Chapter 2
Ibn Sīnā and the Reinvention of Epistemology

2.1 Ibn Sīnā by Himself

Abū ʿAlī al-Ḥusayn Ibn ʿAbd Allāh Ibn Al-Ḥasan ibn ʻAli Ibn Sīnā was born in 980 CE/370 AH in his mother's home village of Afshana near Bukhārā, the capital of the Sāmānid dynasty in central Asia (present day Uzbekistan). He came from a middle class family. His father ʿAbd Allāh was from Balkh, an important town of the Sāmānid empire in what is today Afghanistan, but moved to Bukhārā to serve as a governor of a nearby village. A few years after the birth of his brother, Ibn Sīnā's family settled in the capital Bukhārā where he received his entire education. He died in 1037 CE/428 AH in Hamadhān, Iran. After introducing himself very briefly in his autobiography, Abū ʿAlī spells out the course of his education in the intellectual capital by dividing it into two main stages. The first stage was a basic education consisting of non-Greek or Arabic-Islamic sciences: Qur'ān, Arabic literature, fiqh or Islamic law and jurisprudence including *al-jadal*, arithmetic and algebra.[1]

> A teacher of the Qur'ān and a teacher of literature were provided for me, and when I reached the age of ten I had finished the Qurān and many works of literature, so that people were greatly amazed at me. […] Then he [my father] sent me to a vegetable seller who used Indian arithmetic, I was thus learning from him. […] Before his [al-Nātilī] arrival I

---

[1]Ibn Sīnā tells us later that he has mastered the mathematical sciences, which means that he also learned algebra, as evidenced by his mathematical works. As will be explained later, many bio-bibliographers such as al-Bayhaqī are right when they point out:

> When he was ten years old, he knew certain fundamental texts of literature by heart. His father was studying and reflecting upon an opuscule of the Brothers of the Purity. He also reflected over it. His father took him to a greengrocer named Maḥmūd al-Massāḥ who knew Indian calculation and algebra and al-muqābala (Al-Bayhaqī 1946, p. 53).

Ibn al-ʿImād, who quotes Ibn Khallikān, gives this biographical anecdote in similar words: "When he was ten years old, he improved his knowledge in the science of the Glorious Qur'ān, literature, and he knew certain religious foundations by heart, Indian calculation and algebra and al-muqābala" (Ibn al-ʿImād n.d., III, p. 234; see also Ibn Khallikān 1969, II, pp. 157–158).

H. Tahiri, *Mathematics and the Mind*,
SpringerBriefs in Philosophy, DOI 10.1007/978-3-319-25238-4_2

had devoted myself to jurisprudence with frequent visits to Ismāʿīl al-Zāhid about it. I was one of the most agile questioners, having become acquainted with the methods of request and the modes of interception to the respondent (طرق المطالبة و وجوه الإعتراض على المجيب)[2] in

[2]It is particularly interesting that Ibn Sīnā provides these kinds of details in his short autobiography for it shows that he was given advanced training in legal disputation by Ismāʿīl al-Zāhid (d. 1012) who was a ḥanafī jurist. How advanced was it? This is indicated by the expression: the methods of *al-muṭālaba* and the modes of *al-iʿtirāḍ*. Most scholars have struggled to translate *al-muṭālaba* (المطالبة) and *al-iʿtirāḍ* (الاعتراض), and Gohlman rendered them as prosecution and rebuttal, respectively, which is not accurate. The inaccurate translations are due to the fact that the translators seemed to be puzzled by the literal meaning of the words. Literally, *muṭālaba* means asking, but not in the sense of asking questions but of asking for or requesting something and hence is closer to requesting. Furthermore, *iʿtirāḍ* means interception. Literally then, the sentence should be translated as methods of request and modes of interception. But, request what? And interception of what? The literal meaning seems puzzling but, in fact, it is not so; it only sounds incomplete because the two terms do not have the same technical meaning in English as in Arabic. In the *jadal* literature, they are just two from the arsenal of rules and argumentation techniques that can be used by the questioner (*al-sa'il*) to attack the respondent (*al-mujīb*), and their use is governed by the rules of the argumentation procedure. As very specific terms of the new logical science, it is not sufficient to suggest the complete translation of the two concepts without explaining their meaning. By the same token, it will become clear why Ibn Sīnā mentioned just these two technical words. I will very briefly explain the underlying idea of *jadal* in 11th century Islamic jurisprudence, which would have been the kind of *jadal* roughly practiced in Ibn Sīnā's time (for an extensive analysis of the state of *jadal* in this period, see Miller 1985, Chap. III, p. 87). My presentation is based on the discussions of the two prominent jurists Abū al-Walīd al-Bājī's *Kitāb al-minhāj fī tartīb al-ḥijāj* and Imām al-Ḥaramain al-Juwainī's *al-Kāfiya fī al-jadal*, the teacher of al-Ghazālī. The argumentation procedure is comprised of two stages. First stage: understanding the point of the dispute. This is the aim of the following three moves that should be introduced in the following order: (1) What is your opinion? (the whatness of the opinion); (2) What is your evidence?" (the whatness of the evidence); (3) *Muṭālaba bi-tabyīni ad-dalīl* (request for clarification of the evidence). This third move is introduced by al-Juwainī, who defined it as "challenging the opponent to clarify his proof (مؤاخذة الخصم بتبين الحجة)" (al-Juwainī 1979, p. 68), and is of two types: (*i*) request for clarifying the basis (*aṣl*) of proof (*dalāla*) and its establishment and (*ii*) request for clarifying the mode of proof (*wajh ad-dalāla*)." (ibid.). Al-Bājī calls this third move *muṭālaba bit-taṣḥīḥ* or request for verification of the proof (al-Bājī 1978, pp. 40–41), which illuminates al-Juwainī's label since the request for clarification is aimed at verifying that such and such is the case. We choose this way of representing the third move because it gives us a better idea of how to characterize this entire phase, for it turns out that the aim of the three successive questions is to make sure that the opponent understands the actual position of the proponent. While keeping *muṭālaba bi-tabyīni ad-dalīl* (*muṭālaba* can in fact be dropped) for the third move, the three questions can be grouped under the general name *Muṭālaba bit-taṣḥīḥ* or *Muṭālaba bit-taḥqīq* which can then be translated as a Request to establish the facts. Only after making sure that the questioner understands the respondent's thesis and the evidence on which it is based can he begin the process of its refutation. But, as pointed out by al-Juwainī, the questioner can go directly to the refutation stage if the respondent's thesis and the evidence on which it is based are widely known to him (al-Juwainī 1979, pp. 79–80). Second stage: the refutation procedure. The principle of *jadal* is what al-Juwainī calls *al-ilzām bil-muqābala* or the necessity for exchanging arguments, i.e. the respondent is obliged to answer all legal attacks on his evidence. The questioner is then provided with all kinds of techniques that can be used to defeat the evidence of his opponent. *Iʿtirāḍ* is just one of these techniques and is defined by al-Juwainī as follows: "opposing (*muqābala*) the opponent's claim by an argument that prevents him from attaining his goal; and it is said: preventing (*mumāna*ʿa) the opponent from proving his point by opposing an argument of equal probative force

> the manner which the practitioners of it [jurisprudence] follow.[3] (Gholman 1974, pp. 19–21)

In the second stage, he was introduced to Greek science and philosophy by another teacher, al-Nātilī, in the following order as mentioned by him—logic beginning with the book of *Isagoge,* the book of Euclid*, The Almagest*—before he went on to study natural sciences (الطبيعيات), metaphysics (الإلهيات) and medicine by himself.

This rapid survey shows that Abū ʿAlī received a much richer interdisciplinary education than that which the Arabic-to-Greek tradition wants us to believe, and the Aristotelian corpus does not constitute the basis of his education as this latter is dominated by the mathematical sciences, mastery of which is unusual in the formation of a traditional philosopher. Is this the case by chance? By no means, as we shall see. What follows represents what seems to be a significant development in Ibn Sīnā's thought, which has had major implications for philosophy. After "mastering the logical, natural and mathematical sciences", we are surprised by the young prodigy's bold admission that he simply could not make sense of what a great classicist calls "the Mount Everest of Aristotle's treatises"[4]:

(Footnote 2 continued)

(*bi-musāwātihi*)". (ibid., p. 67). He justifies his definition by recalling that it is close to its intuitive meaning (لغة): someone intercepts me (*iʿtaraḍanī*) on my way means he prevents me (*manaʿanī*) from following it. After providing a second example, he concludes: "since the opponent prevents the proponent from reaching his goal by opposing an argument with equal probative force, it is called interception in close proximity to the natural language." (ibid.). Literally then, it can be translated as interception of the proof (of the thesis). More generally, *iʿtirāḍ* represents the kind of arguments that attack the rules of inference used by the proponent to establish the conclusion of his claim. In our modern argumentation theory, some formal systems term this kind of attacks as undercutting an inference (Prakken 2002, p. 231). Al-Bājī stresses its importance by describing *iʿtirāḍ* as the argument that attacks "the heart of the evidence (*nafs ad-dalīl*) rendering it void (*bimā yubṭiluhu*)" (al-Bājī 1978, p. 41) and, by this, he means the internal structure of the evidence. He discusses fifteen types of *iʿtirāḍ*, whereas al-Juwainī examines nine. With just two keywords, Ibn Sīnā succeeds in capturing the essence of *jadal* as practiced in the eleventh century. Ibn Sīnā seems to be proud of his courses in Islamic sciences and it appears that he was a formidable opponent. His excellent training in law, jurisprudence and *jadal* was thus very important in his intellectual formation as they contributed to sharpening his philosophical investigations as evidenced in his refutation of the Greeks' conception of number. His refutation of Aristotles' *Metaphysics* (books M and N) in particular confirms that he learned one of the basic lessons of *jadal* of the jurists, i.e. any successful refutation should be preceded by a comprehensive understanding of the opponent's theory. This is just one of many examples, and there are undoubtedly others that show the knowledge of the two powerful traditions, juridical and *kalām*, are essential to understanding the development of his philosophical thought and of the history of Arabic philosophy in general.

[3]The pagination refers to Gohlman's parallel Arabic-English text.

[4]Ross (1995, p. ix).

> I read the *Metaphysics*, but I could not comprehend its content, and the aim of its author [واضعه][5] was confused to me, to the point I reread it forty times and ended up having it memorized. In spite of this I still did not understand it nor what was intended by it, and I despaired of myself and said, "there is no way of understanding this book." (ibid., pp. 32–33)

It looks as if Ibn Sīnā's exceptional correct guessing failed him time and again in attempting to penetrate, let alone recover, Aristotle's most cherished heritage. Why was he unable to understand the content of *Metaphysics* despite mastering the new sciences of his time? Why does *Metaphysics* above all other works prove to be so recalcitrant? And what more does he need to learn to enable him to understand the metaphysics of the ancients? The precocious teenager was probably not aware that he was attacking the founder of the powerful Aristotelian tradition by declaring that not only was he unable to understand the content of one of his best works, but even that "the aim of its author [واضعه] was confused to me".

This was not the first time that Ibn Sīnā experienced difficulties with metaphysics. He tells us that he first heard about it when he was a child:

> My father was one of those who responded to the call of the Egyptians and was considered one of the Ismā'īliyya. From them, he, as well as my brother, heard the account of the soul and the intellect in the special manner in which they speak about it and know it. Sometimes they used to discuss this among themselves while I was listening to them and understanding what they were saying, but my soul would not accept it, and so they began inviting me to respond to it. They were also frequently talking about philosophy, geometry and Indian arithmetic. Moreover he [my father] used to send me to a vegetable seller who used Indian arithmetic, I was thus learning from him. (ibid., pp. 18–21)

This is an extremely important passage as it gives us an idea about the education of his time and the kind of topics discussed in his days that significantly impacted his education. He specifically tells us that he was not satisfied with the doctrines circulating in his milieu about the soul and the intellect. And he further points out that it is in the context of the discussion on the relationship between philosophy and mathematics that his father took special care to provide his brilliant son with some basic mathematical training. But why mathematics first? And how can we explain that he was taught two fundamental mathematical disciplines, unknown to Greek mathematicians and philosophers, by just a greengrocer? Widespread mathematical knowledge was symptomatic of the social revolution brought about by the mathematical revolution. The development of arithmetic and algebra greatly contributed to raising the educational standard of society, which became aware of the necessity of mastering mathematics in order to reach the top of the social hierarchy. Since al-Khwārizmī, algebra was widely taught and used as a practical method of arithmetical problem-solving by civil society including jurists, engineers, secretaries, merchants, surveyors and accountants. As a result, middle class families, and even

[5]Ibn Sīnā had the opportunity to mention Aristotle by name in this passage but did not and, in fact, the latter was not mentioned by his name let alone by his title in his entire autobiography. It is also noteworthy that he sharply rebukes his disciple al-Juzjānī when he "asked him to comment on the works of Aristotle, but he said that he was not free to do so." (ibid., p. 55)

modest ones, were very anxious that their children be taught mathematics, mainly arithmetic and algebra, to ensure a better future for them. A new generation emerged in which knowledge of arithmetic and algebra were part of their culture and the basis of their intellectual formation. The job of the philosopher was consequently transformed, as philosophy could not be practiced as it was because of the emergence of the new sciences and in particular mathematics due to its distinctive epistemic status.[6] Like his predecessors, Ibn Sīnā was part of this new class of philosophers who were first taught the new mathematics before ancient philosophy, and it would only be a matter of time before tensions between the old inherited and newly created conception of knowledge would surface. The education of Ibn Sīnā simply reflects the new scientific and social context that arose in the eighth century from the three innovative disciplines of law, linguistics and mathematics. It is because he mastered the new mathematics that he was able, as he tells us in his autobiography, to occupy important jobs like financial administrative posts. A little-known fact is that he also worked for a while as a jurist:

> I assumed some post in the financial administration of the Sultan. But necessity then led me to abandon Bukhārā and move to Gurgānj, where Abū al-Ḥusayn al-Suhaylī, a lover of the sciences (المحب لهذه العلوم), was a minister. I was presented to the Amīr there, 'Ali Ibn Ma'mun; at that time I was in lawyer's dress (بزي الفقهاء), with a fold of the mantle under my chin. They gave me a monthly salary which provided enough for someone like me. (ibid., pp. 40–41)

It is this highly educated and prosperous society that identified itself with knowledge which provided Ibn Sīnā with the topic of his future project and enabled him to master all the known sciences. In his case, it is his father, a very religious man, who encouraged his prodigy to learn more[7] about the soul and the intellect and had the remarkable insight that mathematics could be very useful in helping clarify his thought regarding such complex religious philosophical issues. What Ibn Sīnā's father did not know is that he was preparing his beloved son for an ambitious project that would transform philosophy using mathematics. This sounds intriguing in many respects, for how can mathematics clarify strange concepts such as the soul and the intellect? How could such clarification lead to the transformation of philosophy? And what does mathematics have to do with the soul and the intellect in the first place? To his amazement, some of the questions that embroiled the young Ibn Sīnā were already being tackled by one of his predecessors who belonged to the same class of new philosophers.

---

[6]Besides the practical benefit of mathematics, its theoretical usefulness was also recognised and very much appreciated. All the philosophical disciplines, including Aristotelian logic, were subject to bitter controversy among the various traditions. Mathematics, on the other hand, was highly respected by society as a whole and unanimously regarded as a model of rigour, certainty and rationality.

[7]According to one of the best known quranic verses: "Say: My Lord increase me in knowledge و قل رب زدني علما " (20: 114).

2.2 Ibn Sīnā Rediscovers al-Fārābī: The Man Whom He Wishes to Meet!

In his aforementioned significant paper in which he shows how philosophy was shaped by the new mathematics, Rashed establishes the link and the relevance of al-Fārābī's mathematical investigations to those of Ibn Sīnā. However, his presentation seems to be so condensed that most scholars have overlooked his important findings. What follows is an elaboration of his profound insights.

2.2.1 From Metaphysics to Mathematical Knowledge

Al-Fārābī's works were ignored for decades by the Baghdad Peripatetics, who dominated the philosophical scene from the ninth century, until he was rediscovered by chance by Ibn Sīnā who made him famous by immediately recognising his originality.[8] The discovery of al-Fārābī by Ibn Sīnā represents one of the sensational moments in the history of philosophy.

[8]To understand what seems to be a curious attempt to transplant the domination of the same old tradition into a different socio-epistemic context as if nothing had changed whatsoever, we need to understand better the context of the translation of Aristotelian works, and the way they were introduced, presented and used, which is as important as the works themselves because of its important consequences. For philosophy to emerge as an independent tradition in a knowledge-based society and survive in the highly competitive intellectual milieu, it needed to rely on a strong figure. The best candidate to serve this purpose was Aristotle because of his historical influence in the Greek tradition, although it seems this alone was not enough. To face the formidable challenges posed by the dynamic of the new cultural environment, the philosophical tradition not only found itself systematically dependent on the thought of the ancient philosopher to the extent that all his known works were translated, it also felt the need to get support from the parallel emergence of a secondary literature which erected the Stagirite to an almost infallible authority. The so-called wisdom (*al-ḥikma*) tradition, which seemed to be initiated by translators of the Greek philosophical works such as the influential Ḥunayn ibn Isḥaq (810–873), appears to have been mainly designed for public consumption since they were written in the form of collections of ethical sayings and anecdotes attributed to past philosophers. It was an attempt to strengthen and enhance the position of the philosophical tradition in the intellectual scene by showing that the ancient Greek philosophers also shared the same values as those of the Arabic Islamic society. Some of these authors aimed to achieve more by claiming that philosophy was completed by what they call the first teacher who should continue to be eternally followed, as eloquently reported by an unknown scribe:

> Aristotle is the first teacher, the seal of the ancient philosophers (*al-ḥukamā'* الحكماء) and the model of the learned people who followed their path. He organised wisdom and established it, he refined philosophy and wrote it down exactly; it is he who put logic at the top and set the foundation for all the rest of sciences. He thus became the medium through which the ancients were to benefit the moderns, and the means by which the successors were to acquire the benefits of the ancestors. No sooner had he content himself to pour upon the latter generation what the former one had caught than he added to every kind of knowledge many times what they had come up with (أضاف إلى كل نوع أضعاف ما أتوا به), bringing it thereby to completion and perfection (فصيره بذلك أتم و أكمل). (ms. Arabe 202, f. 29r)

> But one day in the afternoon when I was at the booksellers' quarter, a salesman approached with a book in his hand which he was calling out for sale. He offered it to me, but I refused it with disgust, believing that there was no benefit in this science. But he said to me, "buy it, because its owner needs the money and so it is cheap. I will sell it to you for three *dirhams*." So I bought it and it turned out to be Abū Naṣr al-Fārābī's book *Fī Aghrāḍ kitāb mā ba'da aṭ-ṭabī'a* (*On the Aims of the Metaphysics*.) I returned home and was quick to read it, and instantly the aims of that book became clear to me so much so I had it memorized by heart. I rejoiced at this and the next day gave much in alms to the poor in gratitude to God Exalted. (ibid., pp. 32–35)

The Aims of the Metaphysics seems to be the first work of al-Fārābī which Ibn Sīnā read; a four page review in which al-Fārābī confirms the difficulty of Aristotle's work.

> None of the ancient philosophers has properly commented on this book as they have done of his [other] books, but what can be found at most is an incomplete commentary on chapter Lambda by Alexander [of Aphrodisias] and a complete one by Themistius. As for the other chapters, either they have not been commented upon or no commentary has

---

(Footnote 8 continued)

This passage taken from an Arabic manuscript entitled *Al-Kalimāt al-Mukhtāra min Kalām al-Ḥukamā' al-Arba'a al-Mutaqaddimīn* (preserved in La Bibliothèque Nationale de France and archived as Arabe 202 f.16–36) seems to be a copy of one of the earliest writings in the genre since it presents, as the title indicates, a selection of sayings of only four ancient Greek philosophers: Pythagoras, Socrates, Plato and Aristotle. Besides being the first teacher and the classifier of all sciences, another anecdote, which is currently little known but widely reported at the time in the wisdom literature, identifies Aristotle with Intellect itself (العقل!). As a result philosophy as a complete and closed corpus came to be identified with Aristotle. It seems then that the aim of the project of translating the philosophical works was to perpetuate an old tradition by continuing to generate a series of generations of Aristotelians. This inherited traditional approach to philosophy, which was at odds with the dynamic scientific context in which the translation took place, would, sooner rather than later, inevitably lead to an internal struggle with the rise of a new philosophical movement led by al-Kindī aimed at grounding philosophy in the latest advancements of knowledge. However, the emergence of a generation of new philosophers such as al-Fārābī, who was working in that Aristotelian circle, would find it difficult to openly criticize the first teacher. Even a scholar such as Badawi, who edited Ḥunayn's *Ādāb al-falāsifa* (*The Ethic of the Philosophers*), noted that he had distanced himself from the Aristotelian tradition that tended to make its founder the teacher that could never be surpassed:

> There are some similarities between what al-Fārābī wrote about the school of the philosophers and what Ḥunayn mentioned in the beginning of his book but it seems that al-Fārābī' sources are different from those of Ḥunayn because of the clear differences between their discourse; differences that indicate that al-Fārābī did not base his reports on Ḥunyan. (in Ḥunayn 1985, p. 27)

To challenge the Aristotelians, the philosopher of Bagdad then had to innovate to show them that their teacher was far from having the last word on every topic. His strategy, as we shall see, consisted of expressing his significant non-Aristotelian ideas without making explicit his criticism of Aristotle's views. This was the strategy, followed later by Ibn Sīnā, that would trigger the long but unstoppable process of change.

survived to our times—since upon examining the books of the later Peripatetics,[9] it may be assumed that Alexander did comment on the entire book.[10] (Al-Fārābī 1890, p. 34)

As a result, Aristotelians reduce metaphysics to theology by ignoring topics they find hard to understand:

Many people have the preconceived notion that the purport of this book and its content is to discuss the Creator (may He be glorified and exalted), the intellect, the soul, and other related topics, and that the science of metaphysics and the science of the uniqueness of God (علم التوحيد) are one and the same thing. Consequently, we find that most people who study it are perplexed and misguided by it, since we find that most of the talk in it is devoid of any such aim, or rather, we find that the only talk specifically related to this aim is that in the eleventh chapter, that is, the one designated by the letter Lambda.[11] (ibid., p. 34)

Hence his "intention (قصدنا) in this treatise is to indicate the aim that is contained in the book of Aristotle known as *Metaphysics* and the primary divisions (الأقسام الأولى) which it has." (ibid., p. 34). If theology is not the sole subject of *Metaphysics* but only one of its chapters, what are the other chapters about? Answering this question was one of al-Fārābī's aims and the difficult task was to clearly identify the non-theological parts of metaphysics. The approach he adopted was to try to point out in what way the latter differed from two closer prominent theoretical disciplines, i.e. natural philosophy and mathematics. It seems as if the constitution of metaphysics as an independent discipline depends on some kind of metatheory that distinguishes and classifies the various scientific disciplines according to their subject matter. This task is entrusted to what seems to be the powerful metatheoretical discipline that remains implicit in the Aristotelian system, i.e. the so-called classification of the sciences. Through primary divisions, al-Fārābī means to provide the main distinction between the three theoretical disciplines of natural philosophy, mathematics and metaphysics by comparing their subject matter. In doing so, he was led to characterising mathematics in a rather unusual fashion.

Although mathematics is higher than natural science—since its objects are abstracted from matter—it most certainly should not be called the science of metaphysics because its

---

[9]In this interesting passage, al-Fārābī shows us that not only was he fully aware of the domination of the Aristotelian tradition since he speaks of "the later Peripatetics" but, more importantly, of its long stagnation since philosophy had made little progress since its founding; a clear indication that he did not consider himself as one of them, as confirmed by Ibn Sīnā:

As for Abū Naṣr al-Fārābī, we must have a very high opinion of him and he should not be put on the same group of people (و لا يُجْرى مع القوم في ميدان) for he is all but the most excellent of our predecessors. May god facilitate the meeting with him (لعل الله يُسَهِّل الالتقاء معه), so it shall be useful and beneficial. (in Badawi 1978, p. 122).

[10]لا يوجد للقدماء كلام في شرح هذا الكتاب على وجهه كما هو لسائر الكتب بل إن وجد فلمقالة اللام للإسكندر غير تام و لثامسطيوس تاما. و أما المقالات الأخر فإما لم تشرح و إما ان لم تبق إلى زماننا على أنه يظن إذا نظر في كتب المتأخرين من المشائين أن الإسكندر كان قد فسر الكتاب على التمام.

[11]The pagination refers to the Arabic text edited by F. Dieterici in *al-Fārābī's Philosophische Abhandlungen*, 1890.

> objects are abstracted from matter only by [*human] imagination*, not in existence.[12] (ibid., p. 36)

The emphasised sentence exemplifies the originality of al-Fārābī for in it he characterises the specificity of mathematics in a way which cannot be found in Aristotle's *Metaphysics*. In his effort to distinguish the subject matter of metaphysics from that of mathematics, al-Fārābī argues that while the objects of both disciplines are abstracted entities, mathematical objects are abstracted from sensible objects as a result of a specific operation of the mind. By making mathematics a product of the mind, al-Fārābī's analysis illuminates Ibn Sīnā's thought on the epistemic role of the soul in the production of knowledge. This distinction between metaphysics and mathematics had far-reaching implications for it motivated the philosopher of Bagdad to provide his own revolutionary classification of the sciences.[13]

2.2.2 *Algebra: The New Universal Science*

In *al-Aghrāḍ*, al-Fārābī demonstrates that he perfectly grasped Aristotle's *Metaphysics*, to the extent that one would expect him to have followed the latter's classification only to see him instead abandoning it altogether in his *Iḥsā' al-'ulūm* or *The Enumeration of the Sciences*, a book in which he drastically overhauls the classification of the sciences.[14] He ignores Aristotle's two major principles of classification. The first is the sharp theoretical/practical dichotomy which seems to be modelled on the value-based Greek social system in which purely theoretical studies are highly valued at the expense of practical arts which are viewed with disdain.[15] Fully aware of the new society which rather divides itself into

---

[12] و العلم التعاليمي و إن كان أعلى من علم الطبيعة إذ كانت موضوعاته متجردة عن المواد فليس ينبغي أن يسمّى علم ما بعد الطبيعة لأن تجرد موضوعاته عن المواد وهمي لاوجودي.

[13] Al-Fārābī of course also expressed his original ideas elsewhere other than in his *Aghrāḍ* and Ibn Sīnā certainly read whatever of al-Fārābī's work he could find once he had discovered his first work. To show precisely that it was the new mathematics that led to the discovery of a new philosophical approach, we only present his views about some important questions closely connected to *al-Aghrāḍ*, leaving for future research a systematic study of his theory of knowledge and the extent of its influence on Ibn Sīnā.

[14] It seems so revolutionary that posterity has added little to it as pointed out by one of his successors Sa'īd ibn Aḥmad al-Qurṭubī (d. 1070): "He [al-Fārābī] composed a noble work [the *Iḥsā' al-'ulūm*] in which he enumerated the sciences and indicated the object of each; this treatise, the like of which had never before been composed and the plan of which had never been adopted by any other author, is an indispensable guide to students in the sciences." (in Farmer 1932, p. 561).

[15] Greek society was a slave-based society. The small wealthy intellectual elite did not work and did not need to work and so that they could devote their time to leisure in gymnasiums and museums like the Academy and the Lyceum. Aristotle famously justifies the established social class system by speculating that it was necessary for the emergence of the theoretical sciences such

khaṣṣa or elite and *'amma* or common class, it is not surprising that al-Fārābī adopted criteria forged by the new epistemic context of his time which did not separate theory from action through a new understanding of usefulness. Due to the strong social demand for knowledge, education became a profitable market, attracting many people to the teaching sector. And to help parents provide the best education for their children and identify the pseudo-experts in their subject domain, al-Fārābī established a modern detailed programme according to the most basic knowledge that should be acquired by any student.

> What is contained in this book will be useful to the man if he wants to learn some science among these sciences and examines it, he will know what he will undertake, what he will examine, in what way his examination will benefit him, what benefit it has and by what virtue it will be attained, so that he will be able to engage in what he will undertake in the sciences with knowledge and insight and not blindness and deception. With this book, a person can compare the sciences, he will then know which of them is the best, the most useful; the most perfect, reliable and powerful, and which of them the feeblest, the frailest and the weakest. It will also be useful in finding out he who claims he is familiar with some science among these sciences while it is not the case.[16] (Al-Fārābī 1996, p. 16)

He then classified the sciences into five chapters[17]:

(Footnote 15 continued)

as geometry: "The mathematical arts first took shape in Egypt for there the priestly caste was privileged to have leisure" (*Metaphysics* A. 1. 981b). In stark contrast, the development of science in the Arabic tradition was based on a different model as it was financed by public-private mixed funding. Scientists of the famous institution *bayt al-ḥikma* for example were financed by the political body, other scientists such as astronomers were financed by *al-waqf*, an Islamic-specific mode of financing. The third class of intellectuals were able to live from their own works due to the strong demand for knowledge from a prosperous and well-educated society. Ibn Sīnā, who came from a modest family, was one of them and, as he tells us in his autobiography, his career began at an early age (21 years old) when he began composing a wide variety of books commissioned by middle class people who were willing to pay to get the latest advancements in knowledge before joining the top elite. This socio-epistemic model ensured two kinds of interaction: between the intellectuals themselves who belonged to various traditions, and between the intellectual elite and the rest of society. This is an entirely different social and cultural setting than what we find in Ancient Greece.

[16] وينتفع بما في هذا الكتاب الانسان إذا أراد أن يتعلم علما من هذه العلوم وينظر فيه، علم على ماذا يقدم، و في ماذا ينظر، وأي شيء سيفيده نظره، وما غناء ذلك، وأية فضيلة تنال به، ليكون إقدامه على ما يقدم عليه من العلوم على معرفة وبصيرة، لا على عمى و غرر. وبهذا الكتاب يقدر الإنسان على أن يقيس بين العلوم ، فيعلم أيها الأفضل ، وأيها أنفع ، وأيها تقن وأوثق وأقوى ، وأيها أوهن و أوهى و أضعف. و ينتفع به أيضا في تكشف من ادعى البصر بعلم من هذه العلوم ولم يكن كذلك.

[17] This is the new programme that al-Fārābī proposes to reform philosophical studies. Here is not the place to discuss the modernity of his classification. Sufficient for our purpose is to make some important remarks that are symptomatic of the reinvention of knowledge brought about by the surge in social disciplines since the eighth century. The first is the representation of three powerful traditions of his time, i.e. the linguistics, legal and *kalām* traditions. Logic is separated from natural philosophy and metaphysics and placed at the intersection between linguistics and mathematics. It is particularly interesting to note that the new philosopher relegates the two main traditional philosophical disciplines of natural philosophy and metaphysics to fourth place while linguistics comes first. At least two reasons that he simply could not ignore explain this decision, which confirms once again that the philosopher was a thinker of his time. His detailed treatment shows how far linguistics had developed since the eighth century to become one of the most advanced

The first chapter on the science of language and its parts (في علم اللسان و أجزائه);
The second chapter on the science of logic and its parts (في علم المنطق و أجزائه);
The third chapter on the science of mathematics and its parts, comprising the science of number, geometry, optics and astronomy, the science of music, the science of weights and the science of "ingenious devices" (to this we shall return later);
(في علوم التعاليم و أجزائه و هي: العدد، والهندسة، وعلم المناظر، و علم النجوم التعليمي، وعلم الموسيقى، وعلم الأثقال، و علوم الحيل)
The fourth chapter on the science of nature and its parts and on theology and its parts;
(في العلم الطبيعي و أجزائه، و في العلم الإلهي و أجزائه)
The fifth chapter on the science of politics and its parts, on the science of jurisprudence, and the science of *kalām* (في العلم المدني وأجزائه، و في علم الفقه، و علم الكلام).

As for the classification of the theoretical disciplines, al-Fārābī ignores Aristotelian criteria of motion and separation from matter. We will examine his classification of just two of the mathematical sciences he enumerated: the science of number and the science of ingenious devices. He tells us that the science of number is made up of two branches: the science of practical number and the science of theoretical number and begins by defining them as follows:

> The science of practical number examines the numbers inasmuch as they are accountable numbers that needs to determine the number of bodies and similar things such as men, horses, dinars, dirhams or other things which are accountable. They are the numbers that people use in merchant transactions and civil life.[18]
>
> As for the science of theoretical number, it examines the numbers *absolutely* as abstracted from the bodies *in the mind* (مجردة في الذهن), and from *all* what is counted. It examines them *purified* (ملخصة) from all what could be counted with by sensible things. And this is the science which is involved in *all* the sciences (جملة العلوم).[19] (ibid. p. 50, my emphasis)

(Footnote 17 continued)

scientific disciplines as shown by the sheer number of volumes on the Arabic language (series of monographs, dictionaries and encyclopedias) produced by grammarians, lexicographers, litterateurs, etc. Even today, research on the Arabic language is one of the most dynamic and fascinating lines of research on the Arabic tradition. Linguistics not only developed before philosophy but, more importantly, it has greatly influenced the latter by making philosophers aware of the importance and relevance of language analysis in philosophical studies; a fact that was fully grasped by al-Fārābī in the composition of his books such as *Kitāb al-ḥurūf* which, in fact, should rather be called the book of meaning since meaning (الدلالة), an important juridical notion, is its central topic. Linguistics also offers a rich vocabulary that unifies all kinds of knowledge, making it de facto the first lingua franca for centuries to come.

18 فالعملي يفحص عن الاعداد من حيث هى أعداد معدودات تحتاج إلى أن يضبط عددها من أجسام و غيرها، مثل الرجال أو أفراس أو دنانير أو دراهم أو غير ذلك من الأشياء ذوات العدد، و هي التي يتعطاها الجمهور في المعاملات السوقية و المعاملات المدنية.

19 وأما النظري فانه يفحص عن الاعداد بإطلاق على أنها مجردة في الذهن من الأجسام ، وعن كل معدود منها، وإنما ينظر فيها ملخصة عن كل ما يمكن أن تعد بها من ا لمحسوسات، ومن جهة ما يعم جميع الأعداد التي هي أعداد محسوسات؛ وهذا هو الذي يدخل في جملة العلوم.

For al-Fārābī, whose appreciation of arithmetic is reminiscent of that of the nineteenth century German mathematician Gauss (1777–1855), pure number theory is queen of the sciences. In his next paragraph, he explains that the science of theoretical number involves studying numbers as a structure by examining their properties, such as the property of being even or odd, their relationships and the various operations that can be performed on them. In short, what al-Fārābī calls the science of theoretical number is simply number theory.[20] This distinction is of paramount importance for two reasons: firstly, in itself because of its modernity and, secondly,

[20]Al-Fārābī does not put arithmetic and geometry on the same epistemic level since his description of geometric objects is substantially different from the one he previously provides for the natural numbers.

> Theoretical geometry examines lines and planes of bodies absolutely and generally and in a general manner that subsumes the planes of all bodies (على وجه يعم جميع الأجسام). For the geometrician represents in his mind (يصور في نفسه) lines in the general manner regardless (لا يبالي) of whatever body in which they are; and he represents in his mind planes, being square (التربيع), circularity (التدوير), being triangular (التثليث) in the most general manner regardless of whatever body in which they are (بالوجه الأعم الذي لا يبالي في أي جسم كان); and he represents the physical objects in the most general manner regardless of whatever body in which they are and of whatever matter and sensible thing in which they are. He rather represents them absolutely without making reside in his mind (يقيم في نفسه) a physical body be it a wood, a physical body be it a wall or a physical body be it an iron, but the general physical body of these. (ibid., pp. 51–52)

In this paragraph, al-Fārābī does not simply try to identify theoretical geometry as the study of shapes for example, he is rather aiming to determine its epistemic status. Two points of great importance are discussed. The first: in what way geometry is theoretical? His answer is that geometry is theoretical by the nature of its objects: geometric concepts are general, but what does it mean for a concept to be general? Unlike theoretical arithmetic whose objects have little to do with experience, al-Fārābī is struggling here to determine the relationship between particular objects and the concept to which they give rise. And it is in the first sentence where he captures how a concept is related to its particular objects: the concept is general in the sense that it is represented in the mind of the epistemic subject as subsuming all particular objects. And he goes further by spelling out what this specific mental representation amounts to. In the last sentence where he speaks of the mind as a location ("reside in the mind") for the general concepts, al-Fārābī suggests that the mind is an ontological region in which its objects can thus be regarded as having a mental existence. This new understanding of ontology paves the way for a major transformation of ancient metaphysics by making a simple presence in the mind a new form of existence and a mental representation an act of creation. In the second point, he describes the cognitive process involved in the formation of the general concept. Regarding the creation of geometric objects, he explains that the mental act that generates the general concept consists in the identification of an invariant in the perception of physical objects. He uses the concept of "disregard" to indicate that that specific mental act which perceives the invariant property inherently involves reference to an external reality. Because of its shape, the concept of triangle for example refers to all physical objects which have the property of being triangle; and by extension geometric constructions point to some extramental objects leaving the question of their realisation, i.e. their actual construction, to practical geometry. Though geometry is theoretical since its objects are general concepts, its subject matter, in which shape is an integral part, is what makes it essentially linked to objects of experience; that is what al-Fārābī is trying to say: unlike number theory which is a pure science, geometry is an empirical science.

because of its historical significance. As will be shown in the next chapter, Aristotle finds it hard to clarify the nature of mathematics since he seems to confuse two important conceptual terms by calling theoretical what is empirical. As an alternative to the underlying conception of the Pythagorean mathematical practice,[21] the author of *Metaphysics* suggests that their manipulation of numbers assumes a reference to some concept serving as a counting unit and on which their existence is dependent: “one means a measure of some plurality, and number means a measured plurality and a plurality of measures” (*Metaphy.*, N. 1088a), which implies that one is not in itself the substance of anything but rather of a particular and definite substance or, as he puts it, “a number, whatever it is, is always a number of something” (ibid., N. 1092b). Al-Fārābī disagrees and refutes Aristotle’s claim by arguing that numbers are objects of the mind and, as a result, they do not need to refer to an implicit counting unit since number theory examines numbers themselves as unities and not made of unities. But how does he deal with the question of their existence? For an answer, we have to go to his *Kitāb al-ḥurūf* where he discusses the ontological issue:

> The thing can be said of every thing that has a quiddity, whether it is external to the soul or [merely] conceived of in any way… Whereas the existent is always said of every thing that has a quiddity, external to the soul, and cannot be said of a quiddity merely conceived of. For this reason the thing is more general than the existent.[22] (Al-Fārābī 1970, p. 128).

Al-Fārābī is thus fully aware of many of the problems of *Metaphysics*, one of which is the enormous struggle faced by the doctrine of its author to account for the extant mathematical ontology that was further extended by al-Khwārizmī’s algebra. As al-Fārābī pointed out in *The Aim*, according to Aristotle, metaphysics is a universal science since its subject is ‘being’ in general. But al-Fārābī attacks the old metaphysics by surprisingly arguing that the thing is more general than the existent. It looks as if what can be called the science of the thing is more universal than the

---

[21]Aristotle launched a strong attack on the Pythagoreans by emphasising that they “use stranger principles than those of the natural philosophers, because they take them from the non-sensible, unchangeable world of mathematics. Yet all their discussions are about nature” (*Metaphys.* A 989b)—an odd philosophical approach that he attacks further when it encroaches on his poaching territory, natural philosophy, by claiming that numbers are the cause of things:

> Number, then, whether it be number in general or number composed of units, is neither the cause as agent, nor as matter, nor as ratio and form of things. Nor, of course, is it the final cause. (ibid., N. 1092b)

This strong hostility against the Pythagoreans has further implications for the Aristotelian epistemic status of mathematics. Aristotle showed us that his theory of causes can defeat the Pythagoreans bizarre doctrine, but the author of *Metaphysics* believes that he has established more in terms of the limited relevance of mathematics to philosophy by conceding elsewhere that at best astronomy is “the nearest to philosophy of all the mathematical sciences, since it studies substance which is eternal, whereas the others are concerned with no kind of substance, e.g. the sciences of arithmetic and geometry.” (ibid., Λ. 1073b).

[22]والشيء قد يقال على كلّ ما له ماهيّة مّا كيف كان، كان خارج النفس أو كان متصوَّرا على أيّ جهة كان... فإنّ الموجود إنّما يقال على ما له ماهيّة خارج النفس ولا يقال على ما له ماهيّة متصوَّرة فقط، فبهذا يكون الشيء أعمّ من الموجود.

science of being. Is there a science whose object is the thing? A traditional philosopher whose knowledge is limited to the Greek scientific and philosophical output would be puzzled to hear that there is such a thing as the science of the thing; he would be lost if he tried to find it in al-Fārābī's classification of the sciences as he would not be able to recognise that the author of *Iḥsā' al-'ulūm* had put it under a new category called "the science of ingenious devices"

> In arithmetic, the ingenious devices involve, among other things, the science known by our contemporaries under the name of algebra and al-muqābala, and what is similar to it. But this science is common both to arithmetic and geometry.[23] (ibid., pp. 63–64).

This is just another example that shows that philosophy can make little progress if it ignores major scientific achievements. The emergence of new mathematical branches like arithmetic, algebra, the science of ingenious devices and the surge in major social disciplines like law and linguistics have made obsolete the old Aristotelian classification of the sciences.[24] He further explains how algebra comes to wreck the Greek conceptions of arithmetic and geometry by giving rise to a new unifying mathematical concept, i.e. the algebraic quantity, the thing.

> It includes the ingenious devices to determine the numbers that we try to determine and use, those which are rational and irrational the principles of which are given in Euclid's *al-Usṭuqusāt* 10th book, and those which are not mentioned by Euclid. Since the relation of rational to irrational numbers — to one another — is like the relation of numbers to numbers, each number is thus homologous with a certain rational or irrational magnitude. If we determine the numbers which are homologous with magnitude ratios, we then determine these magnitudes in a certain manner. That is why we postulate certain rational numbers to be homologous with rational magnitudes, and certain irrational numbers to be homologous with irrational magnitudes. (ibid., p. 64)

This text is of great historical importance for it demonstrates that al-Fārābī not only was aware of the latest developments in the mathematics of his time, he had also fully grasped the far-reaching philosophical implications of the reinvention of mathematics. It was al-Khwārizmī who created a new formal ontology[25] by making the "thing" such a powerful mathematical concept to refer to both rational and irrational numbers, as Rashed explains:

---

[23] فمنها [علوم الحيل] الحيل العددية، و هي على وجوه كثيرة: منها العلم المعروف عند أهل زماننا بالجبر والمقابلة و ما شاكل ذلك. على أن هذا العلم مشرك للعدد والهندسة.

[24] Al-Khwārizmī's algebra undermines one of the major Aristotelian metaphysical assumptions underlying his classification of the sciences that can be called the incommensurability of kinds, i.e. the impossibility of passing from one kind to another. Like living species, Aristotle has denied the possibility of interaction between not only the various sciences, like mathematics and physics, but also within a given scientific discipline like mathematics, confidently concluding: "Therefore it is impossible to prove a fact by transition from another genus, e.g. a geometrical fact by arithmetic. […] The axioms may be the same; but where the genus is different, as of arithmetic and geometry, the arithmetical proof cannot be applied to prove the attributes of spatial magnitudes." (*An. Post.*, 75a38).

[25] See also al-Khwārizmī (2007, p. 48).

> If algebra is in fact common to arithmetic and geometry, without in any way giving up its status as science, it is because its very object, the "algebraic unknown", that is, the "thing (الشيء, res)" can refer indifferently to a number or to a geometric magnitude. More than that: since a number can also be irrational, "the thing" designates then a quantity which can be known only by approximation. Accordingly the algebraists' subject matter must be general enough to receive a wide range of contents; but it must moreover exist independently of its own determinations, so that it can always be possible to improve the approximation. The Aristotelian theory is obviously unable to account for the ontological status of such an object. So a new ontology has to be made to intervene that allows us to speak of an object devoid of the character which would none the less enable us to discern what it is the abstraction of; an ontology which must also enable us to know an object without being able to represent it exactly. This is precisely what has been developing in Islamic philosophy since al-Fārābī: an ontology which is "formal" enough, in a way, to meet the requirements mentioned above. (Rashed 2008, pp. 168–169)

We have presented just some views of al-Fārābī's philosophy of mathematics, which represent a major breakthrough that few were capable of recognising at his time. The problem that gave so much trouble to Ibn Sīnā on the nature of the relationship between the soul and mathematics was clarified by his predecessor, i.e. that mathematics is created by the human mind. The significance of al-Fārābī's original ideas is that it opened a new field of research the exploration of which would lead his successor to reinvent epistemology. It is remarkable that since childhood, one specific entity intrigued Ibn Sīnā's thought and dominated his investigations, which distinguished his project from those of all his predecessors; namely, the soul and its relationship to the world. And to the question: What is the soul? He does not try to answer it in the old fashioned way by writing an abstract treatise on the subject. Instead, he relates it to its production by considering the soul as a powerful creation of knowledge—a new topic that generates some unprecedented epistemic questions such as: what is knowledge?; how is it created by the soul?; what is possible for it to know?; how does it know that it knows?; and, more importantly, how can it know more than what it knows? He then used mathematics, the knowledge par excellence, as an instrument of investigation and analysis of the mind by characterising the various mental operations that could explain the production of mathematical knowledge and contribute to its progress through various levels of understanding.

By calling him the second teacher, Aristotelians seem to be proud of al-Fārābī's contributions to the understanding of the Aristotelian doctrine, an admission that, before him, philosophy was in disarray not to say in crisis. The irony is that they did not realise that he was in fact leading a major change in philosophical tradition. This is the subtle point that Ibn Sīnā grasped once he read the first book that fell into his hands, which happened to be *The Aim of the Metaphysics*.

> The purpose of its author [واضعه] was confused to me… it turned out to be Abū Naṣr al-Fārābī's book *Fī Aghrāḍ kitāb mā baʻda aṭ-ṭabīʻa*. I returned home and was quick to read it, and instantly the aims of that book became clear to me. (Gohlman 1974, pp. 33–35)

This is the passage that makes al-Fārābī famous and by which it became known that his *al-Aghrāḍ* helped his successor understand Aristotle's *Metaphysics*. But what is little recognised is the underlying major conceptual change that was communicated.

Aristotle's treatise becomes clear to Ibn Sīnā only after reading al-Fārābī's work, whereas beforehand as he admitted he was confused. It turns out that what al-Fārābī's work made him understand was why he was confused, helping him to understand issues that the Stagirite had failed to tackle let alone clarify, i.e. how mathematical knowledge is created by the epistemic subject. In other words, after reading al-Fārābī's *Aghrāḍ*, Ibn Sīnā was able to identify some of the problems that make *Metaphysics* the Mount Everest of Aristotle's treatises, its refutation was just around the corner. Al-Fārābī had already started the process of breaking down the old metaphysics by highlighting some of the weaknesses of Aristotle's ontology, especially regarding mathematics. In *al-Ilāhiyāt* or the new metaphysics, Ibn Sīnā finishes the job by being considerably more precise in his attacks. As a result, the differences within the philosophical tradition became much more serious than had ever been the case and the gap between representatives of the stagnating Aristotelian tradition in particular and Ibn Sīnā's increasingly non-Aristotelian tendency became much wider. Since he belonged to the philosophical circle dominated by the traditional philosophers, he gave them the benefit of the doubt by initially trying to accommodate them. But the ever-increasing gap only made matters worse, and ultimately it became so wide that he could not even have a basic meaningful discussion with them. This persistent dialogue with the deaf made him realise that the old generation were so deeply entrenched in the Aristotelian tradition that they would never change as he was hoping. He learned from his experience that, like his predecessor, his thoughts were so revolutionary that they would only be understood by the generations to come. It is for them that he wrote his *Logic of the Orientals* in which he takes the landmark historical decision to publicly announce the beginning of a new era.

http://www.springer.com/978-3-319-25236-0

Mathematics and the Mind
An Introduction into Ibn Sīnā's Theory of Knowledge
Tahiri, H.
2016, IX, 76 p., Softcover
ISBN: 978-3-319-25236-0