

Preface

This book is the offspring of the summer school “Macroscopic and Large Scale Phenomena: Coarse Graining, Mean Field Limits and Ergodicity.” The school was held in 2012 at the University of Twente, the Netherlands, under the aegis of the national cluster *Nonlinear Dynamics in Natural Systems* (NDNS+). We assumed the initiative to organize that school to bring certain aspects of particle systems theory to the attention of junior applied mathematicians at large and to applied analysts in particular. To that effect, the main focus was on the modeling, multiscale analysis, and continuum limits of particle systems.

This emphasis is reflected in the book at hand, which collects in print an extension of the material presented in that summer school. Each of the four chapters here is based on a set of lectures delivered at the school, yet all authors have expanded and refined their contributions. The first three of these discuss mean field limits and (evolutionary) Γ -convergence as methods to derive macroscopic model equations from microscopic descriptions. The chapter concluding the volume offers a more abstract perspective, discussing the statistical properties of large-scale dynamics from the standpoint of ergodic and dynamical systems theory. Each chapter concerns one particular aspect of the overall theme and reflects the author’s specific viewpoint and focus. We hope that this book can continue to stimulate interaction between these somewhat separate fields, in the spirit of the school that generated it.

François Golse delivers a chapter on the dynamics of large particle systems in the mean field limit. In doing so, he deals with a central issue in statistical mechanics: the derivation of reduced dynamical descriptions of large systems composed of identical microscopic constituents. In such systems, the effect of each individual particle is weak, but the collective action of the particle ensemble generates a nontrivial mean field acting on each and every particle. An early example of such a reduced description is the kinetic theory of gases, which is the mean field limit of molecular dynamics. According to that theory, the state of a system of a large number of identical gas molecules is reflected in the statistical behavior of one typical particle in the system. Golse surveys the most significant tools and

methods employed to date to establish mean field limits with mathematical rigor. He illustrates their application actively by means of a variety of examples rooted in physical reality, such as regularized variants of the Vlasov–Poisson or Vlasov–Maxwell systems from plasma physics.

Lucia Scardia focuses on the derivation of continuum limits of discrete models using variational methods. The specific focus is on Γ -convergence, a more recent method that has proved very powerful in tackling problems in material science. Fracture mechanics, spin systems, magneto-mechanics, and dislocations are counted among the many successes of this approach. A common feature of all these problems is that good models are known at the discrete level of atoms and dislocations (microscale), but they are missing at the continuum level (macroscale) which is relevant for engineering applications. The notion of Γ -convergence furnishes a mathematically rigorous way to bridge those scales effectively. Scardia starts with an introduction to this concept, illustrated by concrete examples of discrete-to-continuum upscalings for simple systems. The latter part of the chapter deals with a more advanced application in dislocation theory.

Alexander Mielke's contribution focuses on the multiscale modeling and rigorous analysis of generalized gradient systems. Specifically, he examines the new concept of evolutionary Γ -convergence for systems driven by a dissipation mechanism for a functional, such as an energy or entropy. The focus in this work is on one-parameter families of such vector fields, where the (small) parameter quantifies the multiscale structure of the system in question. In this setting, the central challenge is to describe the limiting behavior of solutions to the detailed model in terms of solutions to a simplified, appropriately chosen, macroscopic one. In this context, the static concept of Γ -convergence provides a framework in which to understand and perform such upscalings. The presentation connects to physical and engineering reality by numerous evocative examples relating, for instance, to periodic homogenization or to the passage from viscous to dry friction.

Martin Göll and Evgeny Verbitskiy conclude this volume with a review of recent advances in a modern branch of ergodic theory. Statistically motivated approaches lie at the heart of ergodic theory and the study of complex dynamical systems. Their numerous offshoots have embraced a diverse array of mathematical fields including number theory, algebra and geometry. The relevant portion of the summer school in Twente discussed fundamental notions of ergodic theory and demonstrated applications of ergodicity in nonlinear dynamics. The chapter in this volume goes much further and reviews recent developments in the study of homoclinic points for certain classes of discrete dynamical systems. From our point of view, the most striking feature is that these relate certain particle systems to algebraic dynamical systems via ergodic properties of, e.g., lattices configurations. The authors motivate the abstract notions and theory by numerous examples, in particular, spatial dynamical systems. Having defined homoclinic points arising through principal algebraic actions, they review existing results and demonstrate the use of the theory in computing the entropy and in studying probabilistic properties of these discrete systems.

We express our gratitude to the Netherlands Organisation for Scientific Research (NWO), the Dutch Applied Mathematics Institute (3TU.AMI), and the Dutch Research School in Mathematics (WONDER) for supporting the summer school financially. We also thank the authors, who kindly agreed to contribute to this volume, as well as the school participants who proved eager to learn about these different approaches. Last but not least, we thank the series editors and publishers for realizing this book.

Eindhoven
Bremen
Twente
June 2015

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Macroscopic and Large Scale Phenomena: Coarse
Graining, Mean Field Limits and Ergodicity

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2016, XIII, 295 p. 15 illus., 13 illus. in color., Softcover

ISBN: 978-3-319-26882-8