

Preface

It is a well-known fact that there exist functions that have nowhere first order derivative, but possess continuous Riemann-Liouville and Caputo fractional derivatives of all orders less than one, e.g., the famous Weierstrass function, see Chap. 1, [9, 18], p. 50.

This striking phenomenon motivated the authors to study Newton-like and other similar numerical methods, which involve fractional derivatives and fractional integral operators, for the first time studied in the literature. All for the purpose to solve numerically equations whose associated functions can be also non-differentiable in the ordinary sense.

That is among others extending the classical Newton method theory which requires usual differentiability of function.

In this monograph we present the complete recent work of the past three years of the authors on Numerical Analysis and Fractional Calculus. It is the natural outgrowth of their related publications. Chapters are self-contained and can be read independently and several advanced courses can be taught out of this book. An extensive list of references is given per chapter. The topics covered are from A to Z of this research area, all studied for the first time by the authors.

The list of presented topics of our related studies follows.

Newton-like methods on generalized Banach spaces and applications in fractional calculus.

Semilocal convergence of Newton-like methods under general conditions with applications in fractional calculus.

On the convergence of iterative methods with applications in generalized fractional calculus.

A fixed point technique for some iterative algorithm with applications to generalized right fractional calculus.

Approximating fixed points with applications in k -fractional calculus.

Generalized g -fractional calculus and iterative methods.

A unified convergence analysis for a certain family of iterative algorithms with applications to fractional calculus.

A convergence analysis for extended iterative algorithms with applications to fractional and vector calculus.

A convergence analysis for a certain family of extended iterative methods with applications to modified fractional calculus.

A convergence analysis for secant-like methods with applications to modified fractional calculus.

Semilocal convergence of secant-type methods with applications to modified g -fractional calculus.

On the convergence of secant-like algorithms with applications to generalized fractional calculus.

Generalized g -fractional calculus of Canavati-type and secant-like methods.

A convergence analysis for some iterative algorithms with applications to fractional calculus.

Convergence for iterative methods on Banach spaces of a convergence structure with applications to fractional calculus.

Local convergence analysis of inexact Gauss–Newton method for singular systems of equations under majorant and center-majorant condition.

The asymptotic mesh independence principle of Newton’s method under weaker conditions.

Ball convergence of a sixth order iterative method with one parameter for solving equations under weak conditions.

Improved semilocal convergence of Broyden’s method with regularly continuous divided differences.

Left general fractional monotone approximation theory.

Right general fractional monotone approximation.

Univariate left general high order fractional monotone approximation.

Univariate right general high order fractional monotone approximation theory.

Advanced fractional Taylor’s formulae.

Generalized Canavati type fractional Taylor’s formulae.

The last two topics were developed to be used in several chapters of this monograph.

The book’s results are expected to find applications in many areas of applied mathematics, stochastics, computer science, and engineering. As such this monograph is suitable for researchers, graduate students, and seminars in the above subjects, also to be in all science and engineering libraries.

The preparation of the book took place during 2014–2015 in Memphis, Tennessee and Lawton, Oklahoma, USA.

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