Chapter 2
Hydraulics: The Importance of Observations and Experiments

Abstract Before the expansion of analysis, studies of fluid mechanics were necessarily confined to experiments although a complete body of technical applications of hydraulics had developed starting with the most ancient civilizations around the Mediterranean Sea and in Asia. Greeks, Egyptians and Romans, in particular, conceived all kinds of elementary machines but also well designed systems of distribution of water, whether in the fields or in the supply of cities. The true experimental aspects concerning this matter had to await the birth of the modern scientific spirit of the Renaissance period. The notion of pressure, the most elementary form of stress, appears with experiments performed by Torricelli, Pascal, and Mariotte. With these scientists and Newton, the first formulas appear, some just reporting pure empirical results, others founded on some rational reasoning. In the eighteenth century one witnesses parallel developments in the works of mathematicians (mostly, Clairaut, the Bernoullis, d'Alembert, Euler and Lagrange) and in the careful experiments of a group of gifted engineers (e.g., Borda, Bossut, du Buat,..). The role played by viscosity, but still in laminar flows, will be best captured by Poiseuille, Hagen, and others in experiments, and obviously by Navier, Saint-Venant and Stokes in analysis, before the consideration of turbulence. Accordingly, this contribution places the emphasis on experiments that were decisive in the perused evolution of continuum mechanics, from ancient times to the nineteenth century.

2.1 Introduction

Our personal vision of continuum mechanics is essentially theoretical and mathematical. It must be realized that before the works of our grand predecessors (the Bernoullis, d'Alembert, Euler, Lagrange, Cauchy, etc.) in the eighteenth century and early nineteenth century,[1] continuum mechanics may be identified in

[1]The reader may consult our book (Maugin 2014) for the history of that period.

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experimental developments and engineering material realizations, for instance, in so far as the statics and flow of some fluids are concerned. The first natural fluid of this kind was obviously **water**, from which the word **hydraulics** naturally follows from the combination of two Greek words, “hydra” (for water) and “aulos” (for pipe). This sounds very much like the flow of water in conduits, channels, and rivers that all ancient Egyptians, Mesopotamians, Persians, Greeks, Romans and other people around the globe had to consider of necessity in practical applications. The choice of water as the main object of study was particularly salient because it is a good example of a practically incompressible fluid that can transmit without much loss what will become known as the *pressure*, a force per unit area, and obviously the simplest form of our modern notion of stress. Thus hydraulics in its most elementary form certainly is the backbone of the fruitful development of notions that provided the basic ingredients in the theory of hydrodynamics and then continuum mechanics. Per force, very few relevant mathematical expressions were formulated before the eighteenth century, i.e. before sufficient expansion of mathematical analysis.

In accordance with its basic etymology, hydraulics is usually characterized as dealing with the physical behavior of water at rest or in motion, while the *Encyclopaedia Britannica* is more severe in speaking about “the practical applications of liquids in motion” and thus puts the emphasis on an applied field that for a long time was considered a part of civil engineering. Engineering and experimental sides cannot be overlooked as they provided a support for all inclusive ideas that developed in time between the ancient Greeks and the eighteenth century. Here we need not go back in detail to these ancient times for which there exist nice histories and reviews (e.g., Mays 2008). All the reader will retain are expressions like running water, water power, pumps, hydraulic machines, water supply to cities, etc., and the role of luminaries such as Archimedes (287–212 BC) in hydrostatics. What is really surprising to our modern scientific mind is that contrary to our common view of the development of a science, hydraulics first went through technological applications, then much later through a phase of experiments, and finally reached a theoretical level in hydrodynamics or, more generally, fluid dynamics. Thus, after a rapid perusal of technological developments where one cannot totally forget the material realizations of the Romans and of the Hellenistic period of which we inherited, one then usually jumps directly to the Renaissance period with true experiments that mark the birth of the practice of modern science.

2.2 Ancient Times: Hydraulic Technology

In ancient times, with the development of sedentary civilization and the associated agriculture, and the advent of some kind of urbanism, needs appeared to control the distribution of water to the fields in the form of organized irrigation, as also to bring water to cities. In both cases water had to be directed from some natural spring or a river to a specific place, usually at a lower level, and to bring it in the best conditions of delivery without too much loss on the way. Greeks and especially Romans—whom we consider the inventors of civil engineering—developed a special taste for the related technology, having determined the best conditions to have a descending flow, creating machines if necessary to take water to a higher elevation, and possibly devising true mega hydraulic projects for which the remnant Roman aqueducts in Italy, Spain and France provide the most spectacular examples. Cisterns and other means of water collecting and containment, the design of drainage channels, the construct of public fountains and the invention of animated fountains all are illustrations of this vivid development. Among the most impressive oldest inventions we cannot avoid citing the water screw by Archimedes (287–212 BC) in Syracuse (the founder of hydrostatics with his principle of buoyancy), and the force pump (or water lifting device) by Ctesibius or Ktesibios (circa 270 BC) in Ptolemaic Alexandria, Egypt, and later adapted by Roman engineers. Ctesibius is also supposed to have invented instruments using cleverly the properties of flowing water such as in hydraulic clocks and some musical instruments. All these are well documented in the text of Mays (2008) to whom we refer the reader. Of course we cannot say that these technical achievements came out of the blue. Although we cannot identify here any true scientific method, we must assume that some technical solutions—which in modern terms would involve the notions of pressure, flux through orifices, and certainly a bit of friction—were found only after a long series of trials and errors, a kind of archaic precursor of the scientific method. The Greek, Hellenistic, and Roman engineers greatly influenced those of the Renaissance (cf. Gille 1964, First two chapters), in particular via Vitruvius' teaching (cf. Vitruvius ca 25 BC).

2.3 The Renaissance Experimentalists-Thinkers: Leonardo, Stevin, Galileo Galilei

With the Renaissance period that we grossly delineate by the dates 1453 (fall of Constantinople)—or the invention of printing by Gutenberg circa 1450—and the introduction of the Baroque style in arts or the successful career of Shakespeare in drama or still the death of Galileo Galilei in 1642, we witness a renewed interest in the antic culture and a characterization by the reading and re-evaluation of fundamental texts. This applies well to both science and technology. For us, the symmetric, perfectly geometrically proportionate, representation (cf. Fig. 2.1) of Vitruvius' man by Leonardo da Vinci (1452–1519) is an ideal that directly connects with the preceding section. Indeed, Leonardo is an unavoidable figure including in our present subject of interest, hydraulics. At this point, we must mention the thorough studies of Duhem (1906–1912) on Leonardo. By studying in depth so many original texts from the Middle Ages (from the masters at the Sorbonne or in Oxford, etc.) and the immediate Pre-renaissance Italian scientists, Duhem tried, and sometimes seems to have succeeded, to trace what may have influenced Leonardo,

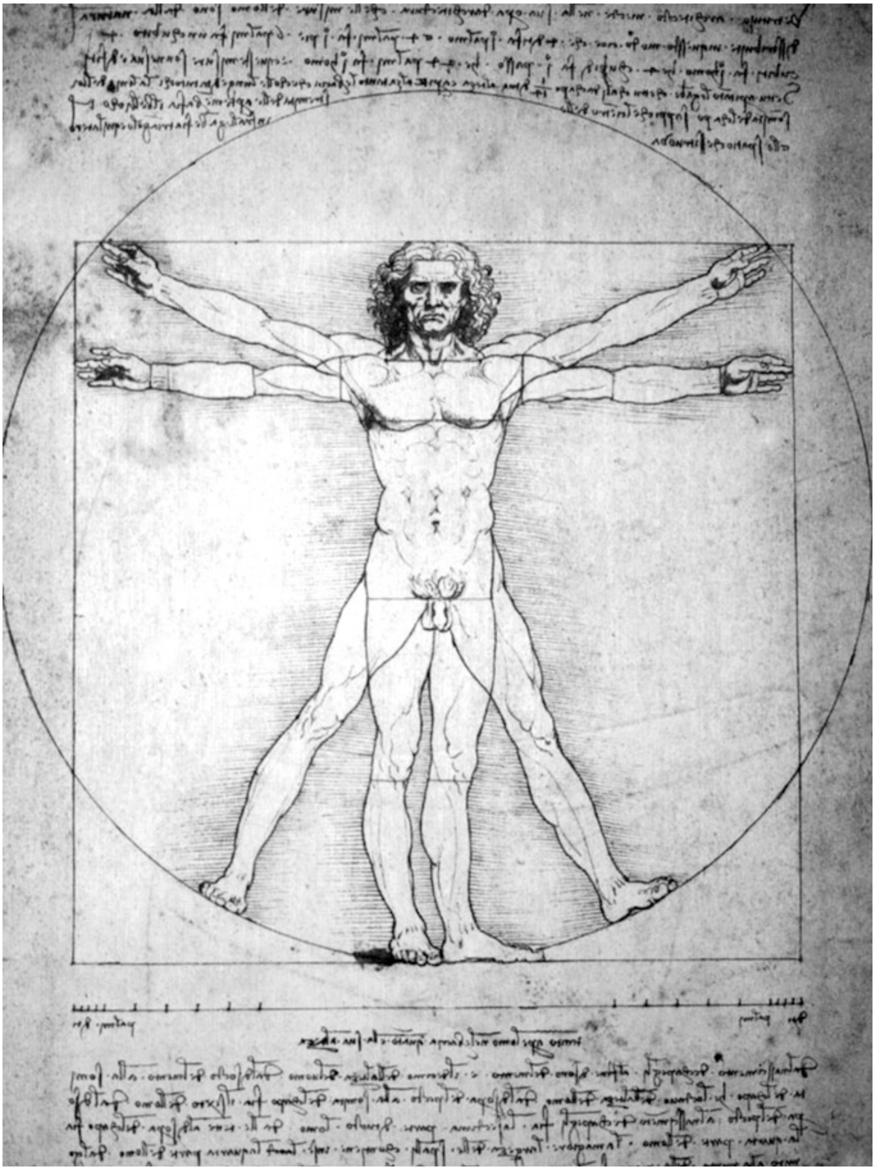

Fig. 2.1 Vitruvius' man by Leonardo da Vinci

among his predecessors in the Middle Ages, and what he may have read, although Leonardo himself does not provide any clues. This has remained a discussed matter among historians of science (including A. Koiré, Th. Kuhn, etc., who are in favour of a clear discontinuous evolution of science such as in the change of paradigm, while Duhem leans toward a greater continuity, from what his thesis follows).

In the present case, Leonardo is not a scientist in the modern sense of the word, but he is a great observer of Nature and a true engineer's mind. We can say with Truesdell (1956, p. 4), that Leonardo is "an observer of the undisturbed nature". His graphic talents are here extremely useful because a good accurate picture can replace many words. Whether his drawings represent a report on a true (well designed) experiment or just a visual observation is not clear, although the second avenue is more likely. But Leonardo, like some Japanese artists of woodprints (e.g., Hokusai) in the eighteenth and nineteenth centuries, captures very well the intricacy of some flows with vortices and fantastic waves. But this is too involved, and one must start with some more quiet situations, as a matter of fact, *hydrostatics*. In so far as theoretical thinking is concerned, Leonardo seems to be somewhat limited, but we must attribute to him some kind of recognition of a *principle of continuity* in a simple form (cf. Truesdell 1956, p. 6). His explanation for the fact that fluids such as water at the top of mountains or blood in the human head can go all this way up is erroneous (cf. comments by Dugas 1950, p. 80). But his vision of the flux of a fluid is more appropriate according to Duhem (1906–1912, First volume, p. 198). In effect, in his "Del moto e misura dell'acqua" Leonardo states that "all motion of water of uniform width and surface will be faster in one place than in another one all the more that this water is less deep in one case than in the other". He also sketches out a theory of hydraulic pumps that, in some sense, prefigures Pascal's findings. So much for Leonardo's accomplishments that we do not want to belittle in any way.

We may consider that Leonardo is at the turning point of the birth of a true modern science. For pedagogical purpose this birth is often attributed to a few individuals such as Francis Bacon (1581–1626), a developer of an empirical theory of knowledge with a specification of rules in the experimental method (See his "Novum Organum", 1620), and Galileo Galilei (1564–1642), a true mathematician with a taste for experiments. But Simon Stevin (1548–1620), a Flemish all round scientist (mathematician, mechanician, engineer and physicist, much concerned with the basic principles of mechanics and the non existence of perpetuum mobile—perpetual motion), is also to be considered. Stevin brought to the science of hydrostatics two fundamental ideas. One is the *principle of solidification* according to which a solid body of any shape but with the same density as water can stay in equilibrium in water in any position without altering in any amount the pressure field in the rest of the fluid. This helped him to determine the pressure exerted on each element of the base of the container by solidifying thus the whole fluid save a fine channel leading from the open surface to this element. As a result the pressure is independent of the shape of the container; it depends only on the weight of the column that fills up the channel. Another consequence of this is the so-called *hydrostatic paradox*. According to this paradox, the total force—resultant sum of pressures—that a fluid can exert on the bottom of a vase can be considerably larger than the weight of the same fluid. Another result of Stevin deals with the resultant of pressures acting on an inclined plane. He used here a reasoning that involves an astute limit procedure by cutting the surface of this plane by a succession of small horizontal slices and then increasing indefinitely the number of slices. He finally related Archimedes' principle to the impossibility of a perpetual motion. He is a direct predecessor of Galileo Galilei although his publications in Flemish—a language of which he was an ardent propagandist—may have been missed by Galileo.

Galileo Galilei is a true mathematician in that he does provide formulas based on mathematical reasoning. He is also a true experimentalist because he performed experiments that were purposefully designed. He is in quest of checking a hypothesis but may also arrive at an unexpected result. This is the quintessence of Francis Bacon's advised method. But regarding hydrostatics, he first borrows elements to Aristotles' mechanics. According to his "Discorso intorno alle cose che stanno in su l'acqua o che in quella si movimento" (Galileo 1612) one such principle is that "two weights moving with equal velocities are endowed with the same power or *momento* in all their operations" (Dugas 1950, p. 138), a statement in which we can see an application of the principle of virtual velocities as conceived by Aristotle. A second principle along the same line is that "the power of gravitation grows with the velocity of the moving object". Accordingly, absolutely equal weights moving at different velocities have unequal *momenti*: the most "powerful" one is that one that moves faster than the other, and this in proportion of its velocity compared to the velocity of the other one. From this one deduces, following Aristotle, that two weights of unequal magnitude can balance reciprocally one another and will be endowed with equal *momenti* each time that their "gravity" will be in the inverse ratio of the velocity of their motion. Galileo applies this reasoning to the siphon where a small quantity of water contained in a narrow vase can equilibrate a large mass of fluid in a wide vase as a small decrease in height in the second vase will correspond to a large level increase in the former. If we are to believe Duhem (1906–1912, Volume 2, p. 214), this prefigures Pascal's vision (see below). Galileo also brings the properties of floating bodies to an application of the principle of virtual velocities. It is noted by Dugas (p. 139) that Galileo still believed in the "horror of vacuum" ("resistenza del vacuo"). But he is pragmatic enough to accept the evidence that a newly developed pump cannot elevate water higher than a certain limit, thus in contradiction with this "horror of vacuum".

2.4 Seventeenth Century Experiments: Torricelli, Pascal, Mariotte

If we keep with our own chronological delineation of the Renaissance, then we must admit that Evangelisto Toricelli (1608–1647) worked at the final period of this prestigious epoch. Although interested in all things mechanical, Torricelli left an indelible trace in the history of hydraulics with his law of fluid flow ("efflux") through a small orifice placed at the bottom of a vase (container, tank). Exploiting an analogy with the fall of heavy objects, he observes that the coming out water cannot by itself rise again at an elevation larger than the one of the fluid in the vase;

while this elevation would be the same if there were no resistance at all in the system. As a conclusion he formulates—but does not prove—his famous law according to which the velocity of the flowing water at the orifice goes like the square root of the height of the fluid in the vase. Both Newton and Varignon (1725) will have to ponder this matter again. But Torricelli's law acted as a true motive power in further developments in hydraulics. Note that Torricelli improved on others' experiments by using mercury (known as quicksilver in his time) instead of water.

The next scientist in our gallery of distinguished "hydraulicians" certainly is Pascal. Blaise Pascal (1623–1662) is a French philosopher and religious thinker. His interests cover a wide spectrum both in humanities and sciences. He may be considered a somewhat unorthodox catholic intellectual,[2] but also an excellent geometer, an inventor (of a computing machine), and an amateur physicist who gave his name to a unit of pressure, the "pascal" (one newton per square meter). It is this last point that requires our attention. The story is told in all books for French teenager apprentice scientists in order to encourage them to engage in a scientific career. Pascal repeated Torricelli's experience in 1644 in Rouen, Normandy, with a friend called Petit. Then convinced that Nature does not have horror of vacuum, he projected in 1647 an experiment that he indeed conducted on September 19, 1648 at the Puy-de-Dôme mount in central France. This experiment proved that, in Pascal's words, "fluids weigh more given their height" since he observed a change in the level of mercury in a vertical tube depending on the altitude at which the experiment was conducted. The result was printed in October 1648 while Pascal's "Traité de l'équilibre des liqueurs" (Treatise on the equilibrium of liquors") was published in 1663. In other words, he demonstrated the variation of atmospheric pressure with altitude. A corollary of this result was the first well established definition of *pressure*. "A vessel full of water is a new machine to multiply a force to any desired degree" since a fluid in a small vertical tube connected to a much larger reservoir can balance the large amount of fluid in the corresponding height in the reservoir. We face here the observation that the cause of equilibrium in all examples treated by Pascal stems from the fact that the "matter that extends from the bottom of a vessel up to its open surface is indeed liquid". This shows the possibility of incompressible fluids such as water to transmit pressure without loss. This is usually referred to as Pascal's principle. It contains the basic idea for the conception of a *hydraulic press*, what was indeed industrially realized by Samuel Bramah (1748–1814), a British inventor, with the deposit of a patent for such a machine in

[2]Pascal is often presented as an opportunist and pragmatic—within the framework of apologetics—who considered a surprising probability reasoning (Pascal was interested in gambling games) and dared to bet on the existence of God (the celebrated "Pascal's wager"): In short, we can say that it is "safer" to believe in the existence of God than in his nonexistence; the odds are better this way since there is nothing to lose in making this bet. In truth this "bet" is best understood as an expression of the mathematical expectation, i.e. the product of gain and probability (a small, nonzero probability can produce a large expectation). The main writings of Pascal in both science and philosophy are collected in his "*Pensées*" (Thoughts). The author was offered a reprint copy of this thick book by his elder brother when he was in high school. No need to say that this was very difficult reading that went much beyond his understanding.

1795. Indeed, the press has two cylinders and pistons of different cross-sectional areas. If a force is exerted on the smaller piston, this will be translated into a large force on the larger piston. The difference in the two forces will be proportional to the difference in area of the two pistons (remember that pressure equals force per unit area). In effect, the cylinders act in a similar way that a lever is used to increase an exerted force. This sounds very much like another statement of the principle of virtual displacements. In modern technology one often meets the word “oil-hydraulics”, what simply means that oil has replaced water in the process (although the Greek root for “water” has remained in the word!).

The Abbé Edmé Mariotte (1620–1684) appears to be a true experimentalist who dealt with solids, fluids and gases. He published on the laws of shocks of bodies (See his “Traité sur la percussion des corps”) as well as on the flow of fluids (see his “Traité du mouvement des eaux” 1684). As a member of a committee in charge of checking Torricelli’s law, he indeed checked this law, but missed the constriction of the emerging flow. This will be corrected by Newton in a second edition of his *Principia*. Mariotte also showed the importance of the deviation of the momentum in the experience of the impinging of a flow on a flat surface. He is a pioneer in considering size effects and introducing thus an embryonic form of a law of similitude in the study of the resistance of fluids where he formulates proportionality between resistance and the squared velocity. He also looked at the velocity of a flow in channels or rivers, and envisaged the action of friction in the flow of water in pipes. These achievements, of course, may appear modest compared to Mariotte’s empirical determination of the famous Boyle-Mariotte law of the elasticity of ideal gases according to which the volume of such a gas varies like the inverse of the pressure at mass and temperature kept constant. Mariotte’s result was obtained in 1676 but Robert Boyle (1627–1691) had proposed the same law earlier in 1662. This empirical result will later on be justified by the kinetic theory of gases.

Of course Isaac Newton (1643–1727) could not have left unexplored any branch of mechanics and physics of his time. No wonder, therefore, that he paid some attention to flows and undertook to validate Torricelli’s law. In his interpretation of the free fall of water, each particle acquires its velocity in free fall so that Torricelli’s law is verified. But his solution concerning a vertical vase with an orifice at its base contradicts the principles of hydrostatics (Dugas 1950, p. 216). It is Pierre Varignon (1654–1722), a French Jesuit and mathematician, better known for his proposal of composition of forces and his analytic definition of velocity and acceleration, who gave a more natural explanation of Torricelli’s law (Varignon 1725). According to this explanation recalled by Dugas (1950, pp. 216–217), each fluid particle receives instantaneously a finite velocity at the exit of the vase through the effect of the column of water above it. Accounting for the flux, the so generated quantity of motion is proportional to the square of the velocity. But the weight of the fluid column is proportional to its height h, from which there follows the law $h \approx v^2$.

2.5 Eighteenth Century Theoreticians: Clairaut, Daniel Bernoulli, D'Alembert, Euler[3]

With the post-Newtonian and Leibnizian mathematical achievements, a new era starts that will harmoniously mix analysis and experiments, perhaps with an advantage for the former that we examine first.

Although much less known in continuum mechanics than in mathematics, Alexis Clairaut (1713–1765)[4] contributed to hydrostatics through his marked interest in geophysics and the equilibrium figure of the earth in particular (see his book: Clairaut 1743). Of necessity he had to envisage the equilibrium of water volumes under the action of the earth's gravity. The equilibrium will decide on whether the earth is a perfect sphere or is more or less flattened at the poles (in agreement with Newton) since there existed conflicting theories about the actual shape of the earth. In looking at this problem, Clairaut applied the strategy proposed by Stevin of the solidification of fluid volumes, considered fluid masses in motion (rotation) and practically introduced the notion of *potential* in mathematics (cf. Dugas 1950, p. 273). As a matter of fact, the general purpose of Clairaut's research was to find the laws of hydrostatics that are in accord with all hypotheses on gravity (work of 1734). Lagrange (1788, p. 128) emphatically said that Clairaut had "changed the face of hydrostatics to create out of it a new physical theory". The equilibrium principle enounced by Clairaut that provides a necessary and sufficient condition is that "a fluid mass can be in equilibrium only in so far as all parts that are contained in a channel of any form that traverses the whole mass, do not destruct each other" (Clairaut 1743, p. 1). From the application of this almost trivial principle, Clairaut deduces that the total weight of a column of fluid of constant section and density extending from the center of the earth to the equator or to a pole is none other than the "effort" on the column. This is obtained by integration over the height of the column. Applied to a short element of a channel this yields that the gradient of the "pressure" is balanced by the components of gravity. This result must be independent of the precise form of the channel, from which it follows that the spatial variation of pressure should be an exact differential. In a rotating mass, this transforms into the fact that the sum of the potential of gravitation—without specific hypothesis on its expression—and of the potential of the centrifugal force is a constant on a level surface. The vanishing of the variation of this potential yields the surface of equilibrium that is more than often perpendicular to the action of gravity as suggested earlier by Huygens. The spheroid itself will be such a level surface. On using the Newtonian gravitation law this indeed provides the flattening of the earth at the poles.

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[3]The pioneering works on the foundations of continuum mechanics in the eighteenth century were surveyed in our book (Maugin 2014).

[4]For the life of Clairaut, see Brunet (1952). For Clairaut's original contribution to hydrostatics, see Passeron (1994) and the corresponding extract of 2009. The analysis of Dugas (1950, Chap. 7) is also very informative.

The expedition to Laponia led by Maupertuis—but supported by Clairaut—in 1736–1737 proved that a degree of an arc of meridian is larger at the north pole than in France, so that the hypothesis supported by Newton and Clairaut seems to be correct, but with a flattening of the order of 1/300 th only, i.e., much below Newton's estimate. This will be corroborated by more recent precise measurements. Clairaut, however, concluded from the discrepancy with Newton' proposal that the earth is probably inhomogeneous with a density increasing with distance from the centre. It is of interest to note in conclusion that Lagrange (1788, p. 129) finds in Clairaut's analysis the idea that pressure is the same in all directions, i.e., is isotropic although it is Euler who really formulated this essential property.

Practically contemporary of Clairaut we find Bernoulli (1700–1782) whose name remains for ever attached to a famous theorem which in modern standard notation and for a stationary flow reads

$$p + \frac{1}{2}\rho v^2 + \rho g h = const. \tag{2.1}$$

Here we have reinstated the pressure p that Bernoulli did not conceived as such. No need to emphasize the tremendous role played by this equation in all further developments in theoretical fluid mechanics, simple applications, and aeronautics. As noticed before (Maugin 2014, p. 10), this was not the only result of Daniel who produced a fundamental book on hydrodynamics written in 1734 (Hydrodynamica 1738). To the bewilderment of many, this scientific achievement by his own son caused a burst of envy and jealousy from the shameless John (Johann) Bernoulli (1667–1748). He hastily published a competing book with the title *Hydraulica* in (1739). He even anti-dated the date of writing this opus to 1732 to pretend to a (false) priority! But, to the credit of John, we must acknowledge that his book also had many merits. In particular, it presented the first successful use of the *balance of forces* to determine the motion of a deformable body. This was possible for John because he had recognized that "the fluid on each side of an infinitesimal slice pressed normally upon that slice, with a varying force which was itself a major unknown" (Truesdell 1968, p. 121). With this we are very close to the notion of *internal pressure* and a concrete view of *contiguity* of action in continuum mechanics in a line that both Euler in the period 1749–1752 and Cauchy in 1823–1828 will expand. Also, John was the first to practically give the above modern form to his son's theorem. As to Daniel, we must also note that he combined the notions of "pressure" and motion, being guided by the conservation of living forces in the sense of Leibniz. He also considered a motion by slices that are perpendicular to the direction of motion, all particles in the same slice having the same velocity that is inversely proportional to the considered section. This is very close to our modern view of the flow problem in a tube. Daniel again discussed the principle of living forces at the time of the death of his father (cf. D. Bernoulli 1748), a principle to which he was strongly committed.

The great Leonhard Euler (1707–1783), perhaps more than the Bernoullis, is to be credited for many definitive results in hydraulics and continuum mechanics in a general manner. This, of course, is the viewpoint of Truesdell (1984, pp. 212–217) whose real god in mechanics is none other than Euler. As emphasized earlier, *pressure* is the main constructive concept in hydraulics. Nobody did more to clarify this notion than Euler for whom, much influenced by the recent progress in the theory of hydraulics by the Bernoullis, father and son, *pressure* can be seen as the action "*from all sides* and *from neighbouring elements of fluid on an isolated element of fluid*" (a "particle"). In modern terms, it is *isotropic* and, with Euler, will be viewed as normal force acting on an element of surface. This means that Euler has reached the idea of pressure in a quasi-modern sense—while Lagrange attributed it to Clairaut-, and that the notion of *contiguity* is thus definitely formulated. Furthermore, pressure becomes a true *field* that depends on both space and time in the general case of dynamics. With the additional inception by (Euler 1757) of the balance equations in Eulerian format, we have then at hand the basic formulation of the field theory of perfect fluids. This brief discussion on fluids may make the reader believe that Euler had no notion of a tangential force. But this is not true because Euler himself dealt with this notion in a problem of solid mechanics—the elastica—that had been examined by James (Jacob) Bernoulli a long time before. The high value of Daniel Bernoulli's and Euler's seminal works in hydraulics is emphasized by Truesdell (1956). Daniel conducted some experiments in parallel with his mathematical approach, while Euler was for ever a pure theoretician.

In referring to Euler we have bypassed Jean Le Rond d'Alembert (1717–1783) who left a quantity of innovative works in the science of Mechanics in his young age. Perhaps not in the same class of mathematicians as Euler with whom he had many discussions and exchanges of correspondence, and certainly not as clear as Euler in his writing and proof reports, d'Alembert must be singled out for his originality and his general view of mechanics in a Leibnizian tradition. Thus in 1744, this gentleman, well educated in the best college in Paris, but mostly self-taught in mathematics, published his own book on the emerging fluid mechanics (D'Alembert 1744) after his publication of a celebrated Treatise on Dynamics in 1743. Furthermore, in 1752 d'Alembert obtained correct partial differential equations for axially symmetric and plane flows (of the type now called irrotational flows; cf. D'Alembert 1752). This is one of the first considerations of a two-dimensional motion of a continuum. He had already introduced for the first time the notion of *partial differential equations* in a previous work of 1743 on the mechanics of a heavy hanging rope. As noticed by Truesdell (1968, p. 228), d'Alembert does not speak of "pressure" but of "forces" that are viewed as "reversed accelerations". This fits well in d'Alembert's vision of reducing hydrodynamics to hydrostatics in accord with the general approach to mechanics he had given in his treatise of 1743. D'Alembert was also responsible for the introduction of the notion of *stream function* while Euler had introduced that of *velocity potential*. And we cannot avoid mentioning the d'Alembert paradox for the absence of drag on a cylinder placed in a perfect fluid flow. This was proved by d'Alembert in 1750. Of course, this theoretical result is contrary to common experience. A resolution of this paradox could be given only with the introduction of discontinuities in the flow field and the notion of wake.

In a general manner Truesdell has an exaggerate tendency to belittle the importance of d'Alembert's works as compared to Euler's. Thus he claims concerning d'Alembert's treatise of 1744 that (Truesdell 1968, p. 227) "this entry of a newcomer [he means d'Alembert] in the field added nothing to the subject". He repeated the same kind of argument in his review of the book by Rouse and Ince (1957) on the History of Hydraulics (Truesdell 1984, pp. 212–217). There he emphasized the inaccuracies introduced by Lagrange (1788) in his historical introduction on the history of fluid mechanics; he claims that Lagrange wrongly attributes successes to d'Alembert and also Clairaut, and doing so does not hesitate to introduce some innuendos (e.g., tactic fully making a remark that d'Alembert was the "patron of the career of Lagrange in France"; p. 213) or demonstrating (p. 215) that his translation of the Latin text of Daniel Bernoulli is much better than the one produced by others (e.g., Dugas—who certainly knew his Latin like all people of Dugas' calibre highly educated before the First World War—or Rouse and Ince). In spite of these harsh criticisms, we note that d'Alembert provided the bases on which Lagrange was going to build his grand scheme of mechanics.

Lagrange (1736–1813) is the last great theoretician of mechanics in the eighteenth century. His beautiful contributions to fluid mechanics need not be reported here again. Suffice it to remind the reader of the Lagrangian formulation of (perfect) fluid mechanics and his solutions of various wave problems in fluids such as the flow in a shallow channel of small slope and uniform width for which Lagrange has shown that the speed of propagation of a wave is proportional to the square root of the depth of the channel.

Toward the end of the eighteenth century we still note the contribution of Lazare Carnot (1753–1823), successful politician, military organizer, but also mechanician of value. He exploits the principle of solidification of Stevin and Clairaut in his consideration on hydraulics which chronologically precedes the treatise of Lagrange although published only in Carnot (1803). Carnot contributed to the proposal of the empirical Borda-Carnot equation (see below).

2.6 The True Experimentalists: Borda, Bossut, and Du Buat

We qualify Jean-Charles de Borda (1733–1799), Abbé Charles Bossut (1730–1814) and Pierre du Buat (1734–1809) of true experimentalists as these scientists devised special purposeful experiments and studied the influence of various parameters. With these scientists we arrive at the study of the resistance of fluids on their flow in real situations, what is obviously of great importance in applied hydraulics. With the new account of essential friction effects this contrasts with the theoretical studies of d'Alembert, Euler and Lagrange.

The "Chevalier" de Borda—as he is often called—in fact was a military naval engineer with a taste for mathematics and physics who fulfilled various state missions throughout the world as a navigator and hydrograph (cf. Mascart 2000). He demonstrated remarkable qualities as an experimentalist in fluid mechanics in works performed in the 1760s and 1770s. Among his successful studies we must record his study on the resistance offered by air to a moving material surface. This effect is a global one that cannot be obtained by integration over simple elements. The global resistance is carefully shown to be proportional to the square of the velocity and to the sine of the incidence angle (not to the squared sine as proposed earlier by Newton). In the case of water he checked the law of proportionality to the squared velocity. He observes that this resistance decreases with depth in water and that it may grow faster than the square of the velocity at the surface. To explain the phenomenon he exploits a theory of loss of living forces that is not entirely convincing. In all, his conclusion is that the theory put forward by Newton (who used the image of the shock of fluid particles on the obstacle) is not valid and should not be applied to the motion of boats. The considerations about the loss of living forces are of special interest because they followed from a divergence from the famous Bernoulli formula (2.1). First of all the idea is original. Second, the result is corroborated by experiments. We remind the reader that this well known problem concerns the flow of a fluid by an orifice standing for a sudden expansion or contraction of the flow, say in a horizontal pipe. Carnot revisited the problem, hence the often given name to the obtained formula, the *Borda-Carnot equation*. Using modern notation, this equation can be stated in the following simple form (for decreasing velocity):

$$\Delta E = \frac{1}{2}\xi\rho(v_1 - v_2)^2, \tag{2.2}$$

where ΔE is the loss of mechanical energy by the fluid—or dissipated kinetic energy-, ρ is the fluid density, v_1 and v_2 are the mean flow velocities before and after expansion, and ξ is an empirical nondimensional coefficient with value between zero and one (in fact equal to one for an abrupt and wide expansion). The energy quantity ΔE is none other that the variation in the “constant” in Daniel Bernoulli’s equation (2.1) along a streamline. For an open channel flow we have

$$\Delta E = \rho g \Delta H, \quad H := h + v^2/(2g), \tag{2.3}$$

where $h = z + p/(\rho g)$ is called the total *head* in hydraulics, i.e., the free surface elevation above a reference height. In practice, the flow separates and one observes a *turbulent* recirculating zone after expansion. Separating recirculating zones are also observed at the entrance of a narrower pipe in the case of contraction. The flow is contracted between the separated flow areas and then re-expands to cover the full pipe section. The contraction coefficient for a sharp-edge contraction was experimentally determined by Julius L. Weisbach (1806–1871) in the nineteenth century (cf. Weisbach 1855). The original bold idea of Borda (1766) was to assimilate the hydraulic phenomenon to a shock with loss of living force, that is, a shock between *hard* bodies according to the terminology of the early eighteenth century (in the controversy between momentum and vis-viva as the conserved quantity in mechanics). Borda also contributed to the theory of hydraulic wheels (cf. Borda 1767, 1786).

The Abbé Charles Bossut is a personality who differs completely from that of Borda. We do not know of any adventures through the world by this quiet immovable priest. We noted in our book (Maugin 2013, p. 7) that he was "*a disciple of d'Alembert and a specialist of hydrodynamics* (cf. Bossut 1771)*, but also an underestimated historian of mathematics* (cf. Bossut 1810), *and a remarkable pedagogue. His course of mathematics at the Military school of Mézières was first published in 1781. Its last edition* (1800) *was in seven volumes, of which two were devoted to differential and integral calculus in the Leibniz notation. This had a decisive influence on the rebirth of mathematical physics in the UK. He was a colleague of Laplace and Lagrange at the Paris Academy of Sciences, but not in the same class as these two mathematicians-mechanicians from the point of view of creativity*". He was a long time examiner (1796–1808) in mathematics at the Ecole Polytechnique after the creation of this school. However, the surprise here is provided by the outstanding qualities he demonstrated as a great experimentalist in a series of crucial experiments in fluid dynamics.

In truth, a team composed of d'Alembert, Nicolas Caritat de Condorcet (1743–1794)—another protégé of d'Alembert—and Abbé Bossut at the Paris Academy of Sciences was asked by the King's government to investigate the means of improving the navigation of ships. But the corresponding experiments were conducted, and the resulting report written, only by Bossut (cf. D'Alembert et al. 1777) and a few helpers. Bossut performed his experiments in the water basin available at the Royal Military School in Paris. The experiments involved as many as twelve different models of ships, and were repeated a total number of 300 times, 200 of these in a practically indefinite space and about 100 in an artificially realized channel in the basin, with depth and width adjustable at will. He also accounted for corrections due to the friction in the pulling apparatus due to a falling weight (of course no other type of motor was available at the time) and due to the part of the model in contact with air. We can say that the theoretically minded Bossut followed a remarkable experimental protocol. Among the conclusions reached we single out: (1) the resistance to a similar cross-sectional surface but at different speeds follows approximately the square law in velocity; (2) the resistance of fully submerged bodies is less than for those of partly submerged ones; For bodies equally submerged in water but with different widths, the resistance varies in the same direction as the surface; (3) the squared sine law (of Newton) is not well verified for small angles of incidence. In fact, Bossut assumed a tentative law involving a sine at a power n, and experimentally found n between 0.66 and 1.79. In the conclusion by Bossut, the resistance perpendicular to a plane surface in an indefinite fluid equals the weight of a column of fluid of which the section is that of the said surface and the length is the height determined by the speed at which the interaction with the fluid (for him a "shock") takes place. Clearly, Bossut is interested by the role played by the friction that the ship suffers along its length. This friction is rather small once the ship has been set in motion. A final remark is that the observed resistance is larger in a channel of restricted width so that he advised channels as deep and wide as possible for easier navigation, but for the limitation in cost. Bossut also advised on the construction of canals. It is said that the results of his experimental work influenced the design of ships by the American inventor Robert Fulton (1765–1815) when the later experimented with steam ships on the Seine river in Paris (Bossut, Carnot and Prony witnessed these well publicized events).

Pierre Du Buat was a French engineer specialist of hydraulics. Educated at the Ecole Royale du Génie de Mézières (Royal School of Engineering in Mézières, the forerunner of the Ecole Polytechnique), he obtained his diploma in 1750. He is the author of a celebrated book on the Principles of Hydraulics (Du Buat 1779; second edition 1786) that will remain a kind of "bible" in the field until the publication of the course of Bélanger (1790–1874) delivered at the Ecole des Ponts et Chaussées (1850s). Du Buat may be considered a follower of d'Alembert and Abbé Bossut but he is more applied than them. Barré de Saint-Venant (1865) expressed his admiration for him (See Fig. 2.13 herein after). He proved to be as good an experimentalist as Bossut. His research on the drag exerted on ships is documented in his book. The major step taken by du Buat, both experimentally and intellectually, is that he wants to account for friction and viscosity. According to his vision, water resistance is due to the friction of the fluid on solid walls. This allowed him to propose a corrected formula for the flux of fluid in terms of: (i) the width of the river, (ii) its depth, and (iii) the slope of the bed of the river. This also included the acceleration of gravity and a proportionality coefficient that depends on the roughness of the banks. This formulation marks a deep progress in sophistication compared to formulas proposed earlier by hydraulicians such as Antoine Chézy (1718–1798) who gave a simple equation for the mean velocity of the flow. With Du Buat, viscosity is viewed only as intervening indirectly in the "delay" caused by the presence of walls. His experiments are conducted in artificially constructed wooden canals or metallic or glass tubes. He introduced the original and fruitful notion of *mean radius* for such conducts. Like some of his predecessors, he interpreted the mechanical interaction with walls through the notion of shocks of fluid particles with the walls especially when considering the sinuosity of the river. As a matter of fact, he envisaged all accidents that a river can meet during its flow all the way to the sea. Concerning the flow around a cylindrical object, he noted the presence of an overshoot of pressure at its head and a lowering of pressure and a suction effect at the rear. He multiplied the number of experiments by varying the various parameters, and he wisely concluded that much more was needed on the experimental side. With such a conclusion we can say that du Buat was remarkably honest and a true experimentalist, indeed.[5]

Two remarks are in order at this stage of our presentation. First, in the line of Newton, many of the discussed scientists view the interaction of water with obstacles as shocks of particles with these obstacles. But, second, the great progress

[5]There is some irony that the etymology of the name "Buat" brings us back to water. Indeed, a possible etymology is that "buat" stands for a water conduit, or a "washing place" ("lavoir" in old French) and that even in modern French "buandrie" means a covered place or special room devoted to washing household linen and clothes.

is that this interpretation results in *friction* and a new era will expand with account of viscosity in one way or another.

We can conclude this section that dealt mostly with the eighteenth century by mentioning that some more practical books had a strong influence on the civil engineering applications of hydraulics. This is the case of the treatise by the engineer Bélidor (1697–1761) where all developments since the ancient Greeks and the Romans are taken into account (Bélidor 1737–1753). This met great success and was translated into foreign languages.

2.7 The Role of Viscosity: Poiseuille, Hagen

At all times scientists and engineers have noticed the negative influence of friction on the motion of objects in contact with other objects (and the associated feeling of expanded heat) or on the flow of fluids in conducts. They had the feeling that something was lost. Was it momentum, energy or else; anyway, notions that were not well defined? But with experiments in hydraulics conducted at the end of the eighteenth century, the actuality of the problem was pregnant and more thought was needed about it. The French military engineer Charles Augustin de Coulomb (1736–1806), a pioneer in geotechnical engineering, may be considered the creator of the *science of friction* in the late eighteenth century. Of course a correct inclusion of the phenomenon of friction in irreversible thermodynamics had to await the second part of the twentieth century to find a satisfactory formulation. But Coulomb, a gifted and careful experimentalist, provided the corner stone on which the theory could be built. This endeavour was undertaken by Coulomb after the publication of a competition prize offered by the Paris Academy of Sciences s in 1781 with subject: "To produce new and large scale experiments applicable to machines used in the Navy, e.g., pulleys, capstans, and inclined planes". Coulomb won the prize. He realized a very thorough series of experiments by varying all kinds of parameters such as the nature of surfaces in contact (wood, metal), with or without lubricant, the roughness of these surfaces, the pressure exerted on these surfaces, their size, the duration of contact, the more or less large speeds of the contacting objects, and the humidity and dryness of the apparatus. Rarely had such complete studies been made before. Among the reached conclusions we note: the proportionality of friction to the pressures; heterogeneous surfaces yield markedly different results; and friction increases with the velocity (with an arithmetic progression in friction for a geometric progression in velocities. In order to pull a weight P on a horizontal plane Coulomb finds that one must expand a force (in our notation)

$$T = A + Pa, \tag{2.4}$$

where A is a small constant that depends on the "coherence" of the surfaces in contact, and a is a friction coefficient that depends on the nature of the contacting surfaces. Considering an inclined plane (as is the case during the launching of a new ship) he finds that the force needed to hold the ship is given by the following formula that generalizes (2.4):

$$T = \frac{A + P(a\cos n + \sin n)}{\cos m + a\sin m}, \tag{2.5}$$

where n is the angle made by the plane with the horizontal, and m is the angle made by the pulling force with the inclined plane. T is minimum for $a = \tan m$. Equations (2.4) and (2.5) contain the main message left by Coulomb.

In his long memoir where he collects his results of research for the period (1838–1844) Poiseuille (1846) says that "I began my investigations because progress in physiology demands knowledge of the laws of motion of the blood…in small-diameter pipes". Poiseuille (1797–1869) received first the standard engineering-science education of the day at the Polytechnique School, but he decided to shift to medical studies to devote his whole life to the transposition of physical laws to the laws of physiology. As a first success in his doctoral thesis in medicine (1828) he proposed the first scientific means of measuring the blood pressure with a manometer using mercury to be called a hemodynamometer.[6] However, his most well known research results are those that pertain to his delicate studies of the flow of liquids in tubes of very-small diameters (i.e., capillaries) and are now summarized in the well-known Poiseuille law. A detailed history of this law and a description of the apparatus used by Poiseuille are to be found in Sutera and Skalak (1993). This law is often called the Hagen-Poiseuille law—especially in Germany—although Hagen's (1839) studies are much less extensive and accurate than those by Poiseuille. There was no contact between these two scientists and they published in different languages, but their researches were indeed contemporary. Hagen (1797–1884) was a Prussian hydraulician with an administrative career as a construction official (cf. Uhlemann 2009).

Although viscosity was duly introduced by Navier in his landmark work on fluid mechanics in 1823, Poiseuille does not use the expression "coefficient of viscosity", but his work indeed relates to the flow of viscous fluids, of which blood truly is a good representative. Using a modern notation that was obviously foreign to Poiseuille, Poiseuille's law for the velocity field of the *laminar* flow of a liquid in a circular tube of radius R is given in cylindrical coordinates by the following expression:

$$v(r, \theta, z) = v(r) = v_{\max}\left(1 - \frac{r^2}{R^2}\right), \tag{2.6}$$

[6]Poiseuille's perduring influence of this technique was such that we are still giving the blood pressure in units of height of mercury.

where the maximal velocity is given in terms of the pressure gradient in the flow direction, the radius R and the viscosity coefficient η by

$$v_{\max} = \frac{R^2}{4\eta}\left|\frac{dp}{dz}\right|. \tag{2.7}$$

The proof of this equation requires assuming that the velocity of the fluid vanishes at the boundary $r = R$, i.e., a no-slip condition. Equation (2.6) is a neat progress compared to what existed before (like in formulas proposed by Chézy—see above—and de Prony), when only a *mean* velocity across the cross section of a tube was introduced. Here the velocity profile is parabolic with a maximum at the centre of the tube. A similar computation can be effected for the flow between two flat fixed walls. But the problem of the flow motion caused by the shearing effect of a moving top plane is another famous problem called the Couette flow.

The original work of Poiseuille did not provide an equation such as (2.6). Like that of Hagen, it purported to obtain the expression of a pressure drop along a pipe as a function of the diameter D or radius R of this pipe of length L. If ΔP is the pressure drop and Q is the volumetric flow rate, then Poiseuille's result in his notation can be written as

$$Q = \frac{K'' D^4 \Delta P}{L} \quad or \quad \frac{\pi R^4}{8\eta}\frac{|\Delta P|}{L}, \tag{2.8}$$

what is compatible with (2.6) and (2.7). Poiseuille's constant K'' equals $\pi/128\,\eta$. The determination of this constant (in fact depending on temperature) by Poiseuille produced the dynamic viscosity of water to a remarkable accuracy[7] (cf. Sutera and Skalak 1993). The great Stokes could have easily deduced the result (2.6)–(2.7) if he had indeed implemented the no-slip condition at the boundary but he still had some doubt about this condition. This result was deduced from the Navier-Stokes equations by Eduard Hagenbach (1833–1910) from Basel in 1860. He is the one who chose to call it the Poiseuille law without reference to Hagen.[8]

[7]According to Bingham (1922), Poiseuille's value is accurate to 0.1 %.

[8]The priority question between Poiseuille and Hagen does not require much development although Hagen in 1869 claimed priority because of a paper he had published in 1839 (see Szabó 1977, pp. 269–273; Hagen 1869). But Poiseuille published continuously his results in notes in the Comptes Rendus of the Paris Academy of Sciences between 1838 and 1844, the final publication of his long memoir of 1846 being due to the delay in publication of contributions by non-members of the Academy (so called "savants étrangers") in a special series of memoirs.

2.8 Summary and Conclusion

More than two thousand years elapsed between the original works of Archimedes and the study of blood flow by Poiseuille. A wealth of practical applications of hydraulics, whether in irrigation or the supply of water to cities, was developed using only an empirical approach. The much awaited breakthrough came with the consideration of true experiments with the Renaissance and the next period of the seventeenth and eighteenth centuries. This is due to a few scientifically-minded engineers who acknowledged the protocol of a good experiment (That is: consider the main facts and design the experiment accordingly, identify the most relevant parameters and vary them as far as possible, do measurements, and propose simple mathematical relations, that would be justified later on by a true analysis when this technique had been constructed). Before analysis was used all reasoning presented by experimentalists was vague and most often erroneous, although some of these gifted people had the right intuition.

According to our survey, three basic notions seem to have emerged from these experimental studies. The first of these notions is *pressure*. The emergence of this quantity was facilitated by the a priori consideration of fluids such as water that are nearly incompressible (in modern terms). This led to the idea of making a fluid flow by means of an applied pressure. But this is related to the second notion that played an essential role, the notion of *gravity*. This is materialized by the fact that pressure in its most primitive form is measured by the height of a fluid in a vertical column opened to air, that is, it weight. That is clearly demonstrated by Torricelli's and Pascal's experiments. This relates to a static view of the phenomenon, called *hydrostatics.* In turn, this played a determining role in the study of flows under gravity and the equilibrium figures of the earth. In any case, all these experiments and considerations remain very *earthy*. It will take some time to arrive at a more abstract vision of pressure as *contiguity of action from all directions*, as proposed by Euler; that is, the first step towards the true notion of *contact action* in the framework of continuum mechanics.

It will also take some time to introduce the third notion of interest, *viscosity*. Of course, experimentalists had noticed the loss in the rate flow of fluids in experiments involving a pressure gradient (via a decrease in altitude along an inclined plane). But they experimented with negligibly viscous fluids (e.g., water) and the accuracy of measurements was insufficient. Kinematics enters the picture with the measurement of velocities. Early experimental research provided some velocity of exit from a container as proportional to the square root of the height of water in the container (Torricelli, Borda, du Buat). But one of the most beautiful formulas of physics was proposed by Daniel Bernoulli with his Eq. (2.1). This indeed relates two of our three ingredients, pressure and gravity, together with a velocity field, but for perfect (nonviscous) fluids also called inviscid fluids. With some advocated enthusiasm, we can declare that this equation is formidable. In particular, a straightforward application of it to the flow around an obstacle such as a wing profile provides a direct elementary proof of the lift effect on the wing, hence the theoretical basis of aeronautical flight. Viscosity was first imagined as friction on the bed of rivers or along the walls of a conduct of limited width. This will destroy the constancy of the right-hand side in Eq. (2.1). More precisely, in problems the effect of viscosity will essentially come from the boundary conditions in solving a true boundary-value problem of hydrodynamics while in experiments fluids such as oil or blood will offer a better object for its study. That is why our second favorite formula here is Poiseuille's Eqs. (2.6)–(2.7). It really relates to *hydrodynamics* although in a laminar regime (cf. Darrigol 2005, Jouguet 1924, Tokaty 1971), that is—to be simple—to slow enough flows with regular stream lines. The case of turbulent flows and high velocities will be studied in depth at the end of the nineteenth century, especially through the experimental (Reynolds' number) and theoretical (Reynolds' stress) efforts of Osborne Reynolds (1868–1912). Also, the flow of fluids through porous media of interest in geophysics and civil engineering will benefit from the works of Henry Darcy (1803–1858) and other people. But this is another story of which we shall examine the most important episodes in another contribution (Portraits of the main scientists involved in this chapter are given in Figs. 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.16, 2.17).

Fig. 2.2 Archimedes: By Domenico Fatti (1620) [Museum of Alte Meister, Dresden, Germany]

Fig. 2.3 Leonardo da vinci, self-portrait

Fig. 2.4 Simon Stevin (1548–1626)

Fig. 2.5 Galileo Galilei (1564–1642)

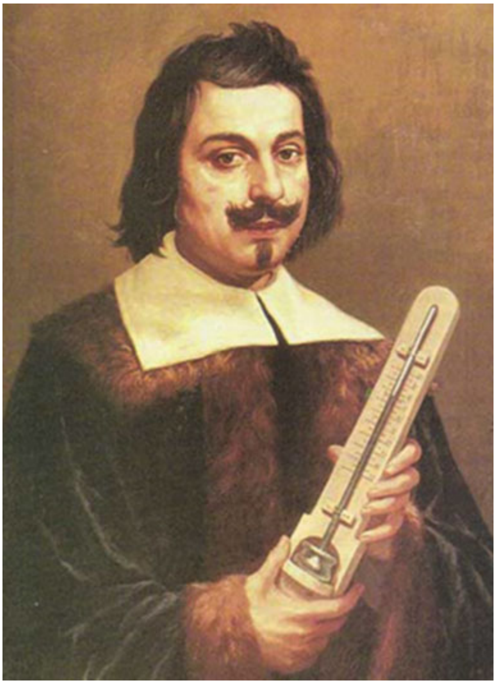


Fig. 2.6 Evangelisto Torricelli ((1608–1647) 62)

Fig. 2.7 Blaise Pascal (1623–1662)

OEUVRES
DE
M^{r}. MARIOTTE,
de l'Académie Royale des Sciences;
DIVISÉES EN DEUX TOMES,
Comprenant tous les Traitez de cet Auteur, tant ceux qui avoient déja paru séparément, que ceux qui n'avoient pas encore été publiez;
Imprimées sur les Exemplaires les plus exacts & les plus complets;
Revûs & corrigés de nouveau.
TOME PREMIER.
A LEIDE,
Chez PIERRE VANDER Aa,
Marchand Libraire, Imprimeur de l'Université & de la Ville.
M D CC XVII.

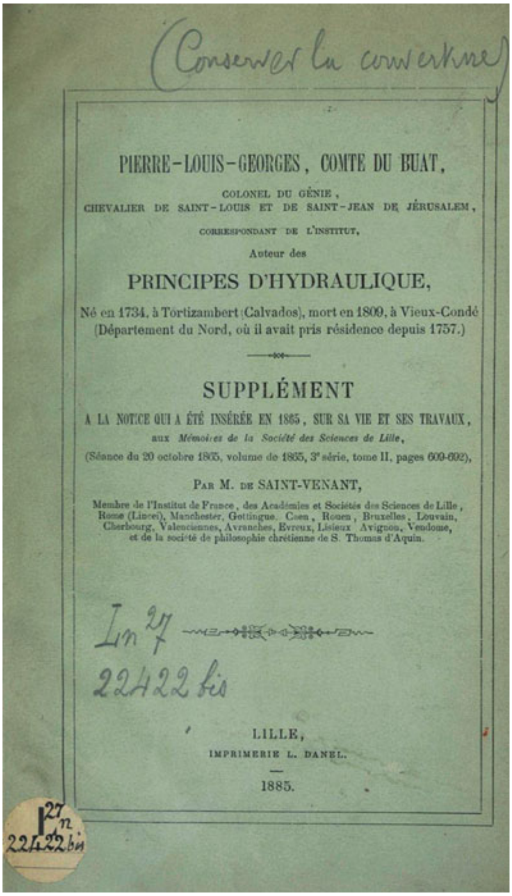
Fig. 2.8 Works by Edmé Mariotte (1717)

Fig. 2.9 Edmé Mariotte (1620–1684)

Fig. 2.10 Alexis Clairaut (1713–1765)

Fig. 2.11 Chevalier de Borda (1733–1799)

Fig. 2.12 Abbé Charles Bossut (1730–1814)



(Conserver la couverture)

PIERRE-LOUIS-GEORGES, COMTE DU BUAT,

COLONEL DU GÉNIE,
CHEVALIER DE SAINT-LOUIS ET DE SAINT-JEAN DE JÉRUSALEM,
CORRESPONDANT DE L'INSTITUT,
Auteur des

PRINCIPES D'HYDRAULIQUE,

Né en 1734, à Tortizambert (Calvados), mort en 1809, à Vieux-Condé
(Département du Nord, où il avait pris résidence depuis 1757.)

SUPPLÉMENT

A LA NOTICE QUI A ÉTÉ INSÉRÉE EN 1865, SUR SA VIE ET SES TRAVAUX,
aux *Mémoires de la Société des Sciences de Lille*,
(Séance du 20 octobre 1865, volume de 1865, 3e série, tome II, pages 609-692),

PAR M. DE SAINT-VENANT,

Membre de l'Institut de France, des Académies et Sociétés des Sciences de Lille,
Rome (Lincei), Manchester, Gottingue. Caen, Rouen, Bruxelles, Louvain,
Cherbourg, Valenciennes, Avranches, Evreux, Lisieux Avignon, Vendome,
et de la société de philosophie chrétienne de S. Thomas d'Aquin.

LILLE,
IMPRIMERIE L. DANEL.

1885.

Fig. 2.13 On Du Buat (1734–1809) by Barré de Saint-Venant (1865, 1885)

Fig. 2.14 Poiseuille (1797–1869)

Fig. 2.15 Julius Weisbach (1806–1871)

Fig. 2.16 Gotthilf Hagen (1797–1884)

Fig. 2.17 Gotthilf Hagen (1797–1884)

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