

Preface

This book brings together the development of a theoretical argument and its translation into an extensive empirical context as a result of the research work carried out by the Problem@Web project. Through the 3 years of the project, we studied youngsters engaged in mathematical problem-solving using digital technologies of their choice and to which they had personal access. Problem@Web, the compact name given to our project, alludes directly to the research context, that of youngsters solving mathematical problems in their own time and with their own choice of digital technology.

The origins of the project arose in the experiences during the SUB12 and SUB14 web-based mathematical problem-solving competitions that have been taking place annually in the south of Portugal since 2005. Organised by the Mathematics Department of the Faculty of Sciences and Technology of the University of Algarve, SUB12 aims at 5th and 6th graders (10–12-year-olds), while SUB14 addresses students in 7th and 8th grade (12–14-year-olds). These are examples of the kind of mathematical competitions that it is important to know more about; they run online, they are inclusive aiming to support wide participation, and they involve what we call “moderate” mathematical challenges. We expand on all these features in the book.

This book is designed to offer new views that extend the knowledge on how today’s youngsters tackle mathematics problems using the technology they have at their disposal. Through our analysis, we want to see how the current generation, who are growing up with digital technologies, have skills and performances that might be quite different from earlier generations. What we uncover is at odds with what some may think; the young people we studied are highly competent in solving mathematical problems through using digital technology and seem to have knowledge that is distinct to their parents and many older educated people.

Opening a window on the world of these young people through offering them challenges where the digital technology is the mediator turned out to be a way to reach them successfully. Mathematical challenges and beyond-school online competitions, such as SUB12 and SUB14, can succeed with young people. In doing so the data we gathered have provided an unparalleled opportunity to better know how youngsters engage in mathematical problem-solving and express their mathematical thinking. This book offers readers the opportunity to know the richness and

quality of mathematical knowledge produced by young people with the digital technologies they freely choose to use.

Primarily looking at a beyond-school context where students explore their “natural” learning resources, some of which may not always be available at school, we investigate what young people show as their spontaneous ways to express ideas and mathematical thinking. We believe that this knowledge can make a valuable contribution to understanding, and foreseeing, the school and the learning of mathematics in a noticeably digital age.

In the first of eight chapters, we begin by providing an introduction to research relating to youngsters’ mathematical problem-solving with technology, together with an overview of the Problem@Web project including its rationale, aims and methods. The chapter also explains the methodological procedures developed in several of the subsequent chapters in which we concentrate on addressing our analysis of real data from youngsters mathematical problem-solving.

The second chapter presents fieldwork data mainly from interviews and digital material on the experiences of the youngsters who were involved in problem-solving within the web-based mathematical competitions. It offers a portrait of such youngsters as technology users, namely, when they utilise commonly available technological tools; it also illustrates the ways in which their technological competences are placed at the service of their mathematical problem-solving and expressing.

The third chapter presents additional data from interviews on the perspectives of the teachers regarding their students’ participation in beyond-school projects that are directly related to the use of technologies in mathematical problem-solving, namely, web-based competitions. Issues related to students’ unforeseen and creative strategies of solving problems and communicating their solutions and, in general, to the development of new teachers’ views on these youngsters in the mathematics classroom are addressed.

In Chap. 4 we develop a theoretical argument about the unity between solving and expressing in problem-solving as a central construct that can be meaningfully correlated with the inseparability between the subject and the digital tool. This theoretical stance evolved from concepts such as humans with media and coaction. In framing the study of a specific and relatively new phenomenon, that of youngsters solving mathematical problems with the digital technologies of their choice, the theoretical tools and constructs that are reviewed and discussed lead to creating new constructs that we use to guide a better interpretation of what young people are able to do in a digital communication context (in the case of the Problem@Web project, finding the solution to a mathematical problem and its explanation within the scope of an online mathematical competition). Our aim is to understand how the youngsters find effective ways to achieve the solution of a problem and to communicate it mathematically, based on the digital resources they have at their disposal in their daily life, most cases in their home environment but also in school, including in the mathematics classroom.

Chapter 5 develops around two fundamental ideas, namely, (1) that the perception of the affordances of a certain digital tool is essential to solving mathematical problems with that particular technology and (2) that the activity thus undertaken

stimulates different mathematising processes which, in turn, result in different conceptual models. Looking thoroughly, from an interpretative perspective, at four solutions to a particular geometry problem completed by youngsters who decided to use the dynamic geometry software *GeoGebra* at some point in their solving activity, our main purpose is to illustrate the ways in which the same tool affords different approaches to the problem in terms of the conceptual models developed by the youngsters for studying and justifying the invariance of the area of a triangle. Their different ways of dealing with the tool and with mathematical knowledge are interpreted as instances of youngsters with media engaged in a solving-with-*GeoGebra* activity, enclosing a range of procedures brought forth by the symbioses between the affordances of *GeoGebra* and the youngsters' aptitudes. The evidence shows that different youngsters solving the same problem with the same media and recognising a relatively similar set of affordances of the tool produce different digital solutions, but they also generate qualitatively different conceptual models for, in this case, the invariance of the area of a geometric shape.

In Chap. 6 we describe and analyse a number of examples of 7th and 8th graders presenting diverse ways of expressing their mathematical thinking in solving algebraic problems with a spreadsheet. Our research purposes are concerned with youngsters' approaches to situations where quantity variation is involved in finding an unknown value under a set of conditions that frame a problem situation. The use of the spreadsheet is thoroughly examined with the aim of highlighting the nature of problem-solving and expressing in the digital tool context as compared to the formal algebraic context; moreover, the ways in which students take advantage of the tool (being guided by and also guiding the spreadsheet distinctive forms of organising and performing variation in columns and cells) are important indicators of their algebraic thinking within the problem-solving activity.

Chapter 7 focuses on a motion problem that concerns the co-variation of displacement and time in a relative motion situation. The decision to focus on a motion problem has to do with the fact that a problem involving motion—while relating the variables space, time and speed—requires some kind of understanding of the dynamic nature of the problem situation and finding suitable models for their representation. We therefore look at the problem-solving and expressing of the youngsters when facing a motion problem, especially how most participants use some form of digital medium to express their thinking.

The final chapter summarises the overall findings of the project and considers the implications. Here we reveal how the youngsters that we studied had domain over a set of general-use digital tools and while they were less aware of digital resources with a stronger association with mathematics, they were able to gain many capabilities by tackling the mathematical problems and seeking expeditious, appropriate and productive ways of expressing their mathematical thinking. In particular, we review how they were able to harness their technological skills while simultaneously developing and improving their capacity to create and use a range of mathematical representations.

In sum, we present in this book an in-depth study of youngsters' mathematical problem-solving strategies and approaches that they demonstrate as they tackle

mathematical problems in their own time and with digital resources of their own choice. Overall, our book provides the following:

- Numerous examples of moderately challenging mathematical problems.
- Many instances of student solutions, together with the students' explanations of how they achieved their solution; these student solutions are both a revelation and a valuable resource showing what youngsters can do in their own time and with their own choice of technology.
- A well-developed theoretical framing that integrates the use of technology into mathematical problem-solving.
- Insightful analysis of the young participants and their teachers and families and of the youngsters' mathematical problem-solving; the latter involving the mathematics of invariance, variation and co-variation.

It is our hope that this book contributes to the continuous development of research in mathematical problem-solving by unveiling the actual ways in which young students engage with challenging mathematical problems with the digital technological devices of their choice.

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Technology

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