

# Preface

Porous materials are found in nature as inanimate objects, such as soils and rocks, in living bodies, such as plants, human flesh, and bones, and as man-made materials, such as polyurethane foams and nanofiltration filters. The pores, or voids, are distributed all over the space the porous body occupies, as cavities, fractures, fissures, vugs, capillary channels, and other interstitial and intergranular spaces. The pores are many, in various shapes and sizes, and are randomly oriented and distributed. For modeling purposes, it is either undesirable, or impossible, to describe their exact geometry and location. As a result, only the averaged, or “homogenized,” material properties and mechanical responses are considered.

When the interstitial space is impregnated with one or more fluids, the mechanical interaction between the solid and the fluid can be interesting and complex, particularly in time-dependent behaviors, both in the quasi-static and the dynamic range. Time-dependent behaviors can manifest in many ways, as the macroscopic porous medium flow in interconnected channels, as grain scale squeeze film lubrication flow, as intergranular frictional sliding of particles, or as solid intrinsic viscoelasticity. In addition to the mechanical forces, a number of other physico-chemical factors can come into play, such as thermal, electrical, electromagnetic, and chemical (osmotic) forces.

The modeling of a full range of porous material responses, from quasi-static to dynamic, from linear to nonlinear, and from partial uncoupling to fully coupled, can be complex and unwieldy. The goal of this book is limited. It focuses largely on the linear theories, as in the classical linear elasticity and porous medium flow (Darcy flow); hence the book is titled *poroelasticity*, rather than a more general term of *poromechanics*. Although the word “Introduction” does not appear in the book title, by virtue of the above-stated scope, the book is essentially an *introduction to linear poroelasticity*.

The book contains 12 chapters. In the “Introduction,” the reader is presented with a set of basic poroelastic mechanisms, such as drained and undrained responses, effective stress, time scale, etc., explained as intuitive physical-mechanical concepts, without introducing the governing equations and mathematical rigor. It is then

demonstrated how these poroelastic mechanisms manifest in a wide range of physical phenomena in geophysics, geomechanics, acoustics, and biomechanics, such as soil consolidation and land subsidence, slope stability, fault slippage, pumping- and injection-induced seismicity, outburst of coal formation, hydraulic fracturing, water wave and sediment interaction, aquifer response to tidal and barometric loading, viscous damping and squeeze film lubrication of articular cartilage, thermal fracturing and burst of cylinder, swelling of clay and shale, and seismoelectric and seismoelectromagnetic phenomena.

For a quick construction of the theory for application, Chap. 2 lays out a continuum theory of poroelasticity, in terms of bulk constitutive equations, using a phenomenological approach, as an ad hoc extension of elasticity. Hooke's law-type linear constitutive equations are semiempirical and are presented through simple mathematical logic, without a deep examination of the underlying physics. The constitutive constants are then correlated to the macroscopically observable phenomena and measured responses.

Chapter 3 examines the micromechanics of porous materials, which isolates the mechanical properties of the solid and the fluid phases, as well as the pore space. Association is made with the phenomenological theory to rationalize its approach and to bring physical insights into Hooke's law coefficients as composite material constants incorporating multiple mechanisms. Special and idealized models are introduced, which provide additional insight into these mechanisms, and lead to bounds for material constants. Laboratory setups that can measure not only the bulk material but also the micromechanical constants are demonstrated.

Chapter 4 gives an even more rigorous examination of the poroelasticity theory, by applying the variational energy minimization principle to the composite material averaging theory. A product of this analysis is a set of intrinsic material constants directly associated with the three fundamental deformation modes of a porous solid: a geometrically similar and porosity-preserving volumetric deformation, a solid shape- and volume-preserving particle rearrangement induced pore space change, and a solid volume-preserving shape change induced pore change. This model gives a clear interpretation of the often used microhomogeneity and microisotropy (ideal porous medium) assumptions. Links between the intrinsic and the micromechanical constants are established. Limiting material behaviors are examined using the asymptotic values of these constants. Laboratory technique that can measure these intrinsic material constants is discussed.

Chapter 5 gives an examination of the material anisotropy, at both the bulk continuum and the micromechanical level. The generalized models are reduced to various models with special symmetry, such as orthotropy, and transverse isotropy. Under the ideal porous medium assumption, the micromechanical analysis shows that not all the bulk continuum general anisotropic material constants are independent to each other. The introduction of one micromechanical constant of solid bulk modulus causes the reduction of six bulk continuum constants, bringing a constraint to the continuum model that cannot be obtained at the bulk continuum level.

All the theories presented so far are constitutive laws that relate the externally applied forces to the deformation of a body. Chapter 6 lays out other physical laws that are required for the modeling of poroelasticity, including the force equilibrium, the fluid mass conservation, and Darcy's law for fluid flow. These laws are assembled into a set of governing equations for mathematical solution. Various physical contact conditions leading to the mathematical boundary conditions are discussed. Provided with a proper set of initial and boundary conditions, a well-posed initial and boundary value problem can be formed, which guarantees the existence of a unique mathematical solution. For the convenience of mathematical solution, several methodologies that reduce the physical variables, such as displacements, stresses, and pore pressure, to a set of "potentials," known as stress and displacement functions, are presented.

Chapter 7 derives a set of analytical solutions, ranging from the one-dimensional geometry to axial and spherical symmetry and from plane strain to generalized plane strain conditions. These simplified problem geometries allow the analytical solutions to be found. A range of problems, many with important practical applications, including soil consolidation, seabed deformation and mass exchange in response to water wave, stability of excavated borehole, in situ stress determination, drill core retrieval, burst of hollow cylinder, and other problems, are investigated.

Chapter 8 presents the singular solutions due to concentrated loads, such as point force, dislocation, fluid source and dipole, etc., in infinite space, also known as fundamental solutions or free space Green's functions. A closely related subject, the singular integral equations of various forms, particularly those that represent the solution of the governing equations, such as the Somigliana, the stress discontinuity, and the displacement discontinuity integral equations, is also presented.

The dynamic problems are addressed in Chap. 9 as poroelastodynamics. The variational Lagrangian formulation is utilized to interpret the inertia and added mass effects. The frequency-dependent dynamic permeability is examined based on conceptual models. Governing equations are laid out in the time and the frequency domain. The wave propagation phenomenon and the phase velocities and attenuation are analyzed, based on the Biot and the extended models, such as the squirt flow model. One-dimensional analytical solutions and solutions of plane wave reflection and refraction on free surface and water and porous medium interface are examined. Fundamental solutions and singular integral equation representation are also derived.

Chapter 10 presents the viscoelastic effects. A brief review of viscoelasticity and the correspondence principle set the stage for the generalization to poroviscoelasticity. In a physically consistent manner, the spring-dashpot models are applied to the micromechanical and intrinsic material constants, which lead to the non-monotonic time evolution behavior of some bulk continuum material constants. Borehole and cylinder problems are examined based on the poroviscoelastic constitutive behaviors. The theory is also extended to poroviscoelastodynamics.

In Chap. 11 the thermal effect is taken into consideration and the theory is extended to porothermoelasticity. The constitutive and governing equations are

derived based on the rigor of the first and second laws of thermodynamics. The irreversible processes of nonequilibrium thermodynamics and transport laws are constructed based on Onsager's reciprocal principle. The fully coupled and partially coupled porothermoelasticity theories are presented. Analytical solutions of one-dimensional geometry, and with axial and spherical symmetry, as well as the fundamental solutions, are derived.

Finally, in Chap. 12 the chemical effect is incorporated to present the theory of porochemoelasticity. The electrochemical effect is introduced as Gibbs chemical potential and free energy. The porous material is considered in three phases, the solid, the fluid, and the bound fluid absorbed and adsorbed to the solid. The chemical effects of the two fluid phases are separately modeled; mechanically, however, the bound fluid is considered as a part of the solid phase. Onsager's reciprocal principle is again used to construct the transport laws. Analytical solutions are provided.

The book also contains extensive appendices, which include a biography section giving biographical sketches of pioneers on whose shoulders the theory of poroelasticity is built.

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