Preface

Agent-based modeling is an interesting tool. It provides model developers with a great degree of freedom for the design of systems in which heterogeneous entities interact with each other and the environment. Agent-based models (ABMs) are therefore a great tool to explore how different assumptions about how individuals behave and interact affect the evolution of social, economic or ecological systems as a whole.

The mathematical formalization of these models, however, is still in its infancy partly due to the fact micro-level heterogeneity or complex interaction structures often lead to effects in the system dynamics which are not easily accounted for by macroscopic formulations of the problem. To address this issue an understanding of the transition from the most informative "atomic" level to the levels at which the system behavior is typically observed is important, because it can help to derive and evaluate models on specific levels, on the one hand, and to understand the temporal and spatial patterns that may emerge in that transition on the other. The book at hand develops a Markov chain approach that allows a rigorous analysis of a class of microscopic models which specify the dynamics of a complex system at the individual level. It provides a general framework of aggregation in agent-based and related computational models by making use of lumpability and information theory in order to link between the micro and macro levels of observation.

The starting point is a microscopic Markov chain description of the dynamical process in complete correspondence with the dynamical behavior of the ABM, which is obtained by considering the set of all possible agent configurations as the state space of a huge Markov chain. This is referred to as micro chain, and an explicit formal representation including microscopic transition rates can be derived for a class of models by using the random mapping representation of a Markov process. The explicit micro formulation enables the application of the theory of Markov chain aggregation—namely, lumpability—in order to reduce the state space of the micro chain and relate microscopic descriptions to a macroscopic formulation of interest. Well-known conditions for lumpability make it possible to establish the cases where the macro model is still Markov, and in this case we obtain a complete picture of the dynamics including the transient stage, the most interesting phase in applications.

For such a purpose a crucial role is played by the type of probability distribution used to implement the stochastic part of the model which defines the updating rule and governs the dynamics. Namely, if we decide to remain at a Markovian level, then the partition, or equivalently, the collective variables, used to build the macro model must be compatible with the symmetries of the probability distribution ω. Microscopic heterogeneity and constraints translate into dynamical irregularities in the micro chain and require a refinement of the aggregation and the corresponding level of observation. This underlines the theoretical importance of homogeneous or complete mixing in the analysis of "voter-like" models at use in population genetics, evolutionary game theory and social dynamics.

The problem of aggregation in ABMs and the lumpability conditions in particular can be embedded into a more general framework which makes use of information theory in order to identify different levels and relevant scales in complex dynamical systems. Lumpability and, respectively, the existence of a higher-level Markovian description is one out of several mutually related criteria which a closed higher-level description should satisfy. Consequently, the application of information-theoretic measures of closure to ABMs allows us to quantify the information that is lost in the transition from the micro dynamics to a particular macro description. The method informs us in this way about the complexity of a system introduced by nontrivial interaction relations. Namely, if a favored level of observation is not compatible with the symmetries in ω, a certain amount of memory is introduced by the transition from the micro level to such a macro description, and this is the fingerprint of emergence in ABMs. The resulting divergence from Markovianity can be quantified using information theory, and the book presents a scenario in which different closure measures can be explicitly computed.

Throughout the book, we mainly rely on two simple models to illustrate these theoretical ideas: the voter model (VM) and an extension of it called the contrarian voter model (CVM). Using these examples, the book shows that Markov chain theory allows for a rather precise understanding of the model dynamics in case of "simple" population structures where a tractable macro chain can be derived. Constraining the system by interaction networks with a strong local structure leads to the emergence of meta-stable states in the transient of the model. Constraints on the interaction behavior such as bounded confidence or assortative mating lead to the emergence of new absorbing states in the associated macro chain and are related to stable patterns of polarization (stable coexistence of different opinions or species). Constraints and heterogeneities in the microscopic system and complex social interactions are the basic characteristics of ABMs, and the Markov chain approach to link the micro chain to a macro-level description (and likewise the failure of a Markovian link) highlights the crucial role played by those ingredients in the generation of complex macroscopic outcomes.

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Leipzig, Germany Sven Banisch

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