

Hidden Attractors in Fundamental Problems and Engineering Models: A Short Survey

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Abstract Recently a concept of self-excited and hidden attractors was suggested: an attractor is called a *self-excited attractor* if its basin of attraction overlaps with neighborhood of an equilibrium, otherwise it is called a *hidden attractor*. For example, hidden attractors are attractors in systems with no equilibria or with only one stable equilibrium (a special case of multistability and coexistence of attractors). While coexisting self-excited attractors can be found using the standard computational procedure, there is no standard way of predicting the existence or coexistence of hidden attractors in a system. In this plenary lecture the concept of self-excited and hidden attractors is discussed, and various corresponding examples of self-excited and hidden attractors are considered. The material is mostly based on surveys [1–4].

Keywords Hidden attractor • Self-excited attractor • Hidden oscillation • System with no equilibria • System without equilibria • Multistability • Coexistence of attractors • 16th Hilbert problem • Aizerman and Kalman conjectures • Aircraft control system • Phase-locked loop • Chua circuit

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1 Analytical-Numerical Study of Oscillations

An oscillation dynamical system can generally be easily numerically localized if the initial data from its open neighborhood in the phase space (with the exception of a minor set of points) lead to a long-term behavior that approaches the oscillation. Such an oscillation (or a set of oscillations) is called an attractor, and its attracting set is called the basin of attraction (i.e., a set of initial data for which the trajectories tend to the attractor).

When the theories of dynamical systems, oscillations, and chaos were first developed researchers mainly focused on analyzing equilibria stability, which can be easily done numerically or analytically, and on the birth of periodic or chaotic attractors from unstable equilibria. The structures of many physical dynamical systems are such that it is almost obvious that attractors exist because the trajectories can not tend to infinity and the oscillations are excited by an unstable equilibrium (see, e.g., the Rayleigh [5], Duffing [6], van der Pol [7], Tricomi [8], Belusov-Zhabotinsky [9], and Lorenz [10] systems). This meant that scientists of that time could compute such attractors by constructing a solution using initial data from a small neighborhood of the equilibrium, observing how it is attracted and, thus, visualizes the attractor. In this *standard computational procedure*, computational methods and the engineering notion of a *transient process* were combined to study oscillations.

2 Self-excited and Hidden Attractors

From a computational perspective, it is natural to suggest the following classification of attractors, which is based on the connection of their basins of attraction with equilibria in the phase space:

Definition 1 [2, 4, 11, 12] An attractor is called a self-excited attractor if its basin of attraction intersects with any open neighborhood of a stationary state (an equilibrium), otherwise it is called a hidden attractor.

The first well-known example of a visualization of chaotic attractor in a dynamical system from the work of Lorenz [10] corresponds to the excitation of chaotic attractor from unstable equilibria. For classical parameters the Lorenz attractor is self-excited with respect to all three equilibria and could have been found using the standard computational procedure (see Fig. 1). Note that the chaotic attractor in the Lorenz system with other parameters may be self-excited with respect to zero unstable equilibrium only, and the possible existence in the Lorenz system of a hidden chaotic attractor is an open problem.

At the same time in the generalized Lorenz system $\dot{x} = -\sigma(x - y) - ayz$, $\dot{y} = rx - y - xz$, $\dot{z} = -bz + xy$ hidden chaotic attractors can be found [4, 13, 14] (see Fig. 2). For negative $a < 0$ the system corresponds to the Rabinovich system, which describes the interaction of plasma waves and was considered in 1978 [15, 16]; for

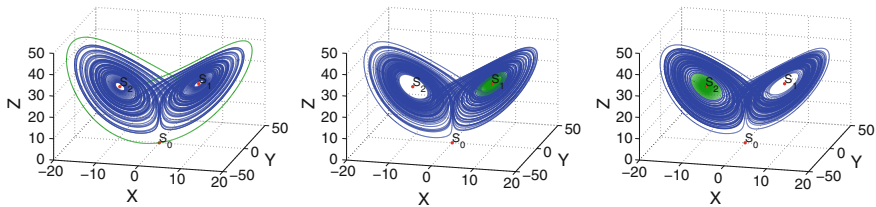


Fig. 1 Numerical visualization of the classical self-excited chaotic attractor in the Lorenz system $\dot{x} = 10(y - x)$, $\dot{y} = 28x - y - xz$, $\dot{z} = -8/3z + xy$. The attractor is self-excited respect to all three equilibria: it can be visualized by trajectories that start in small neighborhoods of any of unstable equilibria $S_{0,1,2}$. Here the separation of trajectory into transition process (green) and approximation of attractor (blue) is rough

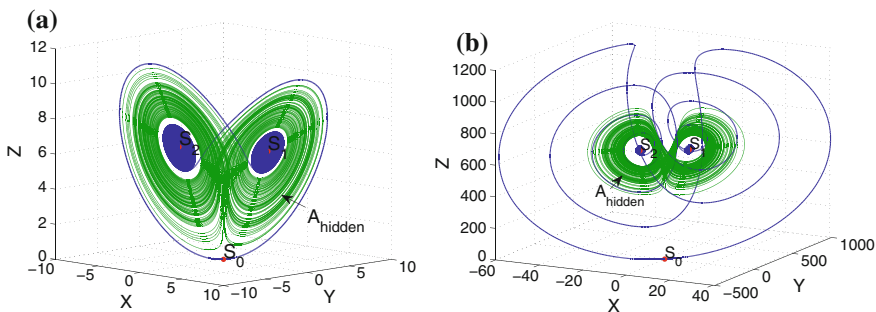


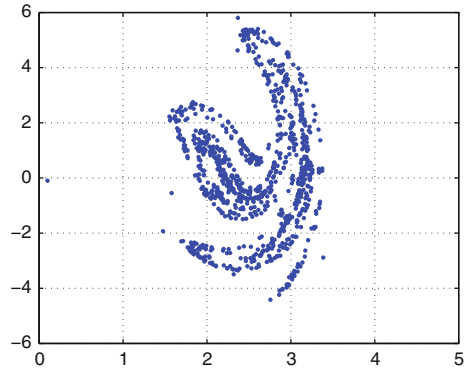
Fig. 2 Numerical visualization of hidden attractor (green trajectory) in the generalized Lorenz system. Outgoing separatrices of the zero saddle equilibrium are attracted to the stable equilibria $S_{1,2}$ (blue trajectories). **a** $r = 6.8$, $a = -0.5$, $\sigma = ra$, $b = 1$, **b** $r = 700$, $a = 0.0052$, $\sigma = ra$, $b = 1$

positive $a > 0$ it corresponds to the Glukhovsky-Dolghansky system, which describes convective fluid motion and was considered in 1980 [17]; also it describes a rigid body rotation in a resisting medium and the forced motion of a gyrostat (see [18]).

The basin of attraction for a hidden attractor is not connected with any equilibrium. For example, hidden attractors are attractors in systems with no equilibria or with only one stable equilibrium (a special case of the multistability and coexistence of attractors). Note that multistability can be undesired in various practical applications. At the same time the coexisting self-excited attractors in multistable systems (see, e.g. various examples of multistable engineering systems in famous book [19], and recent physical examples in [20]) can be found using the standard computational procedure, whereas there is no standard way of predicting the existence or coexistence of hidden attractors in a system.

For nonautonomous systems, depending on the physical problem statement, the notion of self-excited and hidden attractors can be introduced with respect to the stationary states of the system at the fixed initial time or the corresponding system

Fig. 3 An attractor in forced Duffing oscillator $\ddot{x} + 0.05\dot{x} + x^3 = 7.5 \cos(t)$ is self-excited with respect to the zero equilibrium of unperturbed system (without $7.5 \cos(t)$)



without time-varying excitations. For example, one of the classical examples of self-excited chaotic attractors was numerically found by Ueda in 1961 [21] in a forced Duffing system $\ddot{x} + 0.05\dot{x} + x^3 = 7.5 \cos(t)$. To construct a self-excited chaotic attractor in this system (Fig. 3) it was used a transient process from the zero equilibrium of the unperturbed autonomous system (i.e., without $\cos(t)$) to an attractor in the forced system. If the discrete dynamics of system are considered on a Poincare section, then we can also use stationary or periodic points on the section that corresponds to a periodic orbit of the system (the consideration of periodic orbits is also natural for discrete systems). Note that if the attracting domain is the whole state space, then the attractor can be visualized by any trajectory and the only difference between computations is the time of the transient process.

One of the first well-known problems of analyzing hidden periodic oscillations arose in connection with the second part of Hilbert's 16th problem (1900) [22] on the number and mutual disposition of limit cycles in two-dimensional polynomial systems (see, e.g. recent results [2, 23–25] on visualization of nested limit cycles in quadratic systems: $\dot{x} = \alpha_1 x + \beta_1 y + a_1 x^2 + b_1 xy + d_1 y^2$, $\dot{y} = \alpha_2 x + \beta_2 y + a_2 x^2 + b_2 xy + d_2 y^2$).

Later, in the 1950s–1960s, the study of the well-known Aizerman's and Kalman's conjectures on absolute stability led to the discovery of the possible coexistence of a hidden periodic oscillation and a unique stable stationary point in automatic control systems. In 1957 Kalman stated the following [26]: “If $f(e)$ in Fig. 1 [see Fig. 4] is replaced by constants K corresponding to all possible values of $f'(e)$, and it is found that the closed-loop system is stable for all such K , then it is intuitively clear that the system must be monostable; i.e., all transient solutions will

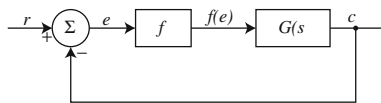


Fig. 4 Nonlinear control system. $G(s)$ is a linear transfer function, $f(e)$ is a single-valued, continuous, and differentiable [26]

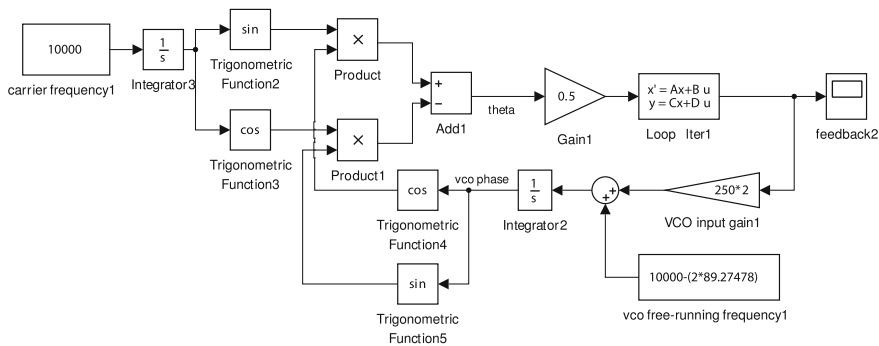


Fig. 5 Model of two-phase PLL with lead-lag filter in MATLAB Simulink. Lead-lag loop filter with the transfer function $H(s) = \frac{1+s\tau_2}{1+s(\tau_1+\tau_2)}$, $\tau_1 = 0.0448$, $\tau_2 = 0.0185$ and the corresponding parameters $A = -\frac{1}{\tau_1+\tau_2}$, $B = 1 - \frac{\tau_2}{\tau_1+\tau_2}$, $C = \frac{1}{\tau_1+\tau_2}$, $D = \frac{\tau_2}{\tau_1+\tau_2}$

converge to a unique, stable critical point.” Kalman’s conjecture is a strengthening of Aizerman’s conjecture [27], which considers nonlinearities belonging to the sector of linear stability. Note that these conjectures are valid from the standpoint of simplified analysis such as the linearization, harmonic balance, and describing function methods (DFM), what explains why these conjectures were put forward. Nowadays various counterexamples to these conjectures (nonlinear systems where the only equilibrium, which is stable, coexists with a hidden periodic oscillation) are known (see [28–35]; the corresponding discrete examples are considered in [36, 37]).

Similar situation with linear stability and hidden oscillations occur in the analysis of aircrafts and launchers control systems with saturation [38, 39]. In [40] the crash of aircraft YF-22 Boeing in April 1992,¹ caused by the difficulties of rigorous analysis and design of nonlinear control systems with saturation, is discussed and the conclusion is made that “*since stability in simulations does not imply stability of the physical control system (an example is the crash of the YF22), stronger theoretical understanding is required*”.

Corresponding limitations, caused by hidden oscillations, appear in simulation of various phase-locked loop (PLL) based systems [2, 41–48]. PLL was designed to synchronize the phases of local oscillator and reference oscillator signals. Next example shows that the use of default simulation parameters in MATLAB Simulink for the study of two-phase PLL in the presence of hidden oscillation (see Fig. 5) can lead to the conclusions concerning the stability of the loop and the pull-in (or capture) range.² In Fig. 6 the model in Fig. 5 simulated with relative tolerance set to “1e-3” or smaller does not acquire lock (black color), but the model with default parameters (a relative tolerance set to “auto”) acquires lock (red color). The same

¹<http://www.youtube.com/watch?v=M6sy-fxIhF0>.

²See discussion of rigorous definitions in [48, 49].

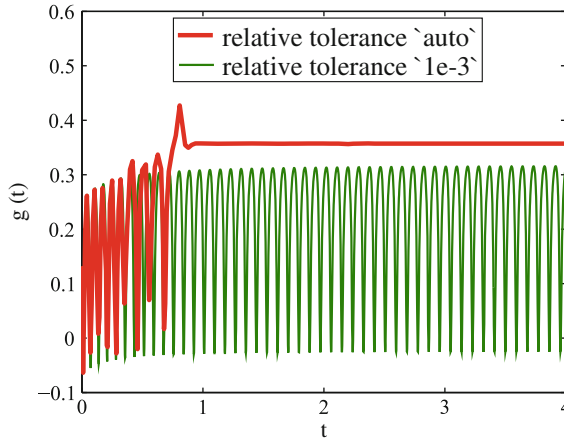


Fig. 6 Simulation of two-phase PLL. Loop filter output $g(t)$ for the initial data $x_0 = 0.1318$ obtained for default “auto” relative tolerance (*red*)—acquires lock, relative tolerance set to “1e-3”(*green*)—does not acquire lock

problems are also observed in SIMetrics SPICE model [41, 45, 46]. From a mathematical point of view, the above case corresponds to the existence of semi-stable periodic trajectory or co-existence of unstable and stable periodic trajectories (here the stable one is a hidden oscillation) [2, 41, 50, 51]. Therefore, if the gap between stable and unstable periodic trajectories is smaller than the discretization step, the numerical procedure may slip through the stable trajectory [45, 47].

Attractor in the systems *without equilibria* are the hidden attractors, according to the above definition. Systems without equilibria and with hidden oscillations appear naturally in the study of various electromechanical models with rotation and electrical circuits with cylindrical phase space. One of the first examples is from paper [52], published in 1902, in which Sommerfeld analyzed the vibrations caused by a motor driving an unbalanced weight and discovered the so-called Sommerfeld effect (see, e.g., [53, 54]). Another well-known chaotic system with no equilibrium points is the Nosè–Hoover oscillator [55–57]. An example of hidden chaotic attractor in electromechanical model with no equilibria was reported in a power system in 2001 [58]. Recent examples of hidden attractors in the systems without equilibria can be found, e.g. in [59–67].

After the concepts of hidden chaotic attractors was introduced first in connection with discovery of the first hidden Chua attractors (see Fig. 7) [11, 12, 68–74], the hidden chaotic attractors have received much attention. Recent examples of hidden attractors can be found in [59–61, 63, 75–112].

See also *The European Physical Journal Special Topics: Multistability: Uncovering Hidden Attractors*, 2015 (see [113–124]).

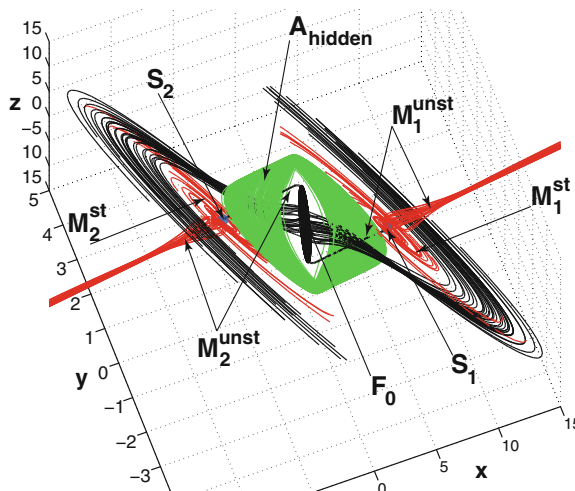


Fig. 7 Hidden chaotic attractor (green) in Chua circuit: $\dot{x} = \alpha(y - x - m_1x - \psi(x))$, $\dot{y} = x - y + z$, $\dot{z} = -(\beta y + \gamma z)$, $\psi(x) = (m_0 - m_1)\text{sat}(x)$. Locally the stable zero equilibrium F_0 attracts trajectories (black) from stable manifolds $M_{1,2}^{\text{st}}$ of two saddle points $S_{1,2}$; trajectories (red) from the unstable manifolds $M_{1,2}^{\text{unst}}$ tend to infinity; $\alpha = 8.4562$, $\beta = 12.0732$, $\gamma = 0.0052$, $m_0 = -0.1768$, $m_1 = -1.1468$

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