Chapter 2
Touschek Effect in Electron Storage Rings

The electrons in a stable beam in a high energy particle accelerator can be lost due to the scattering with residual gas molecules in the beam pipe (elastic or inelastic scattering) or due to the scattering with other electrons of the beam (Touschek effect).

The two effects may be combined in terms of the total lifetime as follows:

$$\frac{1}{\tau} = \frac{1}{\tau_v} + \frac{1}{\tau_t} \tag{2.1}$$

where τ_t is the Touschek lifetime and τ_v is the vacuum lifetime.

Vacuum lifetime depends on the residual gas pressure inside the beam pipe. It varies with time, because the pressure is not constant during the machine operation and it is higher after the shutdowns. In third generation synchrotron light sources, vacuum lifetime is generally longer than Touschek lifetime.

The Touschek effect is a single scattering between two electrons of the bunch. The collision can transfer momentum from transverse to longitudinal motion and both the electrons can exceed the longitudinal acceptance, in which case they are lost.

The high density of particles into a small bunch increases the probability of collisions between particles. The elastic scattering between electrons is called Møller scattering. The Møller differential cross section is given at leading order by [1]:

$$\frac{d\sigma}{d\Omega} = \frac{4r_0^2}{\beta^4}\left(\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta}\right) \tag{2.2}$$

In high energy accelerators, in the bunch frame, the transverse oscillation energy is larger than the longitudinal one. A collision can transfer momentum from transverse to longitudinal motion and the electron can exceed the longitudinal acceptance limit.

The Møller cross section can be integrated on the solid angles which give a momentum deviation larger than the momentum aperture of the ring in a certain

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position. We note that it is assumed that the electron spin has been averaged over in the formula for the cross section. In the case where the beam is polarized, there is a correction term which we consider later.

The resulting Touschek lifetime, in non-relativistic approximation, with a flat beam, i.e. with a very small vertical oscillation, has been derived by Bruck [2, 3].

$$\frac{1}{\tau} = \left\langle \frac{{r_e}^2 c N_b}{8\pi\sigma_x\sigma_y\sigma_l\gamma^2} \frac{1}{\delta_{acc}^3} D(\epsilon) \right\rangle \tag{2.3}$$

where r_e is the classical electron radius, c is the speed of light, N_b is the number of electrons per bunch, σ_x, σ_y and σ_l are the bunch width, height and length, γ is the Lorentz factor, δ_{acc} is the momentum acceptance. $D(\epsilon)$ is defined as:

$$D(\epsilon) = \sqrt{\epsilon}\left[-\frac{3}{2}e^{-\epsilon} + \frac{\epsilon}{2}\int_{\epsilon}^{\infty} \frac{\ln u}{u} e^{-u}\mathrm{d}u + \frac{1}{2}\left(3\epsilon - \epsilon\ln\epsilon + 2\right)\int_{\epsilon}^{\infty} \frac{e^{-u}}{u}\mathrm{d}u\right] \tag{2.4}$$

where the parameter ϵ is:

$$\epsilon = \left(\frac{\delta_{acc}}{\gamma\sigma'_x}\right)^2 \tag{2.5}$$

and

$$\sigma'_x = \sqrt{\frac{\varepsilon_x}{\beta_x}} \tag{2.6}$$

For the ESRF storage ring, assuming $\beta_x = 1$ m, the value of ϵ is given by:

$$\epsilon = \frac{\delta_{acc}^2}{0.57} \tag{2.7}$$

The function $D(\epsilon)$ is shown in Fig. 2.1. The function $\frac{1}{\delta_{acc}^3}D(\epsilon)$ for the ESRF lattice is shown in Fig. 2.2.

Beam sizes, β functions and momentum acceptance are not constant along the ring, so the total lifetime has to be obtained averaging Eq. (2.3) on the whole ring.

Assuming a small variation of δ_{acc} along the ring, a scaling law for the Touschek lifetime can be obtained from Eq. (2.3):

$$\tau_t \propto \frac{\sqrt{\varepsilon_y}\sigma_z}{I_b}\delta_{acc}^3 \tag{2.8}$$

A more precise result has been obtained by Piwinski [4, 5]. Piwinski derivation is valid for a general case, with arbitrary energies in the rest frame of the colliding particles and arbitrary transverse beam envelops.

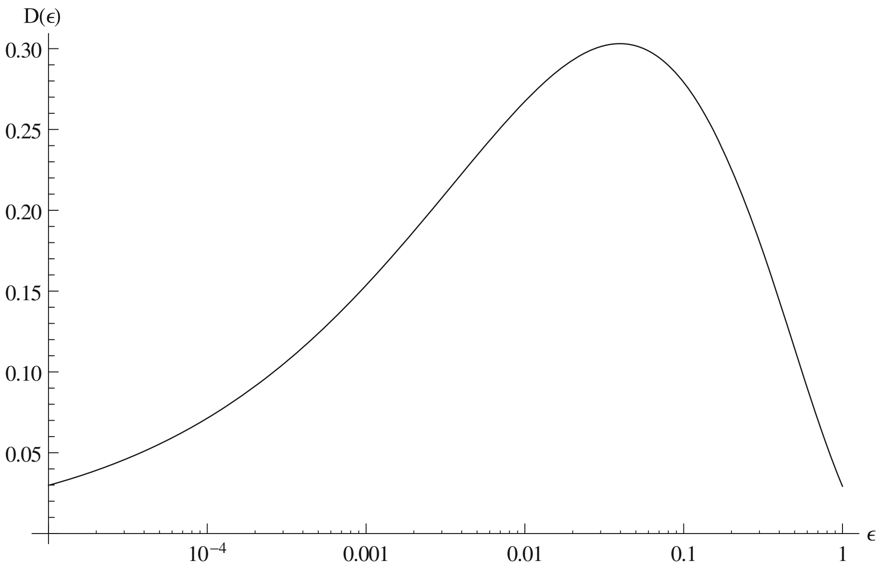

Fig. 2.1 Function $D(\epsilon)$

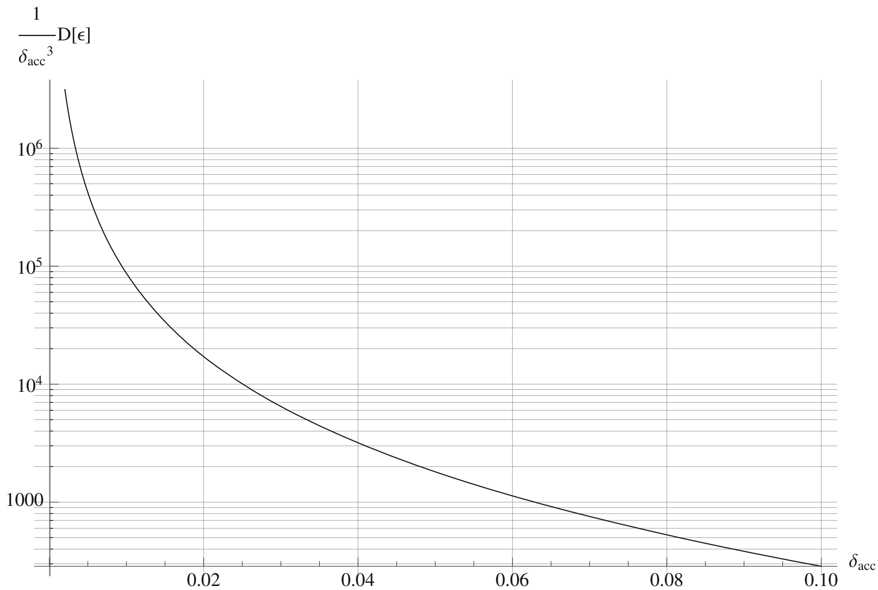

Fig. 2.2 Function $\frac{1}{\delta_{acc}^3} D(\epsilon)$ as a function of δ_{acc}, assuming $\beta_x = 1$ m, for ESRF storage ring

The inverse of Touschek lifetime derived by Piwinski is given by:

$$\frac{1}{\tau} = \left\langle \frac{r_o^2 c N_b}{8\pi\gamma^2\sigma_s\sqrt{\sigma_x^2\sigma_y^2 - \sigma_p^4 D_x^2 D_y^2}\tau_m} F\left(\tau_m, B_1, B_2\right) \right\rangle \tag{2.9}$$

where σ_p is the energy spread, σ_s is the bunch length, D_x and D_y are the dispersion functions, σ_x and σ_x are the beam sizes defined by:

$$\sigma_x = \sqrt{\varepsilon_x\beta_x + \sigma_p^2 D_x^2} \tag{2.10}$$

$$\sigma_y = \sqrt{\varepsilon_y\beta_y + \sigma_p^2 D_y^2} \tag{2.11}$$

and

$$\tau_m = \beta^2\delta_{acc}^2 \tag{2.12}$$

the function $F\left(\tau_m, B_1, B_2\right)$ is:

$$\begin{aligned} F\left(\tau_m, B_1, B_2\right) \\ = \sqrt{\pi\left(B_1^2 - B_2^2\right)}\tau_m \int_{\tau_m}^{\infty} \left[\left(2 + \frac{1}{\tau}\right)^2 \left(\frac{\tau/\tau_m}{1+\tau} - 1\right) + 1 - \frac{\sqrt{1+\tau}}{\sqrt{\tau/\tau_m}} \right. \\ \left. -\frac{1}{2\tau}\left(4 + \frac{1}{\tau}\right)\ln\frac{\tau/\tau_m}{1+\tau}\right] e^{-B_1\tau} I_0\left(B_2\tau\right) \frac{\sqrt{\tau}\mathrm{d}\tau}{\sqrt{1+\tau}} \end{aligned} \tag{2.13}$$

where I_0 is the modified Bessel function. The functions B_1 and B_2 are given by:

$$B_1 = \frac{\beta_x^2}{2\beta^2\gamma^2\sigma_{x\beta}^2}\left(1 - \frac{\sigma_h^2\tilde{D}_x^2}{\sigma_{x\beta}^2}\right) + \frac{\beta_y^2}{2\beta^2\gamma^2\sigma_{y\beta}^2}\left(1 - \frac{\sigma_h^2\tilde{D}_y^2}{\sigma_{y\beta}^2}\right) \tag{2.14}$$

$$B_2^2 = B_1^2 - \frac{\beta_x^2\beta_y^2\sigma_h^2}{\beta^4\gamma^4\sigma_{x\beta}^4\sigma_{y\beta}^4\sigma_p^2}\left(\sigma_x^2\sigma_y^2 - \sigma_p^4 D_x^2 D_y^2\right) \tag{2.15}$$

$\tilde{D}_x$ and $\tilde{D}_y$ are given by:

$$\tilde{D}_x = \alpha_x D_x + \beta_x D_x' \tag{2.16}$$

$$\tilde{D}_y = \alpha_y D_y + \beta_y D_y' \tag{2.17}$$

where D_x' and D_y' are the slopes dispersion functions. σ_h is defined by:

$$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_p^2\sigma_{x\beta}^2\sigma_{y\beta}^2}\left(\tilde{\sigma}_x^2\sigma_{y\beta}^2 + \tilde{\sigma}_y^2\sigma_{x\beta}^2 - \sigma_{x\beta}^2\sigma_{y\beta}^2\right) \tag{2.18}$$

$\sigma_{x\beta}$, $\tilde{\sigma}_x$, $\sigma_{y\beta}$ and $\tilde{\sigma}_y$ are defined:

$$\sigma_{x\beta} = \sqrt{\varepsilon_x \beta_x} \tag{2.19}$$

$$\tilde{\sigma}_x = \sqrt{\sigma_x^2 + \sigma_p^2 \left(D_x^2 + \tilde{D}_x^2\right)} \tag{2.20}$$

$$\sigma_{y\beta} = \sqrt{\varepsilon_y \beta_y} \tag{2.21}$$

$$\tilde{\sigma}_y = \sqrt{\sigma_y^2 + \sigma_p^2 \left(D_y^2 + \tilde{D}_y^2\right)} \tag{2.22}$$

β and γ are the relativistic factors, β_x, β_y, α_x and α_y are the Twiss functions.

Beam sizes, β functions, momentum acceptance are not constant along the ring, so the total lifetime has to be obtained averaging Eq. (2.9) on the whole ring.

2.1 Spin Polarization in Electron Storage Rings

The polarization of an electron beam is the average value of the vertical spin of the electrons.

An electron beam with an initial random distribution of the spin becomes polarized over time due to the Sokolov–Ternov effect [6, 7]. A little fraction of the photons emitted as synchrotron radiation causes a spin-flip to the electrons.

The transition rates between the two possible spin status are:

$$W_{\uparrow\downarrow} = \frac{5\sqrt{3}}{16} \frac{r_e \gamma^5 \hbar}{m_e} \left\langle \frac{1}{\rho^3} \right\rangle \left(1 + \frac{8}{5\sqrt{3}}\right) \tag{2.23}$$

and

$$W_{\downarrow\uparrow} = \frac{5\sqrt{3}}{16} \frac{r_e \gamma^5 \hbar}{m_e} \left\langle \frac{1}{\rho^3} \right\rangle \left(1 - \frac{8}{5\sqrt{3}}\right) \tag{2.24}$$

where r_e is the electron classical radius, γ is the Lorentz relativistic factor, $\hbar$ is the reduced Planck constant, m_e is the electron mass, ρ is the instantaneous bending radius of dipoles. The symbol $\uparrow$ denotes a spin along the magnetic field and the symbol $\downarrow$ denotes a spin opposite to the magnetic field.

These two transition rates are different and so an unpolarized beam becomes polarized. The maximum value of polarization achievable is given by:

$$P_{ST} = \frac{W_{\uparrow\downarrow} - W_{\downarrow\uparrow}}{W_{\uparrow\downarrow} + W_{\downarrow\uparrow}} = \frac{8}{5\sqrt{3}} \simeq 0.9238 \tag{2.25}$$

where P_{ST} is called the Sokolov–Ternov level of polarization.

The polarization follows an exponential law:

$$P(t) = P_{ST}\left(1 - e^{-t/\tau_p}\right) \tag{2.26}$$

The exponential time constant for the polarization effect is given by:

$$\tau_p^{-1} = W_{\uparrow\downarrow} + W_{\downarrow\uparrow} = \frac{5\sqrt{3}}{8}\frac{r_e\hbar\gamma^5}{m_e}\left\langle\frac{1}{\rho^3}\right\rangle \tag{2.27}$$

The average of $1/\rho^3$ is given by:

$$\left\langle\frac{1}{\rho^3}\right\rangle = \frac{1}{C}\oint\frac{1}{\rho^3(s)}\mathrm{d}s \tag{2.28}$$

where C is the total length of the ring and s is the longitudinal coordinate.

Spin Depolarization

The spin of the electrons does a precession around the magnetic field of the dipoles, following the Thomas-BMT equation.

The number of total precessions done by the electrons in a revolution time is the spin tune and it is given by:

$$\nu_{spin} = \frac{g-2}{2}\gamma \tag{2.29}$$

where γ is the Lorentz relativistic factor and $a = \frac{g-2}{2}$ is the electron anomalous magnetic dipole moment. Its value is [8]:

$$a = \frac{g-2}{2} = (1.15965218076 \pm 0.00000000027) \times 10^{-3} \tag{2.30}$$

The precession frequency of the electron spin is proportional to the beam energy:

$$f_{spin} = f_0\,\nu_{spin} = f_0\,a\,\gamma \tag{2.31}$$

where f_0 is the revolution frequency.

If a vertical kick, from an horizontal magnetic field, is applied to the beam, the electron spin vector is rotated by an angle, around the magnetic field direction, given by:

$$\theta_{spin} = \nu_{spin}\,\theta_k \tag{2.32}$$

where θ_k is the kick given by the kicker to the beam in radians.

The kicker can be powered with an angular frequency resonant with the spin tune:

$$\omega_{dep} = (\nu_{spin} \pm m)\,\omega_0 \tag{2.33}$$

where m is an integer and $\omega_0 = 2\pi f_0$.

The rotation angle given to the spin vector, as a function of time, is:

$$\theta_{spin}(t) = \left|A_{spin} \cos\left(\omega_{dep} t\right)\right| \tag{2.34}$$

where $A_{spin} = A_k\,\nu_{spin}$ is the maximum rotation angle given by the kicker to the spin vector and A_k is the amplitude in radians of the kick.

Assuming a beam without energy spread, with nominal energy, it can be depolarized if the spin vectors receive a rotation angle of $\pi/2$. This is achieved when the sum of the angles given to the spins is $\pi/2$:

$$\sum_{n=1}^{N} \theta_{spin}(nT_0) = \frac{\pi}{2} \tag{2.35}$$

The function $\theta_{spin}(t)$ is the absolute value of a sinusoidal function, its average value can be computed:

$$\langle\theta_{spin}\rangle = \frac{2A_{spin}}{\pi} \tag{2.36}$$

The number of turns needed to depolarize the beam, in the simple assumption of beam without energy spread, is given by:

$$N = \frac{\pi/2}{\langle\theta_{spin}\rangle} = \frac{\pi^2}{4A_{spin}} = \frac{\pi^2}{4A_k\nu_{spin}} \tag{2.37}$$

In electron storage rings, the beam has an energy spread and electrons suffer synchrotron oscillations. The spin tune is not constant, it oscillates with energy and there is a spin tune spread. The depolarization of a polarized electron beam can be studied only with a simulation. A simple spin tracking simulation program has been developed and is described in Appendix B.

Either the experimental results from different electron storage rings (LEP [9], SLS [10], Australian Synchrotron [11], Spear3 [11]) and the simulations show that the beam depolarization in real cases, with synchrotron oscillations, is possible only if the frequency of the kicker is in a narrow interval around the value of ω_{dep}, defined in Eq. (2.33).

The size of the spin depolarization resonance has been studied with deuteron beams at COSY synchrotron, experimentally and with simulations [12]. In heavy particles beam, the synchrotron oscillations can be switched off, turning off the cavity, because the synchrotron radiation is negligible. In these experiments, it is confirmed that without synchrotron oscillations the resonance is broadened by the energy spread; with synchrotron oscillations the resonance becomes much narrower, even with energy spread.

Effect of Spin Polarization on Touschek Lifetime

The Møller cross section is smaller if the beam is polarized [13–15]. When the beam is polarized, the Touschek lifetime is longer.

An expression for the effect of the polarization on Touschek lifetime is given in [16]. In particular, if P is the polarization, the Touschek lifetime is given by:

$$\frac{1}{\tau_t(P)} = \frac{1}{\tau_t(0)} + \left\langle R(\epsilon)\frac{1}{\tau_t(0)}\right\rangle P^2 \tag{2.38}$$

where

$$R(\epsilon) = \frac{F(\epsilon)}{C(\epsilon)} \tag{2.39}$$

with

$$C(\epsilon) = \epsilon \int_\epsilon^\infty \frac{1}{u^2}\left(\left(\frac{u}{\epsilon}\right) - \frac{1}{2}\ln\left(\frac{u}{\epsilon}\right) - 1\right) e^{-u}\mathrm{d}u \tag{2.40}$$

and

$$F(\epsilon) = -\frac{\epsilon}{2}\int_\epsilon^\infty \frac{1}{u^2}\ln\left(\frac{u}{\epsilon}\right) e^{-u}\mathrm{d}u \tag{2.41}$$

The quantity ϵ is given by:

$$\epsilon = \left(\frac{\delta_{acc}}{\gamma\sigma_{x'}}\right)^2 \tag{2.42}$$

where δ_{acc} is the momentum acceptance, γ is the Lorentz relativistic factor, $\sigma_{x'}$ is the standard deviation of the beam distribution in horizontal angle.

The value of $-F(\epsilon)/C(\epsilon)$ as a function of ϵ is shown in Fig. 2.3.

Effect of Spin Polarization in Touschek Lifetime of ESRF Storage Ring

In the ESRF storage ring there are 64 compound dipoles, with a hard component and a soft one. The lengths and bending radii are:

$$L_H = 2.1573\,\mathrm{m} \quad \rho_H = 23.37\,\mathrm{m} \tag{2.43}$$

$$L_S = 0.2927\,\mathrm{m} \quad \rho_S = 49.61\,\mathrm{m} \tag{2.44}$$

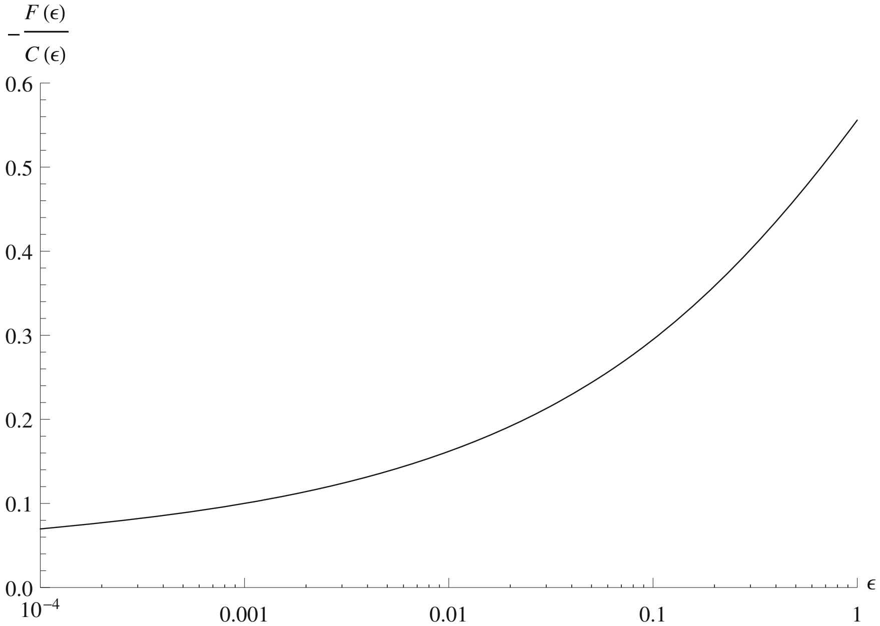

Fig. 2.3 Function $-R(\epsilon) = -F(\epsilon)/C(\epsilon)$

The length *Arc* of the curvilinear trajectory of the particles, given the length L of the rectangular bending magnet and the curvature radius ρ of the particle, is given by:

$$Arc = 2\rho \arcsin \frac{L}{2\rho} \tag{2.45}$$

The average of the third power of the bending radius is given by:

$$\left\langle \frac{1}{\rho^3} \right\rangle = \frac{64}{C} \left(\frac{Arc_H}{\rho_H^3} + \frac{Arc_S}{\rho_S^3} \right) = 1.2997 \times 10^{-5}\,\mathrm{m}^{-3} \tag{2.46}$$

Using Eqs. (2.27) and (2.46), the inverse of the polarization time can be computed. The polarization time (in minutes) for the ESRF storage ring is:

$$\tau_p = 15.75\,\mathrm{min} \tag{2.47}$$

Using the Sokolov–Ternov level of polarization, $S = 0.9238$, the beam sizes of the ESRF storage ring and a constant value of the momentum acceptance, the functions C and F in Eqs. (2.40) and (2.41) have been computed by mean of `mathematica` [17]

Table 2.1 Touschek lifetime increase due to spin polarization, for five different constant momentum acceptances, for the ESRF storage ring

δ_{acc} (%)	Lifetime increase (%)
1.5	12.20
2.0	13.75
2.5	15.09
3.0	16.27
3.5	17.30

and the lifetime increase has been computed. In Table 2.1, the increasing in Touschek lifetime for five different constant momentum acceptance are shown, assuming the maximum polarization P_{ST}.

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