# Preface to the Second Edition

As I wrote the first edition of the book during 2001/2002, one of the goals was that at least the list of references should contain most of the frame literature. Now, 14 years later, even this very modest goal cannot be reached anymore. During the last 20 years frames have experienced an ever-increasing popularity, and they show up in many different contexts, explicitly or implicitly. Considering just four of the key topics, namely, (i) "Frames in finite-dimensional spaces," (ii) "Frames in general Hilbert spaces," (iii) "Frames in Gabor analysis," and (iv) "Frames and wavelet analysis," each of these topics could easily fill a book of the same size as the current book. Therefore one of the major decisions during the work on the second edition has been what to include – and at what level of details. My choice has been to follow the line from the first edition and present the core material (and frequently less known material that should belong to the core) in great detail, while other topics are treated as research topics with more focus on the connections between the results than the proofs. The fact that many recent and advanced results are presented without proofs made it possible still to give a quite broad picture of the frame theory; but clearly it also leaves a gap open for other authors who would like to give a detailed presentation focusing on one of the topics.

The new material mainly occurs in new chapters and sections, but of course the entire book has been updated with additional results and comments. On very compressed form the main additions can be described as follows:

- Sections 1.2 on tight frames and dual pairs of frames in finite-dimensional spaces. Section 1.8 on fusion frames. Section 1.9 on applications of frames. Finally, Section 1.10 which relates the properties of the harmonic frames to the ongoing research within finite frame theory.
- Extension and rearrangement of Chapter 2. Many results from the former appendix now appear here.
- Section 3.7, a new section on Riesz sequences; and Section 3.10 on sampling an analog-digital conversion.
- Several updates and additions in Chapter 4, which motivate the step from bases to frames.
- Section 5.2, a new section on frame sequences.
- Chapter 6, a new chapter that collects results about tight frames and dual pairs of frames in general Hilbert spaces.
- Section 7.2, a new section about relations between frames and their subsequences, with focus on the "strange" behavior of the lower frame bounds for finite subfamilies. And Section 7.7, a short section on the Feichtinger conjecture.
- Chapter 8, a new chapter on selected topics in general frame theory. It contains sections on G-frames, localization of frames, the R-dual sequences, a frame-like theory via unbounded operators, as well as a discussion of frames in the context of signal processing.
- Section 9.4, Section 9.5, and Section 9.7: new sections about the canonical dual of a frame of translates and oblique duals, as well as applications of frames of translates within sampling theory.
- Chapter 10, a new chapter on shift-invariant systems (parts of the material previously appeared within the presentation of Gabor frames).
- Extensions and updates in Section 11.6 on Gabor frames generated by special functions. Section 11.7, a new section collecting the known connections between B-splines and Gabor frames, as well as discussions about open problems.
- Chapter 12, a new chapter on dual pairs of Gabor frames and tight Gabor frames.
- Section 13.1, a new section about the duality principle. Section 13.5, a new section about localized Gabor frames. And Section 13.7, a new section on time-frequency localization.
- Chapter 14: new sections concerning duality of Gabor frames in $\ell^2(\mathbb{Z})$ and $\ell^2(\mathbb{Z}^d)$, and explicit construction of such frames based on dual

pairs of Gabor frames for $L^2(\mathbb{R})$. Construction of periodic Gabor frames in $L^2(0, L)$, and description of the transition from a Gabor system in $L^2(\mathbb{R})$ to a finite-dimensional model in $\mathbb{C}^L$.

- Section 15.3, a new section on dual pairs of wavelet frames.
- Chapter 19, a new chapter on selected topics on wavelet frames. Some of the sections also appeared in the first edition of the book, but the sections on the extension problem and signal processing are new.
- Chapter 20, a new chapter on generalized shift-invariant systems.
- Chapter 21, a new chapter on frames on locally compact abelian groups.
- Section 23.3, a new section that yields convergence estimates in the context of finite-dimensional approximations of the inverse frame operator.
- Chapter 24 on Banach frames: the entire chapter has been updated with more recent results.
- Section A.5 and Section A.6, new sections stating the key properties of the modulation spaces and the Feichtinger algebra. Section A.9 and Section A.10, new sections on exponential B-splines and splines on LCA groups.

I would like to thank all the friends, colleagues, and students who have contributed to the current second edition. First and foremost I would like to thank my coauthors, who have definitely inspired me and shaped my view and understanding of frames over the years. Many of the papers with my coauthors were used as the starting point for various sections and chapters. For example, my papers with Hong Oh Kim and Rae Young Kim form the basis for Sections 6.4, 11.7, 12.5, 12.6, and 12.7; similarly, the paper [176] with Say Song Goh was the driving force behind most of the sections in Chapter 21.

I would also like to thank Hong Oh Kim and Rae Young Kim for organizing and supporting about 20 visits to Korea Advanced Institute for Science and Technology (KAIST) over the years, and for the many pleasant hours we spend working on joint problems; and Say Song Goh, whose many invitations to National University of Singapore (NUS) also gave me scientific inspiration and excellent working conditions, with direct influence on the current book.

It is a great pleasure to thank Henrik Stetkær, who used the first edition as textbook in several master courses at the University of Aarhus; this led to the discovery of several misprints and imprecisions, which I have tried to correct.

I thank Jakob Lemvig and Mads Sielemann Jakobsen for giving me access to a note providing a direct proof of the duality principle in Gabor analysis.

During the preparation of the manuscript, I got help from many colleagues and students to spot typing mistakes, bad formulations, etc.; I thank Say Song Goh, Marzieh Hasannasab, Christina Hildebrandt, Mads Sielemann Jakobsen, Jakob Lemvig, Diana Stoeva, and Jordy van Velthoven for their help, which clearly improved the manuscript.

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