Intuitionistic Fuzzy Logic and Provisional Acceptance of Scientific Theories: A Tribute to Krassimir Atanassov on the Occasion of His Sixtieth Birthday

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Abstract This essay attempts to outline some essential features of Atanassov's intuitionistic fuzzy logic within the framework of the philosophy of science. In particular, it aims to highlight the brilliance of Atanassov's conceptual and symbolic originality. It also illustrates the danger of the univocal caricaturing of scientific terminology.

1 Introduction

It is a pleasure to pay tribute to my friend and colleague on the occasion of his 60th birthday. We have been research collaborators for almost thirty years, and he has been a generous host on my twenty visits to his beloved Bulgaria over many years. He was a pioneer in the now burgeoning field of computational intelligence with its use of both traditional fuzzy logic [1] and intuitionistic fuzzy logic [2] in a variety of applications. He has also been an internationally renowned creator of new ideas and an insightful solver of problems for almost forty years, particularly in discrete mathematics [3]. These include his work on index matrices [4] and generalized nets [5]. The latter are a major advance on the first form of neural network [6].

This essay is more discursive and expository, rather than technical, particularly in relation to empirical sciences [7] in order to demonstrate the range and scope of Krassimir's fundamental ideas beyond fuzziness in the development of soft computing alone. "Fuzziness" itself is open to misinterpretation. It goes far beyond Russell's notion of vagueness [8] and his study of symbols (which have themselves been an important part of Atanassov's research as we shall show). This leads to a brief discussion on the equivocal misuse of scientific concepts: analogical caricatures make a weak argument weaker, not stronger!

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2 Science and Pseudoscience

Media reports of recent empirical challenges to the accepted understanding of the nature and speed of light have demonstrated the inadequate critical understanding of the experimental sciences in the popular mind, at a time when that same popular mind is being asked to make important bioethical and technological decisions vicariously through their representatives in parliamentary democracies. Recent public debates to the contrary science is not democratic!

Most science, even mathematics, is conducted in a mode of 'conventionalism' [9], which involves provisional acceptance of hypotheses—the 'probabilism' hinted at by Aquinas [10]. It is a purpose of this note to examine the foundations of this provisional acceptance within the context of intuitionistic fuzzy logic (IFL) [11].

The simplest explanation which fits the facts tends to be the prevailing confirmation in science. Scientists, being human, can be prone to disregard facts which do not fit this prevailing confirmation if their source is from authority less prestigious than the recognized authorities in their field. Argument from authority in science has historically hampered its progress. Scientific progress is usually marked by 'confirmation' [12] or 'refutation' [13], although in practice the working scientist operates within a framework which contains a collection of hypotheses where there can be disagreement between empirical data and individual hypotheses without destroying the theory as a whole [14].

At this working stage of provisional acceptance, somewhere between refutation and confirmation, the empirical support of the theory prevails over any alleged counter-example. Such was Einstein's attitude when he said that "only after a more diverse body of observations becomes available will it be possible to decide with confidence whether the systematic deviations are due to a not yet recognized source of errors or to the circumstances that the foundations of the theory of relativity do not correspond to the facts" [15].

Public intellectuals like Dawkins and Singer, for instance, at times like to use their eminence and expertise in one field to assert authority in another, even seeking though are seeking the truth. Thus I felt in Dawkins' earlier writings that he was almost drifting towards St Anselm's ontological proof for the existence of God. The god that Dawkins and his disciples are now trying to demolish though is the anthropomorphic god of the fundamentalists. His misdirected zeal ironically appeals to anti-theist fundamentalists. Singer too, while more consistently logical, fails to see in his own thinking the faults he attacks in the thinking of others [16]. Not for them the humility inherent in the title of James Franklin's recent book [17], but rather the arrogance of Atkins [18]: "… science has never encountered a barrier, and the only grounds for supposing that reductionism will fail are pessimism on the part of scientists and fear in the minds of the religious". That Dawkins has tried to use mathematical tools in his argumentation and Singer to dismiss them has motivated these comments.

3 Evidence

It is not surprising that moral relativists disparage any “search for certainty”. For them the only absolute is that there is no absolute. “It is as if to seek certainty denoted a lack of character, and were a sign of psychological or intellectual immaturity” [19]. Dawkins makes much of his view that “evidence”, as he defines it, is missing from religious belief. There are, it is true, some truths, such as the mystery of the Trinity which are inaccessible to reason in terms of existence and content. This does not make it unreasonable to believe them. It depends on whose authority we believe them. In any case a God we could fully understand would not be God—to extend St Anselm [20]. Yet scientists themselves believe some things on the basis of their nature rather than observation alone. Thus we believe it is in the nature of humans to be mortal. While nearly every textbook of introductory logic has the statement “all humans are mortal”, we know that all humans who have died must *ipso facto* be mortal, but we do not know it scientifically that all humans are mortal, because, as far as we know, most humans who have ever lived are alive today. We know that we are mortal from the study of natures, which is something we do in mathematics.

Yet for Dawkins the only evidence is scientific evidence, which itself is a metaphysical opinion, not a scientific statement. Moreover, Dawkins has no evidence that there is no evidence. Even the more persuasive Hitchens reduces his evidence to a series of anecdotes [21]. While some might say that these rebuttals are only playing with words, there are more serious underlying scientific issue relevant to the context of this paper.

These have been articulated in a series of papers by McCaughan who distinguishes extrinsic and intrinsic causes to show that even within science confusion of efficient and formality can lead to the domination of physics by mathematics to control all explanation, despite the fact that mathematics can do no more than predict [22]. Statistics too can disguise the existence of goal directed forces, but “goal directed forces eliminate blind chance. In following David Hume, scientists have removed goals or ends from science. This has not eliminated them from nature but left them unrecognised. Blind faith in blind chance just leads to intellectual blindness” [23]. We can see this in the way some evolutionary and generic algorithms are used analogously [24].

4 Genetic Algorithms

Genetic Algorithms (GAs) are an adaptive heuristic search algorithm based on analogies with the evolutionary ideas of natural selection and genetics [25]. Dawkins’ dichotomy is that we can have God or evolution but not both [23: 215] and so his goal is to use these algorithms to prove that we cannot have God. The basic techniques of GAs are designed to simulate processes in natural systems necessary for evolution, especially those that seem to follow the principles first laid down by Charles Darwin of "survival of the fittest". That GAs use evolutionary terms can be a trap for the unwary.

GAs are implemented in a computer simulation in which a population of abstract representations of candidate solutions to an optimization problem evolves toward better solutions [26]. The "evolution" usually starts from a population of randomly generated individuals and happens in generations. In each generation, the *fitness* of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness (optimal) level has been reached for the population. Once the genetic representation and the fitness (optimal) function are defined, GAs proceed to initialize a population of solutions *randomly*, then improve it through repetitive application of *selection* operators.

For instance, a Generalized Net model (which is essentially a directed graph with choices at the nodes) [27] when combined with IFL (which provides for non-membership as well as membership choices) [28], simultaneously evaluates several "fitness" functions and then ranks the individuals according to their fitness to choose the best fitness function in relation to what is being optimized. GAs require only information concerning the quality of the solution produced by each parameter set (objective function value information). The selection operator could be, for instance, a roulette wheel [29]!

Thus, a GA is an algorithm which has a beginning and which is goal directed in order to eliminate blind chance, but Dawkins, for example, has a goal as the end of his evolutionary algorithm but also, in effect, wants to have no beginning. Hawkins wants to have a beginning, but like Dawkins uses science to sidestep God [30].

5 Intuition

Like Dawkins, Peter Singer steps across into mathematics when he says: "…can we really know anything through intuition? The defenders of ethical intuitionism argued that there was a parallel in the way we know or could immediately grasp the basic truths of mathematics: that one plus one equals two, for instance. This argument suffered a blow when it was shown that the self evidence of the basic truths of mathematics could be explained in a different and more parsimonious way, by seeing mathematics as a system of tautologies, the basic elements of which are true by virtue of the meanings of the terms used. On this view, now widely, if not universally, accepted, no special intuition is required to establish that one plus one equals two- this is a logical truth, true by virtue of the meanings given to the integers 'one' and 'two', as well as 'plus' and 'equals', So the idea that intuition provides some substantive kind of knowledge of right and wrong lost its only analogue" [31].

The broad and very loose statement denying intuitionism as a valid form of knowledge in mathematics is difficult to understand and very contradictory, even without the existence of intuitionism in mathematics [32]. Bertrand Russell a hundred years ago attempted to reduce mathematics to 'tautologies' (logical truths) but it proved impossible.

Working mathematicians simply do not deny intuition. For example, the standard presentation of the foundations of mathematics includes the "axiom of infinity", which says "There exists an infinite set". You just have to take it (by intuition) or leave it. In no way is it a logical truth and no-one the least bit informed maintains it is [33]. Moreover, mathematicians do research by intuitive insights rather than by "symbol shoving" or even logic, though they justify their conclusions with logic acceptable to their peers [34].

Likewise, mathematical notation is more than a form of words; it is a tool of thought [35]. For instance, the relationship between powers and subscripts within the umbral calculus reveals ideas latent in the original mathematical language [36]. Here too Atanassov's symbolism has proved to be a powerful tool of thought even if we were only to judge it by the literature it has spawned. To see this we shall touch on some features of IFL.

6 Intuitionistic Fuzzy Logic

We shall now briefly outline the salient features of Intuitionistic fuzzy logic (IFL) by comparison with classical symbolic logic. IFL in many ways is a generalisation of the mathematical intuitionism of Brouwer [32] and the fuzzy sets of Zadeh [37].

In classical terms, to each proposition p, we assign a truth value denoted by *1* (truth) or *0* (falsity). In IFL we assign a truth value, $\mu(p)\ \varepsilon[0,1]$, for the degree of truth, and a falsity value, $v(p)\ \varepsilon[0,1]$ [4]:

$$0 \le \mu(p) + v(p) \le 1$$

This assignment is provided by an evaluation function V, which is defined over a set of propositions S,

$$V: S \to [0,1] \times [0,1]$$

such that

$$V(p) = <\mu(p), v(p)>$$

is an ordered pair. If the values V(p) and V(q) of the propositions p and q are known, then V can be extended:

$$
\begin{aligned}
V(\neg p) &= < \nu(p), \mu(p) > \\
V(p \wedge q) &= < \min(\mu(p), \mu(q)), \max(\nu(p), \nu(q)) >, \\
V(p \vee q) &= < \max(\mu(p), \mu(q)), \min(\nu(p), \nu(q)) >, \\
V(p \supset q) &= < \max(\nu(p), \mu(q)), \min(\mu(p), \nu(q)) >;
\end{aligned}
$$

and, for the propositions $p, q \varepsilon S$:

$$
\begin{aligned}
\neg V(p) &= V(\neg p), \\
V(p) \cap V(q) &= V(p \wedge q), \\
V(p) \cup V(q) &= V(p \vee q), \\
V(p) \to V(q) &= V(p \supset q).
\end{aligned}
$$

A *tautology* and an *intuitionistic fuzzy tautology* (IFT) are then defined respectively by

"A is a tautology" if, and only if, $V(A) = < 1, 0 >$;
"A is an IFT" if, and only if, $V(A) = < a, b > \to a \geq b$.

Provisional acceptance of a scientific theory means that an individual counterexample of empirical evidence can be related to an individual hypothesis within a theoretical framework in order to modify some of the individual constituents of the theory and thus accommodate the disagreement. This can be written as

(a) $T_1 \equiv ((A \supset C) \wedge \neg C) \supset \neg A$,
(b) $T_2 \equiv ((A \wedge B \supset C) \wedge \neg C \wedge B) \supset \neg A$,
(c) $T_3 \equiv ((A \wedge B \supset C) \wedge \neg C) \supset (\neg A \vee \neg B)$,

for every three propositional forms A, B and C. This leads us to

Theorem T_1, T_2, T_3 are IFTs.

Proof In the interests of brevity, we shall consider (b) only, as it is typical of all three parts.

$$
\begin{aligned}
V(T_2) &= [(< \mu_A, \nu_A > \wedge < \mu_B, \nu_B >) \to < \mu_C, \nu_C >] \wedge < \nu_C, \mu_C > \wedge < \mu_B, \nu_B > \to < \nu_A, \mu_A > \\
&= [< \max(\nu_A, \nu_B, \nu_C), \min(\mu_A, \mu_B, \mu_C) > \wedge < \min(\nu_C, \mu_B), \max(\mu_C, \nu_B) >] \to < \nu_A, \mu_A > \\
&= < \min(\nu_C, \mu_B), \max(\nu_A, \nu_B, \mu_C), \max(\mu_A, \mu_B, \nu_C) > \to < \nu_A, \mu_A > \\
&= < \max[\mu_c, \nu_B, \nu_A, \min(\mu_A, \mu_B, \nu_C)], \min[\nu_C, \mu_B, \mu_A, \max(\nu_A, \nu_B, \mu_C)] >,
\end{aligned}
$$

□

and

$$\begin{aligned}&\max[\mu_C, \nu_B, \nu_A, \min(\mu_A, \mu_B, \nu_C)] - \min[\nu_C, \mu_B, \mu_A, \max(\nu_A, \nu_B, \mu_C)]\\&\geq \min(\mu_A, \mu_B, \nu_C) - \min(\nu_C, \mu_B, \mu_A)\\&= 0. \quad \text{Therefore, } T \text{ is an IFT.}\end{aligned}$$

7 Concluding Comments

The existence of an additional working *modus operandi* between refutation and confirmation can clarify the way the empirical sciences work. Moreover, the schematic expression of this provisional acceptance of a theory invites an estimation of the truth values in any particular case so that the following type of analysis can be made. Suppose that for the propositional forms A and B:

$$\begin{aligned}&V(A) \leq V(B) \text{ if, and only if, } (\mu_A \leq \mu_B) \wedge (\nu_A \leq \nu_B),\\&V(A) > V(B) \text{ if, and only if, } (\mu_A > \mu_B) \wedge (\nu_A < \nu_B).\end{aligned}$$

If we assume that $\mu_A, \nu_A,$ the intuitionistic fuzzy values of A are fixed, then from the form of T_2 we see that T_2 is more reliable as the intuitionistic fuzzy truth of B increases, that is, the bigger μ_C and the smaller ν_c are.

The truth value of T_2 can also increase if any of

- $$(V(A) > V(B)) \vee (V(A) > V(\neg C)), \text{ for fixed } \mu_A;$$
- $$(V(A) < V(B)) \vee (V(B) < V(\neg C)), \text{ for fixed } \nu_A;$$
- $$(V(A) < V(\neg C)) \vee (V(B) < V(\neg C)), \text{ for fixed } \mu_B.$$

On the other hand, T_2 will not be changed if any of

- $$(V(A) \leq V(B)) \vee (V(A) \leq V(\neg C)), \text{ for fixed } \mu_A;$$
- $$(V(A) \geq V(B)) \vee (V(B) \leq V(\neg C)), \text{ for fixed } \nu_A;$$
- $$(V(A) \geq V(\neg C)) \vee (V(B) \geq V(\neg C)), \text{ for fixed } \mu_B.$$

Nevertheless, science should be no more exempt from moral evaluation than any other human activity, especially as it lacks the intellectual certitude of metaphysics and mathematics [38]. The logical analysis of 'provisional acceptance' will not make scientists more logical, but it is important that both scientists and the general public are aware of the nature and scope, including limitations, of science and especially the role of models within science. This is a realm open to research in psychology and philosophical anthropology, namely to relate the conceptual connection between intuition and perception as the link between the internal and external senses and the intellect. In the terminology of evolution, it is a missing link in our knowledge of heuristics.

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