

# Application of the Flower Pollination Algorithm in Structural Engineering

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**Abstract** In the design of a structural system, the optimum values of design variables cannot be derived analytically. Structural engineering problems have various design constraints concerning structural security measures and practicability in production. Thus, optimization becomes an important part of the design process. Recent studies suggested that metaheuristic methods using random search procedures are effective for solving optimization problems in structural engineering. In this chapter, the flower pollination algorithm (FPA) is presented for dealing with structural engineering problems. The engineering problems are about pin-jointed plane frames, truss systems, deflection minimization of I-beams, tubular columns, and cantilever beams. The FPA inspired from the reproduction of flowers via pollination is effective to find the best optimum results when compared to other methods. In addition, the computing time is usually shorter and the optimum results are also robust.

**Keywords** Metaheuristic methods • Flower pollination algorithm • Structural optimization • Topology optimization • Weight optimization

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# 1 Introduction

In solving optimization problems, traditional optimization methods such as gradient-based methods may not be able to cope with high nonlinearity and multimodality. Evolutionary algorithms and nature-inspired algorithms tend to produce better results for highly nonlinear problems. Such nature-inspired metaheuristic algorithms often imitate the successful nature of some biological, physical, or chemical systems in nature. They often have several processes as numerical, algorithmic steps in solving an optimization problem. Each metaheuristic algorithm can have different inspiration from the nature and special rules according to the process of the natural systems. Detailed information about several metaheuristic algorithms can be found in the literature [1, 2]. Inspiration and pioneer papers of several metaheuristic algorithms are given in Table 1.

In structural engineering, economy is one of the main goals of the design engineering. The optimum design variables ensuring security measures and the minimum cost cannot be found with linear equations. As the equations and system behavior can be highly nonlinear, iterative numerical algorithms have been employed to find a solution. Using metaheuristic algorithms, the global optimum solution can be found more effectively.

In this chapter, the flower pollination algorithm (FPA) developed by Yang [16] is presented. Several structural optimization problems were investigated using FPA and the optimum results were compared with other optimization methods.

**Table 1** Metaheuristic algorithms and inspirations

Algorithm	Inspiration
Genetic algorithm [3, 4]	Darwinian evolution in nature
Simulated annealing [5]	Annealing process of materials
Ant colony optimization [6]	Behavior of ants foraging
Bee algorithm [7]	Behavior of bees
Particle swarm optimization [8]	Swarming behavior of birds and fish
Tabu search [9]	Human memory
Harmony search [10]	Musical performance
Big bang big crunch [11]	Evolution of the universe
Firefly algorithm [1]	Flashing characteristic of fireflies
Cuckoo search [12]	Brood parasitic behavior of cuckoo species
Charged system search [13]	Electrostatic and Newtonian mechanic laws
Bat algorithm [14]	Echolocation characteristic of microbats
Eagle strategy [15]	Foraging behavior of eagles
Flower pollination [16]	Pollination of flowering plants
Ray optimization [17]	Refraction of light

## 2 Flower Pollination Algorithm

In nature, the main purpose of the flowers is reproduction via pollination. Flower pollination is related to the transfer of pollen, which is done by pollinators such as insects, birds, bats, other animals or wind. Some flower types have special pollinators for successful pollination. The four rules of pollination have been formulated based on the inspiration from flowering plants and they form the main updating equations of the flower pollination algorithm [16].

1. Cross-pollination occurs from the pollen of a flower of different plants. Pollinators obey the rules of a Lévy distribution by jumping or flying distant steps. This is known as global pollination process.
2. Self-pollination occurs from the pollen of the same flower or other flowers of the same plant. It is local pollination.
3. Flower constancy is the association of pollinators and flower types. It is an enhancement of the flower pollination process.
4. Local pollination and global pollination are controlled by a probability between 0 and 1, and this probability is called as the switch probability.

In the real world, a plant has multiple flowers and the flower patches release a lot of pollen gametes. For simplicity, it is assumed that each plant has one flower producing a single pollen gamete. Due to this simplicity, a solution ( $x_i$ ) in the present optimization problem is equal to a flower or a pollen gamete. For multi-objective optimization problems, multiple pollen gametes can be considered.

In the flower pollination algorithm, there are two key steps involving global and local pollination. In the global pollination step, the first and third rules are used together to find the solution of the next step ( $x_i^{t+1}$ ) using the values from the previous step (step  $t$ ) defined as  $x_i^t$ . Global pollination is formulized in Eq. (1).

$$x_i^{t+1} = x_i^t + L(x_i^t - g^*) \quad (1)$$

The subscript  $i$  represents the  $i$ th pollen (or flower) and Eq. (1) is applied for the pollen of the flowers.  $g^*$  is the current best solution.  $L$  is the strength of the pollination, which is drawn from a Lévy distribution.

The second rule is used for local pollination with the third rule about flower constancy. The new solution is generated with random walks as seen in Eq. (2).

$$x_i^{t+1} = x_i^t + \varepsilon(x_j^t - x_k^t) \quad (2)$$

where  $x_j^t$  and  $x_k^t$  are solutions of different plants.  $\varepsilon$  is randomized between 0 and 1. According to the fourth rule, a switch probability ( $p$ ) is used in order to choose the type of pollination which will control the optimization process in iterations.

The details of the optimization process can be seen in the pseudocode which is given for the flower pollination algorithm.

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Objective minimize or maximize  $f(x)$ ,  $x=(x_1, x_2, \dots, x_d)$ 
Initialize a population of  $n$  flowers or pollen gametes with random numbers
Find the best solution ( $g_*$ ) of the initial population
Define a switch probability ( $p$ )
while ( $t < \text{Number of iterations}$ )
  for  $i=1:n$  ( $n$  is the number of flowers or pollen in the population)
    if  $\text{rand} < p$ 
      Global pollination using Eq.(1)
    else
      Local pollination using Eq.(2)
    end if
    Evaluate new solutions
    Update the better solutions in the population
  end for
  Find the current best solution ( $g_*$ )
end while

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Flower pollination algorithm was first proposed for the optimization problems with a single objective. Then, Yang et al. developed a multi-objective approach for FPA [18].

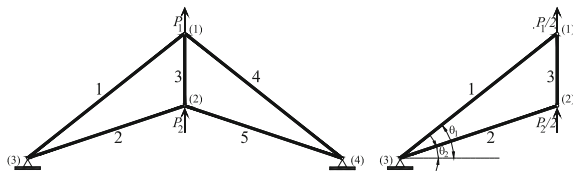
### 3 Numerical Examples

In this chapter, six numerical examples are investigated using the FPA. They are pin-jointed plane frame optimization, truss system optimization, vertical deflection minimization of an I-beam, cost optimization of a tubular column, and weight optimization of cantilever beams (two types of cantilever beams).

#### 3.1 Pin-Jointed Plane Frame Optimization Problem

A pin-jointed plane frame with five members is given in Fig. 1. The system is symmetrical and thus only half of the system with three members is optimized for the minimum weight. Topology optimization is done to find the optimum values of  $\theta_1$  and  $\theta_2$  angles shown in the figure. The system has a fixed base. This example was originally given by Majid [19]. The vertical deflections of joints 1 and 2, used in the design constraints, are defined as

$$|\Delta_1(\theta_1, \theta_2)| \leq \text{Max } \Delta, \quad (3)$$



**Fig. 1** The optimized system [19]

and

$$|\Delta_2(\theta_1, \theta_2)| \leq \text{Max } \Delta, \quad (4)$$

where  $\theta_1$  and  $\theta_2$  are searched within a range defined as minimum ( $\theta_{\min}$ ) and maximum ( $\theta_{\max}$ ) limits.

The members have a constant cross-sectional area ( $A$ ) and an elasticity modulus ( $E$ ).  $P_1$  and  $P_2$  loads are used in the system. The length between the supports is defined as  $l$ .

The lengths of the members are defined in Eqs. (5)–(7) for members 1, 2, and 3, respectively.

$$l_1 = \frac{l}{2\cos(\theta_1)} \quad (5)$$

$$l_2 = \frac{l}{2\cos(\theta_2)} \quad (6)$$

$$l_3 = \frac{l}{2\cos(\theta_1)\cos(\theta_2)} \sqrt{\cos^2(\theta_1) + \cos^2(\theta_2) - 2\cos(\theta_1)\cos(\theta_2)2\cos(\theta_1 - \theta_2)} \quad (7)$$

Since the members of the system have the same cross-sectional area, the total length of the system can be taken as the optimization objective in order to minimize the overall weight. The objective function is shown in Eq. (8).

$$\text{Minimize } f(\theta_1, \theta_1) = \sum_{i=1}^3 l_i \quad (8)$$

If  $\Delta = (\Delta_1, \Delta_1)^t$ ,  $K\Delta = F$  where  $K$  is the stiffness matrix and  $F$  is the load vector. Since  $K$  is equal to  $B^T k B$ , these matrices are given in Eqs. (9) and (10).

$$k = \begin{bmatrix} \frac{EA}{l_1} & 0 & 0 \\ 0 & \frac{EA}{l_2} & 0 \\ 0 & 0 & \frac{EA}{l_3} \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} \sin(\theta_1) & 0 \\ 0 & \sin(\theta_2) \\ 1 & -1 \end{bmatrix} \quad (10)$$

Thus, the stability equation ( $K\Delta = F$ ) of the system can be written as

$$EA \begin{bmatrix} \frac{\sin^2(\theta_1)}{l_1} + \frac{1}{2l_3} & -\frac{1}{2l_3} \\ -\frac{1}{2l_3} & \frac{\sin^2(\theta_2)}{l_2} + \frac{1}{2l_3} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_1 \end{bmatrix} = \begin{bmatrix} \frac{P_1}{2} \\ \frac{P_2}{2} \end{bmatrix} \quad (11)$$

and  $\Delta$  is then solved in the optimization process starting from random  $\theta_1$  and  $\theta_2$  values.

The optimization process has been carried out for the design constants given in Table 2. The results of the flower pollination algorithm were compared with the other methods employing GA [20] and CS [12].

The optimum results together with the results of other approaches are given in Table 3. The table shows that the proposed method is effective to find better solutions.

The numerical example is done by taking the switch probability as 0.5 and the number of population as 5. The optimum solution is found at the 1609th iteration. The convergence of the optimization is seen in the objective function versus iterations as shown in the plot given in Fig. 2.

As seen in the optimum results,  $\theta_1$  is nearly equal to  $\theta_2$ . Since the objective function is the minimization of the total length, the length of the third member is nearly zero. Thus, the method is effective to find the global optimum value.

Also, the optimization process is done for different cross-sectional areas and force values. In all these cases,  $P_2$  is taken as half of  $P_1$ . The optimum results of the

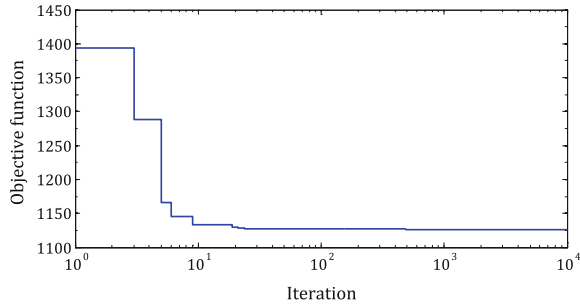
**Table 2** Design constants of numerical example

Max $\Delta$	5 mm
$\theta_{\min}$	0
$\theta_{\max}$	$\pi/3$ (rad)
$A$	100 mm <sup>2</sup>
$E$	200,000 MPa
$P_1$	100 kN
$P_2$	50 kN
$L$	1000 mm

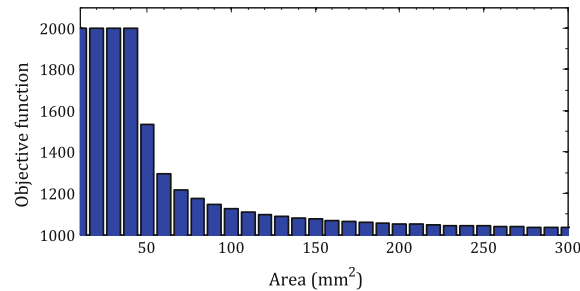
**Table 3** Optimum results of the numerical example

Method	$\theta_1$ (rad)	$\theta_2$ (rad)	$F(\theta_1, \theta_2)$
FPA	0.477634	0.477133	1125.87
GA	0.475784	0.472764	1125.98
CS	0.477459	0.477446	1125.92

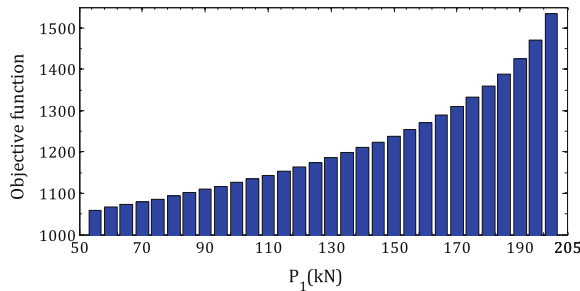
**Fig. 2** Objective function versus iteration



**Fig. 3** Optimum results for different cross sections



**Fig. 4** Optimum results for different forces

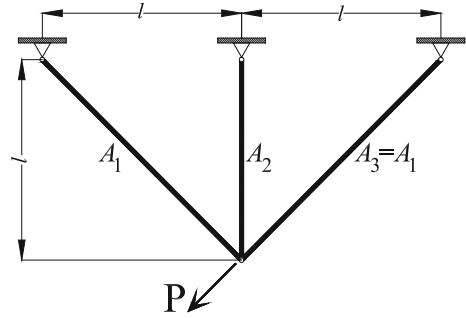


objective function for different cross-sectional areas and forces are given in Figs. 3 and 4, respectively. In Fig. 3, the upper limits of the angles were found as the optima for small cross-sectional areas.

**3.2 A Three-Bar Truss System Optimization Problem**

A three-bar truss structure is given in Fig. 5. This problem was first presented in Nowcki [21]. The objective function is about the minimization of the volume of the truss structure and this function is given in Eq. (12).

**Fig. 5** Truss optimization problem



$$\text{Minimize : } f(A_1, A_2) = (2\sqrt{2}A_1 + A_2)l \quad (12)$$

The design variables are the cross-sectional areas of structural members. Since the system is symmetric, only cross sections shown with  $A_1$  and  $A_2$  are optimized. The optimization problem is carried out for stress constraints. These constraints are

$$g_1 = \frac{\sqrt{2}A_1 + A_2}{\sqrt{2}A_1^2 + 2A_1A_2}P - \sigma \leq 0, \quad (13)$$

$$g_2 = \frac{A_2}{\sqrt{2}A_1^2 + 2A_1A_2}P - \sigma \leq 0, \quad (14)$$

$$g_3 = \frac{1}{A_1 + \sqrt{2}A_2}P - \sigma \leq 0. \quad (15)$$

The cross-sectional areas were searched for the ranges;  $0 \leq A_1 \leq 1$  and  $0 \leq A_2 \leq 1$ . The length, the load, and the stress limit were taken as  $l = 100$  cm;  $P = 2$  kN, and  $\sigma = 2$  kN/cm<sup>2</sup>, respectively. The optimum results are summarized in Table 4 together with the optimum results obtained by other optimization methods.

The result of Tsai [24] seems to be lower than the present results, but the result of Tsai [24] is not acceptable because one of the design constraints (defined by  $g_1$ ) is slightly violated in their study. Using the values of the design variables, the stress on a bar does not obey the stress constraint and the security of the system is not provided.

**Table 4** Optimization results

st_max	Park et al. [22]	Ray and Saini [23]	Tsai [24]	Yang and Gandomi [12]	Gandomi et al. [14]	Present study
$A_1$	0.78879	0.79500	0.788	0.78863	0.78867	0.78853
$A_2$	0.40794	0.39500	0.408	0.40838	0.40902	0.40866
$f_{\min}$	263.8965	264.3000	263.68	263.8962	263.9716	263.8958



### 3.3 Vertical Deflection Minimization Problem of an I-Beam

The FPA is also tested for the problem presented by Gold and Krishnamurty [25]. The optimization objective is to minimize the vertical deflection of an I-beam as shown in Fig. 6.

The vertical deflection of an I-beam is depended to design load ( $P$ ), length of the beam ( $L$ ), and modulus of elasticity which are taken as 600 kN, 200 cm, and 20000 kN/cm<sup>2</sup>, respectively. The load ( $Q$ ) in the other direction is taken as 50 kN.

The deflection of a beam is defined by

$$f(x) = \frac{PL^3}{48EI} \quad (16)$$

and the objective function of numerical example can be written as Eq. (17) when the design constants and the moment of inertia ( $I$ ) of the I-beam are defined in the Eq. (16).

$$\text{Minimize } f(b, h, t_w, t_f) = \frac{5000}{\frac{t_w(h-2t_f)^3}{12} + \frac{bt_f^3}{6} + 2bt_f\left(\frac{h-t_f}{2}\right)^2} \quad (17)$$

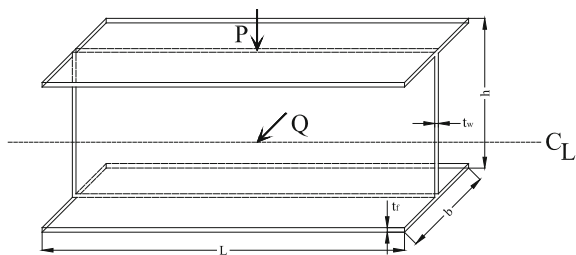
According to the objective function given in Eq. (17), the design variables are  $h$ ,  $b$ ,  $t_w$ , and  $t_f$ . The ranges of these variables are

$$\begin{aligned} 10 &\leq h \leq 80, \\ 10 &\leq b \leq 50, \\ 0.9 &\leq t_w \leq 5 \text{ and} \\ 0.9 &\leq t_f \leq 5. \end{aligned} \quad (18)$$

The cross section of an I-beam must be <300 cm<sup>2</sup> and the allowable bending stress of the beam is 6 kN/cm<sup>2</sup>. In that case, the cross section and stress constraints can be written as Eqs. (19) and (20):

$$g_1 = 2bt_w + t_w(h - 2t_f) \leq 300, \quad (19)$$

**Fig. 6** I-beam problem



**Table 5** Optimum results for I-beam example

	CS	ARSM	Improved ARSM	Present study
h	80.00	80.00	79.99	80.00
b	50.00	37.05	48.42	50.00
$t_w$	0.9	1.71	0.9	0.9
$t_f$	2.3216715	2.31	2.40	2.3217922
$F_{min}$	0.0130747	0.0157	0.131	0.0130741

$$g_1 = \frac{180000h}{t_w(h - 2t_f)^3 + 2bt_w(4t_f^2 + 3h(h - 2t_f))} + \frac{15000b}{t_w^3(h - 2t_f) + 2t_wb^3} \leq 6. \quad (20)$$

The optimum result obtained by the flower pollination algorithm was compared with the results by other methods such as the adaptive response surface method (ARSM), improved ARSM [26], and cuckoo search [12]. The results are presented in Table 5.

The optimum value of FPA has been obtained for 25 pollen agents and 5000 evaluations of design variables. With the increase in iterations, the algorithm stops if further improvement for the optimum results cannot be obtained. In the actual runs of the algorithm, the only improvement is the difference between the worst and best results in this case.

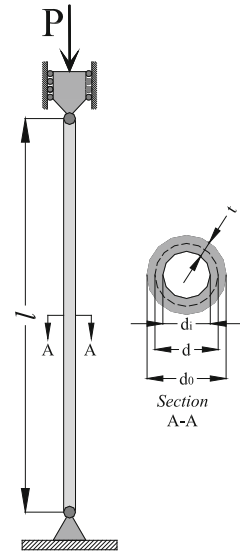
Comparing to the results of CS, a minor improvement of the optimum results can be seen, but in the engineering design, it is not very important. In addition to best optimum results, the convergence and minimization of computational time are also important for metaheuristic algorithms. Also, the same optimum results must be obtained for various runs of the optimization process. For a population of 25 pollens and for a fixed number of 5000 evaluations, the same results were obtained for every run. For 3000 evaluations, the best optimum results are generally found. Even to increase the number of evaluations to 15000, the maximum and minimum objective functions are generally the same. This shows the stability and robustness of the algorithm.

### 3.4 Cost Optimization of Tubular Column Under Compressive Load

The tubular column is shown in Fig. 7 [27]. The tubular column is axially loaded with a load ( $P$ ), and the upper and the lower bounds of the columns are supported from hinged bearings. The design constants of the optimization are shown in Table 6.

The types of constraint about compressive and buckling are important for the column. The compressive stress of the column must be lower than the yield stress of the tubular column. This constraint is given as Eq. (21).

**Fig. 7** Tubular column and A-A cross-section



**Table 6** Design constants of the tubular column

Symbol	Definition	Value
P	Axial force	2500 kgf
$\sigma_y$	Yield stress	500 kgf/cm <sup>2</sup>
E	Modulus of elasticity	0.85x10 <sup>6</sup> kgf/cm <sup>2</sup>
$\rho$	Density	0.0025 kgf/cm <sup>3</sup>
L	Length of column	250 cm

$$g_1 = \frac{P}{\pi d t \sigma_y} - 1 \leq 0 \quad (21)$$

The axial load must be lower than the buckling load of the column defined as the Euler buckling load:

$$P_{kr} = \frac{\pi^2 EI}{l^2} \quad (22)$$

where  $I$  is the moment of inertia of the tubular column section. When Eq. (22) is modified for the column section,  $g_2$ ; the constraint is formed as given in Eq. (23):

$$g_2 = \frac{8PL^2}{\pi^3 E d t (d^2 + t^2)} - 1 \leq 0. \quad (23)$$

The objective function is to minimize

$$f(d, t) = 9.8dt + 2d, \quad (24)$$

which is the sum of the material and construction costs of the tubular column.

The ranges of the design variables found in objective function can also be given as constraints. In this example, the diameter ( $d$ ) of the column must be between 2 and 14 cm, while the thickness ( $t$ ) of the column is a variable between 0.2 and 0.8 cm. These ranges are formulized as the following constraints:

$$g_3 = \frac{2.0}{d} - 1 \leq 0 \quad (25)$$

$$g_4 = \frac{d}{14} - 1 \leq 0 \quad (26)$$

$$g_5 = \frac{0.2}{t} - 1 \leq 0 \quad (27)$$

$$g_6 = \frac{t}{0.8} - 1 \leq 0 \quad (28)$$

The optimization is done for 25 pollens and 200 iterations. The total running time of the optimization algorithm is  $<0.1$  s. The statistical results of the optimization results are shown in Table 7. Nearly the best and the worst results are equal to each other.

The results are compared with the results by other methods and are summarized in Table 8. The FPA-based approach is more effective than the other methods. In addition, the convergence of the algorithm is very effective.

**Table 7** Statistical results of optimization of tubular column example

No. pollen	No. evals.	Best	Average	Worst	St. deviation
25	5000	26.4994969	26.499497	26.4994974	$1.699 \times 10^{-7}$

**Table 8** Optimum results for tubular column example

	Hsu and Liu [27]	Rao [28]	CS [12]	Present study
d	5.4507	5.44	5.45139	5.451160
t	0.292	0.293	0.29196	0.291965
$g_1$	$-3.45 \times 10^{-5}$	-0.8579	0.0241	$9.4343 \times 10^{-7}$
$g_2$	$1.32 \times 10^{-4}$	0.0026	-0.1095	$-4.249 \times 10^{-7}$
$g_3$	-0.6331	-0.8571	-0.6331	-0.6331
$g_4$	-0.6107	0	-0.6106	-0.6106
$g_5$	-0.3151	-0.7500	-0.3150	-0.3150
$g_6$	-0.6350	0	-0.6351	-0.6350
$F_{\min}$	26.4991	26.5323	26.53217	26.49948

### 3.5 Weight Optimization of Cantilever Beams

Two types of cantilever beams are optimized using FPA. In the first example, (Fig. 8) a beam with square section is investigated. Also, the inner part of the section is empty. The second example (Fig. 9) beam has a rectangular cross section.

#### 3.5.1 Weight Optimization of Cantilever Beams (Example 1)

The example given by Fleury and Braibant [29] is optimized by using FPA. The cantilever beam is shown in Fig. 8. The beam is rigidly supported from one end and the other end is free. A vertical load is applied from the free end of the beam. The cantilever beam is optimized for an objective function

$$\text{Minimize } f(X) = 0.0624(x_1 + x_2 + x_3 + x_4 + x_5) \quad (29)$$

subject to

$$g(X) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0. \quad (30)$$

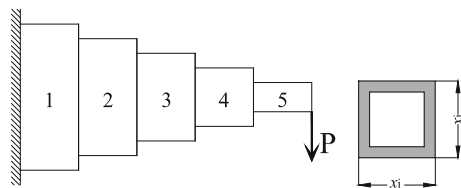


Fig. 8 The cantilever beam (Example 1)

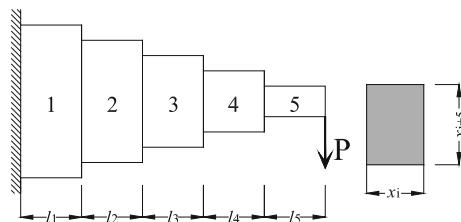


Fig. 9 The cantilever beam (Example 2)

The beam is divided into five steps with different cross sections. The thickness ( $t$ ) is taken as  $2/3$ , and it is fixed for each step of the cantilever beam. For all design variables from 1 to 5 ( $j = 1-5$ ), the following ranges

$$0.01 \leq x_j \leq 100 \quad (31)$$

are also taken into consideration.

The optimum results were compared with the results by CS [12] and other methods [30] as shown in Table 9.

The objective function is the same for all methods because the sensitivity of the results of the other methods is not known. Comparing to CS, it is possible to find the optimum results with 25 pollens and 300 search iterations while CS is performed for 50 cuckoos and 2500 search iterations.

### 3.5.2 Cantilever Beam Optimization (Example 2)

The cantilever beam shown in Fig. 9 contains ten design variables. This example was originally given by Thanedar and Vanderplaats [31]. The cross section of the beam is rectangular. The first five design variables are the width ( $x_1-x_5$ ) of the cantilever beam. The height of the beam ( $x_6-x_{10}$ ) is the other variable. The optimization objective is given below.

$$\text{Minimize } V = \sum_{i=1}^5 x_i x_i + 5l_i \quad (32)$$

The length of a step ( $l_i$ ) is fixed and 100 cm in value. The optimization is done considering 11 constraints formulized as

$$g_1 = \frac{600P}{x_5 x_{10}^2} - 14,000 \leq 0 \quad (33)$$

**Table 9** The optimum results of cantilever beam (Example 1)

Methods	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$F_{\min}$
MMA	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
GCA(I)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
GCA(II)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
CS	6.0089	5.3049	4.5023	3.5077	2.1504	1.33999
Present study	6.0202	5.3082	4.5042	3.4856	2.1557	1.33997

CONLIN CONvex LINearization, MMA method of moving asymptotes, GCA generalized convex approximation

$$g_2 = \frac{6P(l_s + l_4)}{x_4 x_9^2} - 14,000 \leq 0 \quad (34)$$

$$g_3 = \frac{6P(l_s + l_4 + l_3)}{x_3 x_8^2} - 14,000 \leq 0 \quad (35)$$

$$g_4 = \frac{6P(l_s + l_4 + l_3 + l_2)}{x_2 x_7^2} - 14,000 \leq 0 \quad (36)$$

$$g_5 = \frac{6P(l_s + l_4 + l_3 + l_2 + l_1)}{x_1 x_6^2} - 14,000 \leq 0 \quad (37)$$

$$g_6 = \frac{Pl^3}{3E} \left( \frac{1}{I_s} + \frac{7}{I_4} + \frac{19}{I_3} + \frac{37}{I_2} + \frac{61}{I_1} \right) - 2.7 \leq 0 \quad (38)$$

$$g_7 = \frac{x_{10}}{x_5} - 20 \leq 0 \quad (39)$$

$$g_8 = \frac{x_9}{x_4} - 20 \leq 0 \quad (40)$$

$$g_9 = \frac{x_8}{x_3} - 20 \leq 0 \quad (41)$$

$$g_{10} = \frac{x_7}{x_2} - 20 \leq 0 \quad (42)$$

$$g_{11} = \frac{x_6}{x_1} - 20 \leq 0 \quad (43)$$

The solution range are

$$1 \leq x_i \leq 5 \text{ for } i = 1 \text{ to } 5 \quad (44)$$

and

$$30 \leq x_i \leq 65 \text{ for } i = 1 \text{ to } 5 \quad (45)$$

In addition,  $P = 50,000$  kN and  $E = 2 \times 10^7$  N/cm<sup>2</sup>. The optimum results are presented in Table 10. The FPA algorithm is effective to find the minimum objective function value of the numerical example.

**Table 10** The optimum results of cantilever beam (Example 2)

	Thanedar and Vanderplaats [31]	Lamberti and Pappalettere [32]	Huang and Arora [33]	BA [14]	Present study
$x_1$	3.06	–	–	2.99204	2.98211
$x_2$	2.81	–	–	2.77756	2.77002
$x_3$	2.52	–	–	2.52359	2.51546
$x_4$	2.2	–	–	2.20455	2.19861
$x_5$	1.75	–	–	1.74977	1.74722
$x_6$	61.16	–	–	59.84087	59.94777
$x_7$	56.24	–	–	55.55126	55.6512
$x_8$	50.47	–	–	50.4718	50.58328
$x_9$	44.09	–	–	44.09106	44.17907
$x_{10}$	35.03	–	–	34.99537	35.02744
Best objective	63110	65352.2	63108.7	61914.9	61849.9

## 4 Conclusion

From the extensive discussions in this chapter, it can be concluded that the FPA is an effective and suitable algorithm for solving structural engineering problems. It is also easy to implement.

For the pin-jointed plane frame optimization problem, the best optimum results have been obtained by FPA comparing to GA [20] and CS [12]. In addition, the problem has been solved for different loads. As the load increases, the objective function (total length of bars) also increases.

The optimization results of three-bar truss system by the FPA have also been compared with the results by CS [12], bat algorithm [14], and several other methods [22–24]. The results of FPA are slightly better than the results of other methods without exceeding the design constraints.

For the third example, the vertical deflection of an I-beam has been minimized. An important reduction of the existing best results is not provided, but similar results were obtained with a slight improvement using FPA. The major advantage is the shorter computation time and the robustness of the method because the optimum results were obtained for a much lower number of function evaluations than that in CS [13].

The tubular column design under a compressive load is a major structural engineering problem. The results for this example have been compared with the results by CS [12] and several other approaches. The results obtained by FPA are more effective than others.

The last examples are about two types of cantilever beams and the weight optimization of structural elements has been carried out. Comparing with other



methods, the improvement of the result is not physically meaningful for the first cantilever beam example; however, the results are obtained after a much lower number of iterations comparing with those in CS [12]. For the second cantilever beam example with 10 variables and 11 constraints, FPA is very effective and has obtained much better results.

All the above confirm that FPA is a feasible algorithm for optimization in structural engineering by providing better designs with less computing time and improving the robustness of finding the best optimum values. The effectiveness of FPA can be attributed to the fact that it is a good combination of local search (self-pollination) and global search (cross-pollinations). It can be expected that FPA can be used to solve many other optimization problems.

## References

1. Yang, X.S.: Nature-Inspired Metaheuristic Algorithms. Luniver Press (2008)
2. Yang, X.S.: Engineering Optimization: An Introduction with Metaheuristic Applications. Wiley, New York (2010)
3. Goldberg, D.E.: Genetic Algorithms in Search, Optimization and Machine Learning. Addison Wesley, Boston (1989)
4. Holland, J.H.: Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor (1975)
5. Kirkpatrick, S., Gelatt, C., Vecchi, M.: Optimization by simulated annealing. *Science* **220**, 671–680 (1983)
6. Dorigo, M., Maniezzo, V., Colomi, A.: The ant system: optimization by a colony of cooperating agents. *IEEE Trans. Syst. Man Cybern. B* **26**, 29–41 (1996)
7. Nakrani, S., Tovey, C.: On honey bees and dynamic allocation in an internet server colony. *Adapt. Behav.* **12**(3–4), 223–240 (2004)
8. Kennedy, J., Eberhart, R.C.: Particle swarm optimization. In: *Proceedings of IEEE International Conference on Neural Networks No. IV*, 27 Nov–1 Dec, pp. 1942–1948, Perth Australia (1995)
9. Glover, F.: Heuristic for integer programming using surrogate constraints. *Decis. Sci.* **8**, 156–166 (1977)
10. Geem, Z.W., Kim, J.H., Loganathan, G.V.: A new heuristic optimization algorithm: harmony search. *Simulation* **76**, 60–68 (2001)
11. Erol, O.K., Eksin, I.: A new optimization method: big bang big crunch. *Adv. Eng. Softw.* **37**, 106–111 (2006)
12. Gandomi, A.H., Yang, X.S., Alavi, A.H.: Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems. *Eng. Comput.* **29**, 17–35 (2013)
13. Kaveh, A., Talatahari, A.: A novel heuristic optimization method: charged system search. *Acta Mech.* **213**, 267–289 (2010)
14. Yang, X.S., Gandomi, A.H.: Bat algorithm: a novel approach for global engineering optimization. *Eng. Comput.* **29**(5), 464–483 (2012)
15. Yang, X.S., Deb, S.: Two-stage eagle strategy with differential evolution. *Int. J. Bio-Inspired Comput.* **4**(1), 1–5 (2012)
16. Yang, X.S.: Flower pollination algorithm for global optimization. In: *Unconventional Computation and Natural Computation 2012. Lecture Notes in Computer Science*, vol. 7445, pp. 240–249 (2012)

17. Kaveh, A., Khayatazad, M.: A novel meta-heuristic method: ray optimization. *Comput. Struct.* **112–113**, 283–294 (2012)
18. Yang, X.S., Karamanoglu, M., He, X.: Flower pollination algorithm: a novel approach for multiobjective optimization. *Eng. Optim.* **46**(9), 1222–1237 (2012)
19. Majid, K.I.: Optimum design of structures. Newnes-Butterworth, London (1974)
20. Li, J.P., Balazs, M.E., Parks, G.T.: Engineering design optimization using species-conserving genetic algorithms. *Eng. Optim.* **39**(2), 147–161 (2007)
21. Nowcki, H.: Optimization in pre-contract ship design. In: Fujita, Y., Lind, K., Williams, T. J. (eds.) *Computer Applications in the Automation of Shipyard Operation and Ship Design*, vol. 2, pp. 327–338. Elsevier, New York (1974)
22. Park, Y.C., Chang, M.H., Lee, T.Y.: A new deterministic global optimization method for general twice differentiable constrained nonlinear programming problems. *Eng. Optim.* **39**(4), 397–411 (2007)
23. Ray, T., Saini, P.: Engineering design optimization using a swarm with an intelligent information sharing among individuals. *Eng. Optim.* **33**(6), 735–748 (2001)
24. Tsai, J.: Global optimization of nonlinear fractional programming problems in engineering design. *Eng. Optim.* **37**(4), 399–409 (2005)
25. Gold, S., Krishnamurty, S.: Trade-offs in robust engineering design. In: *Proceedings of the 1997 ASME Design Engineering Technical Conferences, DETC97/DAC3757*, 14–17 Sept, Saramento, California (1997)
26. Wang, G.G.: Adaptive response surface method using inherited latin hypercube design points. *Trans. ASME* **125**, 210–220 (2003)
27. Hsu, Y.L., Liu, T.C.: Developing a fuzzy proportional derivative controller optimization engine for engineering design optimization problems. *Eng. Optim.* **39**(6), 679–700 (2007)
28. Rao, S.S.: *Engineering optimization: theory and practice*, 3rd edn. Wiley, Chichester (1996)
29. Fleury, C., Braibant, V.: Structural optimization: a new dual method using mixed variables. *Int. J. Numer. Meth. Eng.* **23**, 409–428 (1986)
30. Chickermane, H., Gea, H.C.: Structural optimization using a new local approximation method. *Int. J. Numer. Meth. Eng.* **39**, 829–846 (1996)
31. Thanedar, P.B., Vanderplaats, G.N.: Survey of discrete variable optimization for structural design. *J. Struct. Eng. ASCE* **121**(2), 301–306 (1995)
32. Lamberti, L., Pappalettere, C.: Move limits definition in structural optimization with sequential linear programming. Part II Numer. Ex. *Comput. Struct.* **81**, 215–238 (2003)
33. Huang, M.W., Arora, J.S.: Optimal design with discrete variables: some numerical experiments. *Int. J. Numer. Meth. Eng.* **40**, 165–188 (1997)

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