.7 Einstein, Podolsky, and Rosen; Schrödinger and Bohm (1935–1963)

In 1935, five years after his clock paradox at the 6th Solvay Conference had failed to demonstrate the inadequacies of the uncertainty principle and Copenhagen quantum mechanics, Einstein made another foray against quantum mechanics. This time, he collaborated with colleagues Boris Podolsky and Nathan Rosen at Princeton's Institute for Advanced Studies in producing a succinct 4-page paper that described two things that he regarded as fatal flaws in quantum mechanics. They published this in the journal *Physical Review* on May 15, 1935 [20]. The work received considerable attention and soon became known as "the EPR paper". The first section of the EPR paper raised an objection to the role of the uncertainty principle in the description of quantum systems. The authors argued that if knowledge of one member of a pair of conjugate physical quantities precludes knowledge of the other quantity, then either (1) the description of reality given by the wave function in quantum mechanics is not complete, or (2) the two conjugate quantities cannot have simultaneous reality.

The second section of the EPR paper raised the issue of making quantum mechanical predictions about the states of two systems that have previously been in physical contact and are then separated. Their arguments, based on momentum conservation, focused on the choice of momentum or position measurements made on one of the systems and its effect on the quantum mechanical state of the other system. The authors argued that the choice of which quantities are measured in one system affects the outcomes of possible measurements made on the other systems. They then argued that "since the systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system." This seeming contradiction leads them to assert that possibility (2) cannot be true, and therefore quantum mechanics must be incomplete. The issues raised in the second section of the paper have become know as "the EPR paradox" and arise from a previously ignored aspect of the quantum mechanics formalism, its *nonlocality* or enforcement of correlations between measurements in spatially separated systems. (See Chap. 3.) In a letter to Max Born [21, 22], Einstein dismissively referred to quantum nonlocality as "spooky actions at a distance".

Niels Bohr responded to the EPR paper by defending the uncertainty principle, focusing on the simultaneous reality and indefiniteness of complementary variables like momentum and position, and discussing the Copenhagen view of wave-particle duality, while essentially ignoring the nonlocality issue that the EPR paper had raised [23]. Heisenberg's reaction to the EPR paper, as reported by his close associate C. F. von Weizsäcker [24], was: "Now, after all, Einstein has understood quantum mechanics. I am sorry for him that he still does not like it."

Schrödinger, on the other hand, took the EPR paradox and the discovery of nonlocality very seriously. He published a two-part paper in 1935-36 in which he agreed that standard quantum mechanics did indeed exhibit the property of nonlocality [25, 26]. He analyzed its aspects and implications in considerable detail. In these papers, he introduced the term *entanglement* to describe the condition of a pair of quantum systems that have interacted and then separated. He concluded that the quantum state of one of an entangled pair of systems cannot be described without making reference to the quantum state of the other member of the entangled pair. He ended by stating that "these conclusions, unavoidable within the present theory but repugnant to some physicists *including the author*, are caused by applying non-relativistic quantum mechanics beyond its legitimate range." In other words, Schrödinger took the demonstrated presence of entanglement and nonlocality in the formalism of quantum mechanics as indications that the theory must be incorrect when applied to systems where nonlocality is important.

In principle, the theory of quantum mechanics, at that point, could have been subjected to experimental testing to determine if it was indeed "being applied beyond its legitimate range". Some conserved quantity like momentum or angular momentum under different localization conditions could have been measured in two subsystems to reveal EPR correlations. In practice, in part because of the formidable experimental challenges of such tests, in part because of the lack of a crisp and falsifiable theoretical prediction, in part because World War II was in the making, and in part because of a lack of interest in such tests among experimental physicists and their sources of funding, it required almost four decades for such testing to begin.

We note, in this context, a missed opportunity: the pair of back-to-back 0.511 MeV gamma rays produced in electron-positron annihilation form a polarization-entangled photon pair because the annihilation process has zero net angular momentum and negative parity. As predicted in a calculation by John Wheeler [27], this leads to planes of linear polarization of the two photons that must be 90° apart (see Eq. 2.2).

Therefore, this system could, in principle have provided a test-bed for the testing and demonstration of EPR quantum nonlocality. C. S. Wu and I. Shaknov in 1950 [28] used a β^+-radioactive ^{64}Cu source to measure the polarization correlations of the gamma ray pair from the e^+e^- annihilation that followed the positron decay. They showed that, after efficiency and geometrical corrections, the ratio of counts with the polarimeter planes perpendicular versus parallel agreed with Wheeler's predictions based on quantum mechanics. However, since this work preceded that of J. S. Bell by a decade and a half, the measurements were only made at polarimeter angle-differences of 0°, 90°, 180°, and 270°, and no one realized the very fundamental significance of the measurements. No connection between these experimental results and Einstein's EPR arguments or nonlocality was made at the time or later in Bell's papers.

About 1951, EPR supporter David Bohm introduced the idea of a “local hidden variable” theory that could replace standard quantum mechanics with a theoretical structure that omitted the paradoxical features of quantum mechanics to which the EPR paper had objected. In Bohm’s hidden-variable alternative to quantum mechanics, all correlations were established locally at sub-light speeds. Position and momentum were permitted to have simultaneous precise values, values that were real but were “hidden” and inaccessible to direct measurement.

Practicing physicists, however, paid little attention to such hidden variable theories. Bohm’s approach was less useful than orthodox quantum mechanics for calculating the behavior of physical systems. Since the theories seemed to make the same predictions, it was apparently impossible to resolve the EPR/hidden-variable debate by performing an experiment, so the general physics community tended to ignore the whole controversy and leave it to the philosophers.

2.8 Bell’s Theorem and Experimental EPR Tests (1964–1998)

In 1964, the testability situation changed. In a series of publications, John Stuart Bell (Fig. 2.13), a Scottish theoretical particle physicist working at the CERN high-energy physics laboratory in Geneva, proved an amazing theorem demonstrating that experimental tests could distinguish the predictions of quantum mechanics from those of any local hidden-variable theory [29, 30]. Bell, following the lead of Bohm [31], had based his calculations not on measurements of position and momentum, the focus of the arguments of Einstein and Schrödinger, but on measurements of the linear polarization of photons of light when considerations of angular momentum conservation constrained them.

Before discussing Bell’s theorem further, we pause to consider the polarization of light. The phenomena we refer to as light: visible light, invisible infrared or ultraviolet rays, radio waves, X-rays, or gamma rays, all are aspects of the same basic physics, differing only in frequency. They are traveling waves produced when electric and magnetic fields vibrate together at right angles to each other as they move through space at the speed of light. The direction in which the electric field of a light wave vibrates determines the *polarization* of the wave. If the electric field vibrates always in the same plane, we say that this is the plane of polarization and that the wave has *linear polarization* in that plane. There is another polarization basis, *circular polarization*, in which the electric field corkscrews through space to the right or left as the light wave moves forward. Circular polarization can be produced by superimposing states of vertical and horizontal linear polarization with appropriate phase, and linear polarization can be produced by superimposing left and right circular polarization. (See Eqs. 6.2–6.5.) Most of the work with Bell’s theorem has been focused on states of linear polarization.

Fig. 2.13 John Stuart Bell (1928–1990)

It is quite easy to measure the linear polarization of visible light. Special optical filters, for example the lenses of polarized sunglasses, absorb light polarized in one direction while transmitting the light polarized in the perpendicular direction. There are also polarization-sensitive splitters that divide one beam of light into two beams, for example with one linear polarization state reflected and the other transmitted.

Using such devices, a particular polarization component of incident light can be transmitted and the other component absorbed or diverted. If a beam of unpolarized light is passed first through one such polarization filter and then through another, the intensity of the transmitted beam varies in accordance with Malus' Law, which states that the transmitted light intensity $I(\alpha)$ is proportional to the square of the cosine of the angle α between the polarization direction of the first filter and that of the second filter, i.e., $I(\alpha) = I_0 \cos^2(\alpha)$, where I_0 is the intensity observed when the polarization directions of the filters are parallel. Malus' Law is shown in Fig. 2.14 This equation tells us that when the planes of polarization of the two filters are at 90°, the crossed filters look black and no light is transmitted. When the planes of polarization make an angle of 45°, half the light intensity passing the first filter is transmitted by the second, and so on.

In an atom, if an orbital electron is kicked from its lowest energy level into a higher orbit by an energetic photon or an electrical discharge, the electron may return to its lowest energy state by a process called a "cascade", a series of quantum jumps to lower orbits, each jump producing a single light photon of a wavelength that depends on the energy gap of the jump. A two-photon cascade in which the atom as a whole begins and ends with no net angular momentum (i.e., no rotational motion) and no change in parity (the mirror-symmetry of the system is unchanged) is of particular interest, because the cascade produces an entangled pair of photons

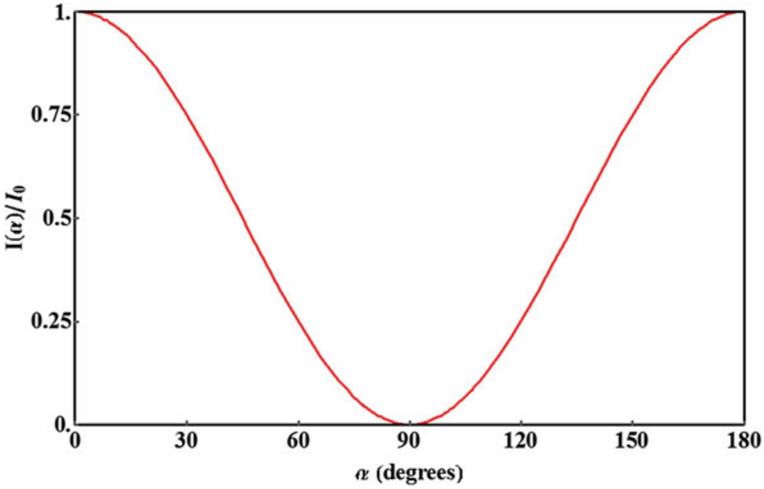

Fig. 2.14 Malus' Law: Linearly polarized light is transmitted through two polarizing filters with intensity $I(\alpha) = I_0 \cos^2(\alpha)$, where α is the angle between the polarizing axes of the filters

that have correlated polarizations due to angular momentum conservation. When the photons from the cascade travel back-to-back in opposite directions, angular momentum conservation requires that if one of the photons is measured to have some definite linear polarization state, the other photon is required to have exactly the same linear polarization state. Kocher and Commins [32] used this technique to produce entangled photon pairs and to demonstrate the polarization correlation predicted by quantum mechanics, but they did not attempt a test of Bell's theorem. The first experimental tests of Bell's theorem in the 1970s, often called "EPR experiments", used the entangled photon pairs from such cascades.

First, a word about notation: in the discussions that follow we will explicitly indicate wave functions ψ using the Dirac "ket" state vector notation; a *ket* is a vertical bar $|$ and an angle bracket $\rangle$ that enclose some symbol that distinguishes one wave function from another. For example, a wave function that is characteristic of system 1 in a state of horizontal linear polarization might be represented by $\psi_{H_1} =| H\rangle_1$, and so on. Later we will also use the complex conjugate of the Dirac ket state vector, which is called a *bra* and for the ket above would be denoted by the symbol $\langle H |_1$.

The wave function that describes such an entangled pair of photons is said to describe a "Bell state", and, for zero initial and final angular momentum, if there is no change in parity it has the symmetric form:

$$| S\rangle_+ = \frac{1}{\sqrt{2}}(| H\rangle_1 \mid H\rangle_2 + e^{i\phi} \mid V\rangle_1 \mid V\rangle_2), \tag{2.1}$$

or if parity changes, it has the anti-symmetric form:

$$| S\rangle_- = \frac{1}{\sqrt{2}}(| H\rangle_1 \; | V\rangle_2 - e^{i\phi} \; | V\rangle_1 \; | H\rangle_2), \tag{2.2}$$

where ϕ is an arbitrary phase angle that depends on geometry and is usually 0 or π, and H and V describe horizontal and vertical linear polarization, respectively, of the entangled photons moving on paths 1 and 2. When a Bell-state wave function described by Eq. 2.1 is collapsed by a linear polarization measurement of the photons, it will be found that either both photons have H polarization or that both photons have V polarization, each with a 50 % probability.

These EPR experiments measured the coincident arrival of entangled photons at opposite ends of the apparatus, as detected by quantum-sensitive photomultiplier tubes after each photon had passed through a polarizing filter. The photomultipliers at opposite ends of the apparatus produce electrical pulses that, when they occur at the same time, are recorded as a "coincidence" or two-photon event. The rate of such coincident events is measured while varying the polarization "pass" directions of the two filters, characterized by transmission-axis angles α_1 and α_2. The two transmission angles are systematically varied and the rate measurement is repeated until a complete map of rate versus the two angles is developed.

Bell's theorem deals with the way in which the coincidence rate of an EPR experiment falls off when the two transmission angles α_1 and α_2 are not equal. Bell proved mathematically that for all local hidden-variable theories [31] the magnitude of the decrease in coincidence rate must be linear (or less) as it depends on the angular difference $\Delta\alpha$ between the two filters. Suppose, for example, that we misalign the angles of the two polarization filters so that the angle between the polarization directions of the two filters is $\Delta\alpha = \alpha_1 - \alpha_2$. We measure the coincidence rate $R(\Delta\alpha)$, as compared to the rate R_0 when the filters are perfectly aligned. That rate drops by an amount $\Delta_1 = R_0 - R(\Delta\alpha)$. Now we double the amount of the misalignment, so that the decrease in rate is $\Delta_2 = R_0 - R(2\Delta\alpha)$. For this situation, Bell's theorem requires that Δ_2 must be less than or equal to twice Δ_1 ($\Delta_2 \leq 2\Delta_1$).

This prediction of Bell's theorem is one of the so-called "Bell inequalities". It can be thought of in the following way. Consider that the coincidence rate R_0 when the polarizing filters are aligned ($\Delta\alpha = 0$) is a "signal", to which "noise" is added when a misalignment is introduced. If the noise Δ_1 introduced by moving one filter an amount α to the right is not correlated with the noise Δ_1' introduced by moving the other filter by the same angle to the left, then at most, when both sources of noise are present, the noise Δ_2 from a 2α misalignment should be twice Δ_1. However, the two uncorrelated noise sources may occasionally cancel, permitting Δ_2 to be less than twice Δ_1. Therefore, the Bell inequality states that Δ_2 must be less than or equal to twice Δ_1.

Quantum mechanics, on the other hand, predicts that the coincidence rate $R(\alpha_1, \alpha_2)$ depends only on the relative angle $\Delta\alpha = \alpha_1 - \alpha_2$ between the two polarization directions, and that $R(\Delta\alpha)$ obeys Malus' Law. In other words, quantum mechanics predicts that $R(\alpha_1, \alpha_2) = R(\Delta\alpha) = R_0 \cos^2(\Delta\alpha)$. Therefore, $\Delta_1 = R_0[1 - \cos^2(\Delta\alpha)]$ and $\Delta_2 = R_0[1 - \cos^2(2\Delta\alpha)]$. When the misalignment angle α is fairly small, this means that Δ_2 is about four times Δ_1, which is clearly much larger than twice Δ_1 (i.e., $\Delta_2 \approx 4\Delta_1$ so $\Delta_2 > 2\Delta_1$). This is a clear violation of Bell's theorem, because the coincidence rate predicted by quantum mechanics falls off much too fast with increasing angle to be consistent with Bell's theorem, which predicts an approximately linear decrease, as shown in Fig. 2.15.

What is the essential difference between quantum mechanics and local hidden-variable theories that causes their distinguishable predictions of the relation between Δ_1 and Δ_2? As pointed out by Herbert [33], in the local hidden-variable theories, the photons leaving the source are required to be in a definite (but possibly random and unknown) state of linear polarization, leading to noise Δ roughly proportional to $\Delta\alpha$. For quantum mechanics, the state of polarization of entangled photons leaving the source is indefinite and is not fixed until a polarization measurement is made, leading to Malus' Law and Δ roughly proportional to $\Delta\alpha^2$. (See the discussion of "realism" in Sect. 6.18). That essential difference between linear and quadratic behavior lies at the root of Bell's inequalities.

We note that Bell did not consider the less-than-100 % efficiency of real single photon detectors and the less-than-perfect behavior of real polarization analyzers. A group led by John Clauser [34] generalized Bell's theorem to take these effects into account, producing the CHSH inequality, which is used in real experimental tests and is essentially Bell's inequality cast in a more realistic experimental context.

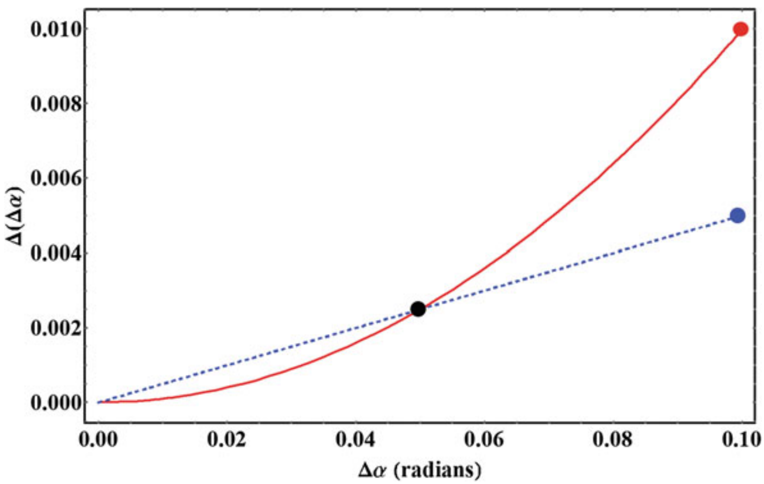

Fig. 2.15 Plot of "noise" rate $\Delta(\Delta\alpha)$ versus polarimeter angle difference $\Delta\alpha$ for small angles. Local hidden variable prediction (*blue/dashed*) rises linearly, while quantum mechanical prediction (*red/solid*) rises quadratically. If the rates are equal at $\Delta\alpha = 0.05$ (*black dot*), the quantum mechanical prediction (*red dot*) is twice the local hidden variable prediction (*blue dot*) at $\Delta\alpha = 0.10$ and four times the rates at $\Delta\alpha = 0.05$

The first unambiguous experimental results from EPR experiments, the pioneering work of John Clauser (Fig. 2.16) and his student Stuart Freedman (Fig. 2.17) at UC Berkeley, was performed in the early 1970s and published in 1972 [17]. They reported a 6.7 standard deviation violation of Bells' inequalities and consistency with the predictions of quantum mechanics. A decade later, in 1982, the EPR measurements of the Aspect group [35, 36] in France used newly developed apparatus and techniques and were able to eliminate several "loophole" scenarios that might constitute unlikely ways of preserving classical locality (see Sect. 3.2). They demonstrated consistency with quantum mechanics and inconsistency with local hidden-variable theories, this time with a 46 standard deviation violation of Bell's inequalities [35, 36]. A more recent EPR experimental example is the 1998 work of the Gisin group in Switzerland [37, 38]. They used fiber-optics cables owned by the Swiss Telephone System to demonstrate the nonlocal connection between EPR measurements made at locations in Geneva and Bern, Swiss cities with a line-of-sight separation of 156 km. Their work constitutes a direct demonstration, if one was required, that not only is quantum mechanics nonlocal, but that such nonlocality can operate over quite large spatial separations.

Such EPR results were initially interpreted as a demonstration that hidden variable theories like those of Bohm had been falsified. That view changed when it was realized that Bell's theorem was based on *local* hidden variable theories, and that nonlocal hidden variable theories could also be constructed to violate Bell's theorem and agree with the experimental measurements. The assumption made by Bell that had been put to the test was the assumption of locality, not hidden variables.

Do these EPR experiments constitute a solid demonstration of the existence of quantum nonlocality? There is more than one way of interpreting the implications of the experimental results, and one can find much discussion in the literature as

Fig. 2.16 John F. Clauser (1942–)

Fig. 2.17 Stuart J. Freedman (1944–2012)

to whether it is locality or “realism” (the objective observer-independent reality of external events) that has been refuted by these EPR measurements. We here adopt the view that reality should be taken as a given, and so we regard these experiments as direct demonstrations of the intrinsic nonlocality of standard quantum mechanics. (See Sect. 6.18 for further discussion of the realism issue.)

The EPR experiments demonstrate the quantum enforcement of a correlation between measurement results in the two separated arms of the experiment. Let us try to clarify the nature of that correlation with an example. Suppose that you are given a gold coin, and you use a fine jeweler’s saw to cut it in half along the plane of the coin, placing the “head” side in one pink envelope and the “tail” side in another. You do the same thing with another coin, separating it with a horizontal cut into a top half and a bottom half and placing these in green envelopes. And you cut a third coin, separating it with a vertical cut into a left half and a right half and placing these in blue envelopes. Now you shuffle the envelopes and send one set of colors to an observer in Boston and the other set to an observer in Seattle. Each observer is allowed to choose one of the envelopes and to open only that one. If the Seattle observer opens his pink envelope and finds the head, he knows that the Boston observer, if he also opens his pink envelope, will find a tail. But he is unable to predict what the Boston observer will find if he opens the green or the blue envelope.

That is classical physics. The difference in the EPR quantum situation is that there is only one coin, and that each observer decides the direction in which his half of the single coin should have been be cut only after the single white envelope has arrived. And yet, he observes the same correlations as described above. If both observers choose the same cut directions, their halves are opposites. If they choose different cut directions, they are unable to predict the observation of the other observer. Here the correlated coin-halves correspond to entangled photons and the cut directions to the choice of measurements of polarization bases (circular right/left, linear vertical/horizontal, linear 45° diagonal/anti-diagonal, and others). See the quantum games described in Appendix C for further analogies to EPR correlations.

We note that the several polarization bases used in these kinds of polarization EPR experiments make it straightforward to demonstrate the quantum nonlocal connections but also make it effectively impossible to use those connections for observer-to-observer signaling, because one would need to deduce from the arriving photons the polarization basis that was being used in the distant measurements. While each observer is free to choose the polarization basis (e.g., circular right/left, linear horizontal/vertical, linear 45° diagonal/anti-diagonal) for the measurement, he is not free to force the photon into a particular state of that basis, as would be required for nonlocal communication. Thus, while polarization-based EPR experiments may be taken as demonstrations that Nature is using some nonlocal mechanism to arrange the correlations of the separated measurements, such a "superluminal telegraph line" is not accessible to the experimenters for sending their own messages. See Chap. 7 for further discussion of the suppression of nonlocal signaling.

To put it another way, the intrinsic nonlocality of quantum mechanics had been tested by the experimental EPR tests of Bell's theorem. It has been experimentally demonstrated that Nature arranges the correlations between the polarizations of the two entangled photons at separated measurement sites by some nonlocal (and perhaps retrocausal) mechanism that violates Einstein's intuitions about the intrinsic locality of all natural processes. What Einstein called "spooky actions at a distance" are in fact an important part of the way Nature works at the quantum level.

References

1. M. Born, W. Heisenberg, Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen. Z. für Phys. **33**, 879–893 (1925). First paper on the matrix mechanics formulation of quantum mechanics
2. M. Born, P. Jordan, Zur Quantenmechanik, Z. für Phys. **34**, 858–888 (1925). Second paper on the matrix mechanics formulation of quantum mechanics
3. M. Born, W. Heisenberg, P. Jordan, Zur Quantenmechanik II. Z. für Phys. **35**(8–9), 557–615 (1926). Third paper on the matrix mechanics formulation of quantum mechanics
4. E. Schrödinger, Ann. der Phys. **79**, 361 (1926)
5. E. Schrödinger, Ann. der Phys. **79**, 486 (1926)
6. E. Schrödinger, Ann. der Phys. **79**, 734 (1926)
7. E. Schrödinger, Ann. der Phys. **81**, 109 (1926)
8. W. Heisenberg, Reminiscences from 1926 and 1927, in reference [39]
9. W. Heisenberg, Z. für Phys. **43**, 172 (1927). Translated in [40], pp. 62–84
10. W. Heisenberg, Z. für Phys. **33**, 879 (1925)
11. W. Heisenberg, The development of the interpretation of quantum theory, in *Niels Bohr and the Development of Physics*, ed. by W. Pauli (Pergamon, London, 1955)
12. N. Bohr, *Atti del Congresso Internazionale dei Fisici Como*, 11–20 Settembre 1927, vol. 2 (Zanchelli, Bologna, 1928), pp. 565–588
13. M. Born, Z. für Phys. **37**, 863 (1926), pp. 52–55. Translated in [40]
14. W. Heisenberg, Daedalus **87**, 95 (1958)
15. M. Jammer, *The Philosophy of Quantum Mechanics* (Wiley, New York, 1974)

16. S.S. Afshar, Violation of the principle of complementarity, and its implications, Proc. SPIE, **5866**, 229–244 (2005). arXiv:quant-ph/0701027
17. S.J. Freedman, J.F. Clauser, Phys. Rev. Lett. **28**, 938 (1972)
18. R. Peierls, In defense of measurement. Phys. World 19–20 (1979)
19. L. Rosenfeld, in [40] (1955), pp. 477–478
20. A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47**, 777–785 (1935)
21. A. Einstein in a 1947 letter to M. Born (see [22])
22. M. Born, A. Einstein, in *The Born-Einstein Letters*, ed. by M. Walker (Born, New York, 1979)
23. N. Bohr, Phys. Rev. **48**, 696 (1935)
24. Th Görnitz, C.F. von Weizsäcker, Copenhagen and transactional interpretations. Int. J. Theor. Phys. **27**, 237–250 (1988)
25. E. Schrödinger, Proc. Camb. Philos. Soc. **31**, 555–563 (1935)
26. E. Schrödinger, Proc. Camb. Philos. Soc. **32**, 446–451 (1936)
27. J.A. Wheeler, Polyelectrons. Ann. N. Y. Acad. Sci. **48**, 157 (1945)
28. C.S. Wu, I. Shaknov, The angular correlation of scattered annihilation radiation (letter to the editor). Phys. Rev. **77**, 136 (1950)
29. J.S. Bell, Physics **1**, 195 (1964)
30. J.S. Bell, Rev. Mod. Phys. **38**, 447 (1966)
31. D. Bohm, *Causality and Chance in Modern Physics*, (Routledge & Kegan Paul and D. van Nostrand, 1957). ISBN: 0-8122-1002-6
32. C.A. Kocher, E.D. Commins, Phys. Rev. Lett. **18**, 575 (1967)
33. N. Herbert, Am. J. Phys. **43**, 315 (1975)
34. J.F. Clauser, M. Horne, A. Shimony, R. Holt, Proposed experiment to test local hidden-variable theories. Phys. Rev. Lett. **23**, 880 (1969)
35. A. Aspect, J. Dalibard, G. Roger, Phys. Rev. Lett. **49**, 91 (1982)
36. A. Aspect, J. Dalibard, G. Roger, Phys. Rev. Lett. **49**, 1804 (1982)
37. W. Tittel, J. Brendel, B. Gisin, T. Herzog, H. Zbinden, N. Gisin, Experimental demonstration of quantum-correlations over more than 10 kilometers, Phys. Rev. A **57**, 3229 (1998), arXiv:quant-ph/9707042
38. W. Tittel, J. Brendel, H. Zbinden, N. Gisin, Violation of Bell inequalities by photons more than 10 km apart, Phys. Rev. Lett. **81**, 3563–6 (1998), arXiv:quant-ph/9806043
39. A.P. French, P.J. Kennedy (eds.), *Niels Bohr, A Centernary Volume* (Harvard UniversityPress, Cambridge, 1985)
40. J.A. Wheeler, W.H. Zurek (eds.) *Quantum Theory and Measurement* (Princeton University-Press, Princeton, 1983)

Springer
http://www.springer.com/978-3-319-24640-6

The Quantum Handshake
Entanglement, Nonlocality and Transactions
Cramer, J.G.
2016, XXV, 218 p. 90 illus., 34 illus. in color., Hardcover
ISBN: 978-3-319-24640-6